Introduction of a Chaotic Dough Mixer, Part B: Chaotic Behavior and Mixing Performance

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Abstract: The aim of this work is to investigate the role of chaotic advection in the mixing performance of the proposed novel mixer in the recent study of Hosseinalipour et.al. (2013). The computed flow domain calculated is used to find material point trajectories needed to calculate mixing measures in the novel dough mixer based on chaos theory. Two characteristics of a Lagrangian chaotic system (strong stretching and folding of material elements and sensitivity to initial conditions) horseshoe maps and also Poincare sections were visualized. Lyapunov exponents which quantify the exponential divergence of initially close state-space trajectories and estimate the amount of chaos presence in a system were also calculated. The results indicated that the flow filed was a combination of coexistence of both the chaotic and non-chaotic zones, with high and poor mixing performance respectively.

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1. Introduction

Unfortunately, due to the high viscosity difficulties usually encountered in food processing industries, rapid mixing produced by turbulence is not available. On the other hand, fluids with long molecular chains, such as synthetic polymers, food and various types of pharmaceutical products, can be damaged by high shear stresses. In these systems the flow regime is often laminar which is considered as a limitation due to the poor mixing performance produced mainly by molecular diffusion. Thus, some other mechanisms must be exploited to enhance mixing.

In the last three decades, the concept of 'chaos theory' has contributed to solve practical problems of mixing highly viscous fluids at low Reynolds numbers. In this way, the advantages of chaotic advection, which provides a natural way of increasing the mixing efficiency of flow, is clearly undeniable. Chaotic advection, or in more precise terms Lagrangian chaos (Peerhossainiet al., 2001), is a flow regime in which chaos is generated in the physical space.

In the context of two dimensional flow fields, it is known that a steady system is called nonchaotic since the relevant velocity field is integrable (Aref, 1984). This means that, for passive injected tracers there are closed streamlines which force tracers to travel through a limited portion of the whole flow domain. Also the rate of stretching in 2D steady flow is linear in time and the resultant mixing is inefficient. A two dimensional flow field has to be made time periodic for chaotic advection to occur and consequently efficient mixing caused by an exponential rate of stretching. However, a three-dimensional steady flow can produce chaotic trajectories. This is due to the nonintegrable nature of their velocity fields. From the Eulerian point of view the flow can be laminar and time-independent; however, the fluid particles follow irregular trajectories different from the Eulerian streamlines, and therefore overcome the virtual barriers which constitute the streamlines for mixing (Peerhossainiet al., 2005).

In this way, an increasing interest in devising new mixers to induce chaotic advection has arisen. In fact, much of the understanding of chaotic mixing can be exploited in the design of novel mixing devices. The literature describes a number of devices designed based on chaos theory (Muzzio et al., 2002; Aref et al., 2000; Hwu, 2008; Chang et al., 2004). These devices fall into one of two categories:

1) Active techniques in which chaotic advection is produced by means of irregularity in the motion pattern of the stirrer or moving part of the mixer, e.g. time-periodic rotating eccentric cylinders (Ottino et al., 1993; Ottino et al., 1990; Kumar et al., 1990; Ottino et al., 1994; Muzzio et al., 1997), blinking vortex (Peerhossainiet al., 2005).

2) Passive techniques in which chaotic advection is produced by means of geometrical perturbations, usually time- periodic configurations of mixers, e.g. partitioned pipe mixer (Mizuno et al., 2002; Yamagishi et al., 2007), twisted pipes (Castelain et al., 2000; Cocero, 1993.).

It should be noted that there are other possible classifications, and that both techniques can be jointly applied in a mixer. In this study, the active technique was employed due to its simplicity in operation, simulation and application.

It should be noted that there are other possible classifications, and that both techniques can be jointly applied in a mixer. In this study, the active technique was employed due to its simplicity in operation, simulation and application.

