Generalized Fermat's Last Theorem(1) $R^n = y_1^3 + y_2^3$

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Abstract: In this paper we prove $R^2 = y_1^3 + y_2^3$ has infinitely many nonzero integer solutions. We prove $R^n = y_1^3 + y_2^3 (n > 2)$ has no nonzero integer solutions.

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In this paper we prove $R^2 = y_1^3 + y_2^3$ has infinitely many nonzero integer solutions. We prove $R^n = y_1^3 + y_2^3 (n > 2)$ has no nonzero integer solutions.

We define the supercomplex number [1,2,3]

$$W = x_1 + x_2 J + x_3 J^2 \qquad (1)$$

where J denotes a $3-th$ root of unity, $J^3 = 1$,

Then from (1)

$$W^n = (x_1 + x_2 J + x_3 J^2)^n = y_1 + y_2 J + y_3 J^2 \qquad (2)$$

Then from (2) we have the modulus of supercomplex number

$$R^n = |x_i|^n = |y_i| \qquad (3)$$

where

$$R^n = x_1^3 + x_2^3 + x_3^3 - 3x_1x_2x_3, \qquad (4)$$

$$|y_i| = y_1^3 + y_2^3 + y_3^3 - 3y_1y_2y_3, \qquad (5)$$

We prove that (3) has infinitely many nonzero integer solutions.

We define the stable group [1,4]

$$G = \{g_2, g_3\} \qquad (6)$$

where

$$g_2 = \begin{pmatrix} 1\,2\,3 \\ 1\,2\,3 \end{pmatrix}, g_3 = \begin{pmatrix} 1\,2\,3 \\ 1\,2\,3 \end{pmatrix} \text{g.}$$

Theorem 1. Suppose $n = 2$ and $y_3 = 0$. Then from (3) and (5)

$$R^2 = y_1^3 + y_2^3 \qquad (7)$$

when $n = 2$ from (2)

$$y_1 = x_1^2 + 2x_2x_3,\ y_2 = x_3^2 + 2x_1x_2,\ y_3 = x_2^2 + 2x_1x_3 \tag{8}$$

which are the homogeneous and irreducible polynomials.

$$g_3:\quad x_1 \to x_1,\quad x_2 \to x_3,\quad x_3 \to x_2$$

$$g_3:\quad y_1 \to y_1,\quad y_2 \to y_3,\quad y_3 \to y_2 \tag{9}$$

$$g_3y_2 = y_3 = 0 \tag{10}$$

If $y_3 = 0$ has nonzero integer solutions, then $y_2 = 0$ also has nonzero integer solutions, and vice versa.

Put $x_1 = P^2$, $x_2 = 2P, x_3 = -2, y_3 = 0$, where P is an odd number.

From (7) and (9)

$$(g_3R)^2 = (g_3y_1)^3 + (g_3y_2)^3, \tag{11}$$

$$R^2 = y_1^3 + y_3^3 \tag{12}$$

Put $x_1 = P^2$, $x_2 = -2, x_3 = 2P, y_2 = 0$, where P is an .odd number.

Suppose $y_1 = 0$ and $n = 2$. From (3) and (5)

$$R^2 = y_2^3 + y_3^3 \tag{13}$$

Put $x_1 = 2P$, $x_2 = -2, x_3 = P^2, y_1 = 0$, where P is an odd number. (7), (11) and (12) are the same equation. We prove that every

$$y_1 = 0,\quad y_2 = 0,\quad y_3 = 0 \tag{14}$$

has infinitely many nonzero integer solutions.

Hence (7), (12) and (13) have infinitely many nonzero integer solutions.

Theorem 2. Suppose $n = 3$ and $y_3 = 0$. Then from (3) and (5)

$$R_1^3 = y_1^3 + y_2^3 \tag{15}$$

when $n = 3$ from (2)

$$y_1 = x_1^3 + x_2^3 + x_3^3 - 3x_1x_2x_3,\ y_2 = 3(x_1x_3^2 + x_2x_1^2 + x_3x_2^2),\ y_3 = 3(x_1x_2^2 + x_2x_3^2 + x_3x_1^2) \tag{16}$$

which are the homogeneous and irreducible polynomials.

From (6)

$$g_3:\quad x_1 \to x_1,\quad x_2 \to x_3,\quad x_3 \to x_2$$

$$g_3:\quad y_1 \to y_1,\quad y_2 \to y_3,\quad y_3 \to y_2 \tag{17}$$

$$g_3y_2 = y_3 = 0 \tag{18}$$

If $y_3 = 0$ has no nonzero integer solutions then $y_2 = 0$ has no nonzero integer solutions, and vice versa [1,5]

Euler prove that (15) has no nonzero integer solutions. Hence y_2 and $y_3 = 0$ have no nonzero integer solutions.

From (15) and (17) we have

$$(g_3R)^2 = (g_3y_1)^3 + (g_3y_2)^3, \tag{19}$$

$$R^3 = y_1^3 + y_3^3 \tag{20}$$

From (18) $y_2 = 0$ has no nonzero integer solutions, Hence (20) has no nonzero integer solutions, Euler prove that (20) has no nonzero integer solutions, hence y_3 and $y_2=0$ have no nonzero integer solutions.

Suppose $n=3$ and $y_1=0$ from (3) and (5)

$$R^3=y_2^3+y_3^3 \tag{21}$$

Euler prove (21) has no nonzero integer solutions, hence $y_1=0$ also has no nonzero integer solutions.
We prove that every

$$y_1=0,\quad y_2=0,\quad y_3=0 \tag{22}$$

has no nonzero integer solutions. Hence we prove that (15), (20) and (21) are the same equation and have no nonzero integer solutions.

Theorem 3. when $n>3$, y_1, y_2 and y_3 are homogenous and irreducible polynomials. Suppose $y_3=0$. From (3) and (5)

$$R_1^n=y_1^3+y_2^3 \tag{23}$$

From (18) $y_3=0$ has no nonzero integer solutions. Hence (23) has no nonzero integer solution.
From (17) and (23) we have

$$(g_3R)^n=(g_3y_1)^3+(g_3y_2)^3, \tag{24}$$

$$R^n=y_1^3+y_3^3 \tag{25}$$

From (18) $y_2=0$ has no nonzero integer solutions, Hence (25) has no nonzero integer solutions.

Suppose $n>3$ and $y_1=0$. From (3) and (5)

$$R_1^n=y_2^3+y_3^3 \tag{26}$$

We prove that every

$$y_1=0,\quad y_2=0 \text{ and } y_3=0 \tag{27}$$

has no nonzero integer solutions.

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