According to the authors' knowledge, there is no work in the literature that has introduced and analyzed a chaotic mixer for dough preparation. Therefore, taking the advantage of high performance mixing in chaotic mixers (for highly viscous fluids) has been the main aim of this study to propose a new chaotic dough mixer. The new model, geometrical and mathematical concepts have been discussed in addition in Hosseinalipour et al. (2013). This paper, thus, focuses on the performance of the proposed dough mixer by employing Lagrangian particle tracing and Lyapunov exponents which reveal clear evidence for the presence of chaotic regimes in the novel dough mixer. In order to achieve a precise assessment of the effects of chaotic advection on mixing efficiency, recording of stretching measurements of fluid elements, were carried out. To gain a more qualitatively fundamental understanding of the process Poincare sections were also used.

2. Proposed model

The proposed mixer includes an eccentric helical rotor rotates around the stator at a constant speed (Figures 1 and 2). In contrast with other eccentric cylinders in which the choatic trajectories can be produced only if either one or both cylinders' angular velocities depend on time, in the proposed mixer, the flow domain will contains the chaotic advection even at constant angular velocity. More details can be found in Hosseinalipour et.al. (2013).

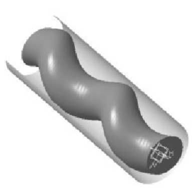


Figure 1. Dough mixer configuration.

3. Mathematical algorithm

Since the rotor has a helical shape and rotation is done around the stator axis, the mesh configuration will change at each time-step. In order to prevent difficulties due to the dynamic mesh, the spatially periodic flow was approximated as piecewise steady flow using Creeping Flow concepts (Hosseinalipour et al., 2013). In fact, the strategy utilized here is to divide the whole transient flow field into numerous separate steady flow fields so that each of them would be compatible with any degree of rotation around the stator axis and consequently, the continuous rotational motion can be calculated as a summation of distinct flows through some discrete degrees. In this regard, equation (1) will provide the adaptable time-step according to which, each flow field should be modeled. This time-step is compatible with different infinitesimal degrees of rotation (Hosseinalipour et al., 2013).

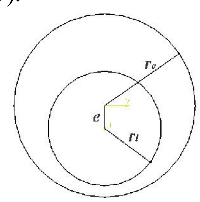


Figure 2. Cross section of the mixer, where $\xi = \frac{r_o}{r_i} - 1$ and $\varepsilon = \frac{e}{r_o - r_i}$

$$\omega = \frac{d\theta}{dt} \tag{1}$$

On the other hand, considering the fact that the mixer is spatially periodic, each section of the flow field can be proportional to one specific degree of rotation which means degrees of rotation can be calculated just with considering specific mixer's cross sections. Equation (2) calculates this location of a cross section that is indicator of a specific rotation degree (Hosseinalipour et al., 2013).

$$dz = Pt\frac{d\theta}{2\pi} \tag{2}$$

Considering all these fact, the numerical method employed in this study is summarized in a mathematical algorithm illustrated in Figure 3. For further details, the reader is asked to refer to (Hosseinalipour et al., 2013).

4. Lagrangian particle tracing

In order to establish the analytical study of chaotic advection effects on mixing performance of the proposed mixer, a robust technique employed as a basic concept of all further calculations is Lagrangian particle tracing. A computer code was developed to track fluid particles as they moved through the flow field. The movement of particles is determined by integrating the vector equation of motion for each particle given by [14]:

$$u = \frac{\partial x}{\partial t},\quad v = \frac{\partial y}{\partial t},\quad w = \frac{\partial w}{\partial t} \tag{3}$$

A combination of an Adams and a fourth order Runge-Kutta integration scheme was adopted for integration of the equation of motion owing to its high accuracy and straightforward implementation

(Muzzio et al., 1997). In integrating equation (3) in the time between t and t+dt, the three dimensional computed velocity field is obtained from the velocity field data according to the degree of rotation θ+dθ previously discussed. More information can be found in (Muzzio et al., 1997).

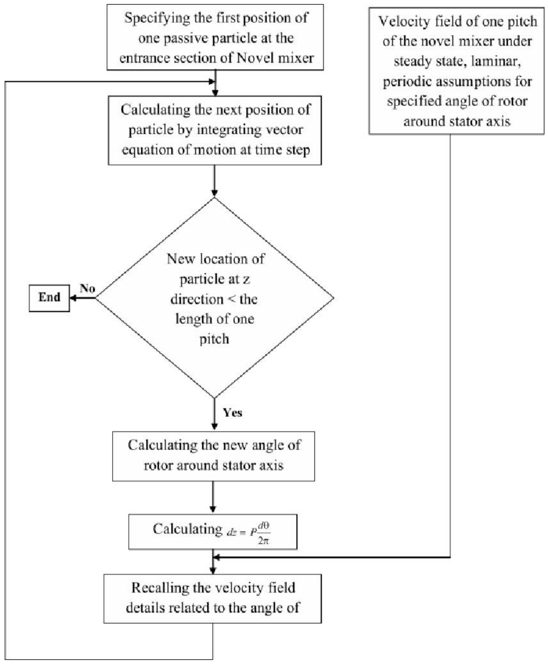


Figure 3. Algorithm for particle tracking (Hosseinalipour et al., 2013).

5. Sensitivity to initial condition

One of the fingerprints of chaos is that two particles, which are initially very close to each other, rapidly diverge exponentially. This special divergence is interpreted as sensitivity to initial conditions which can be considered as a main symptom of the existence of chaotic advection.

As discussed in the Part A of this paper, 576 particles located at the entrance section of the computed flow domain in the arrangement similar to the Figure 4 were traced.

Figure 5 illustrates some results. The results reflect separate regions where different sensitivity to initial conditions can be detected. In some regions trajectories follow completely different paths and diverge rapidly thereby probably representing chaotic regions. However, trajectories of other regions are completely identical. Actually, the flow domain possesses a combination of the coexistence of chaotic and non-chaotic zones in the flow field. Figure 6 and Table 1 show the distribution of initial conditions which yield different mixing behavior. S3 and S2 represent the particle initial conditions which lead to high and low chaotic behavior through the mixer. Overall the sensitivity to the initial conditions of these regions can be compared with each other as:

$$S3 > S4 > S1 > S2 \tag{4}$$

In order to keep the investigation reasonably focused, two significant examples of a chaotic zone (175 degrees) and a non-chaotic zone (130 degrees) were chosen as illustrated in Figures 7 and 8. It is clear that those particles with the initial arrangement of 175 degrees exhibit significant sensitivity to initial conditions. Conversely, those particles in the 130 degrees zone show very low performance in this regard. Therefore the analytical study presented in the following section will focus on assessing and comparing the mixing parameters of these two typical zones.

It should be noted that these regions with different sensitivity to initial conditions, and therefore, different presence of chaotic advection, were observed only for 5760 fluid particles located around 288 small circles at the section mentioned before. Since more particles located at different sections are tracked much more regions can be identified.

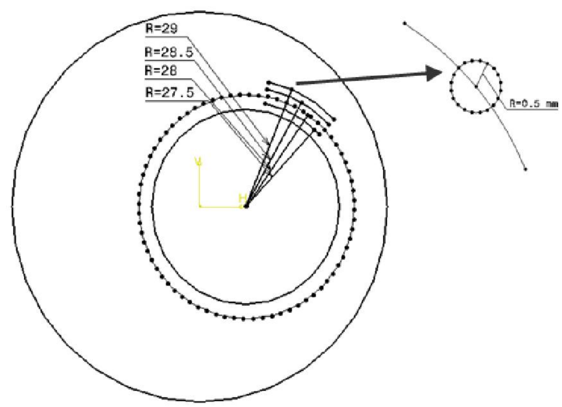


Figure 4. Initial particle arrangements at the entrance section of the proposed mixer.

6. Poincare section

The Poincare map and the stretching distribution provide a conceptual tool for understanding the phenomenon that makes chaotic mixing so efficient. Since stretching is a quantitatively way of assessment of mixing behavior, the Poincare map is qualitative and less computationally expensive to obtain (Kokini et al., 2007). It is often used to visualize the flow structure by superimposing the intersections of several trajectories on a single plane, and is considered to be a practical method in order to evaluate the mixing performance (Peerhossainiet al., 2005). The stretching will be taken up in later section and Poincare map is described more generally here.

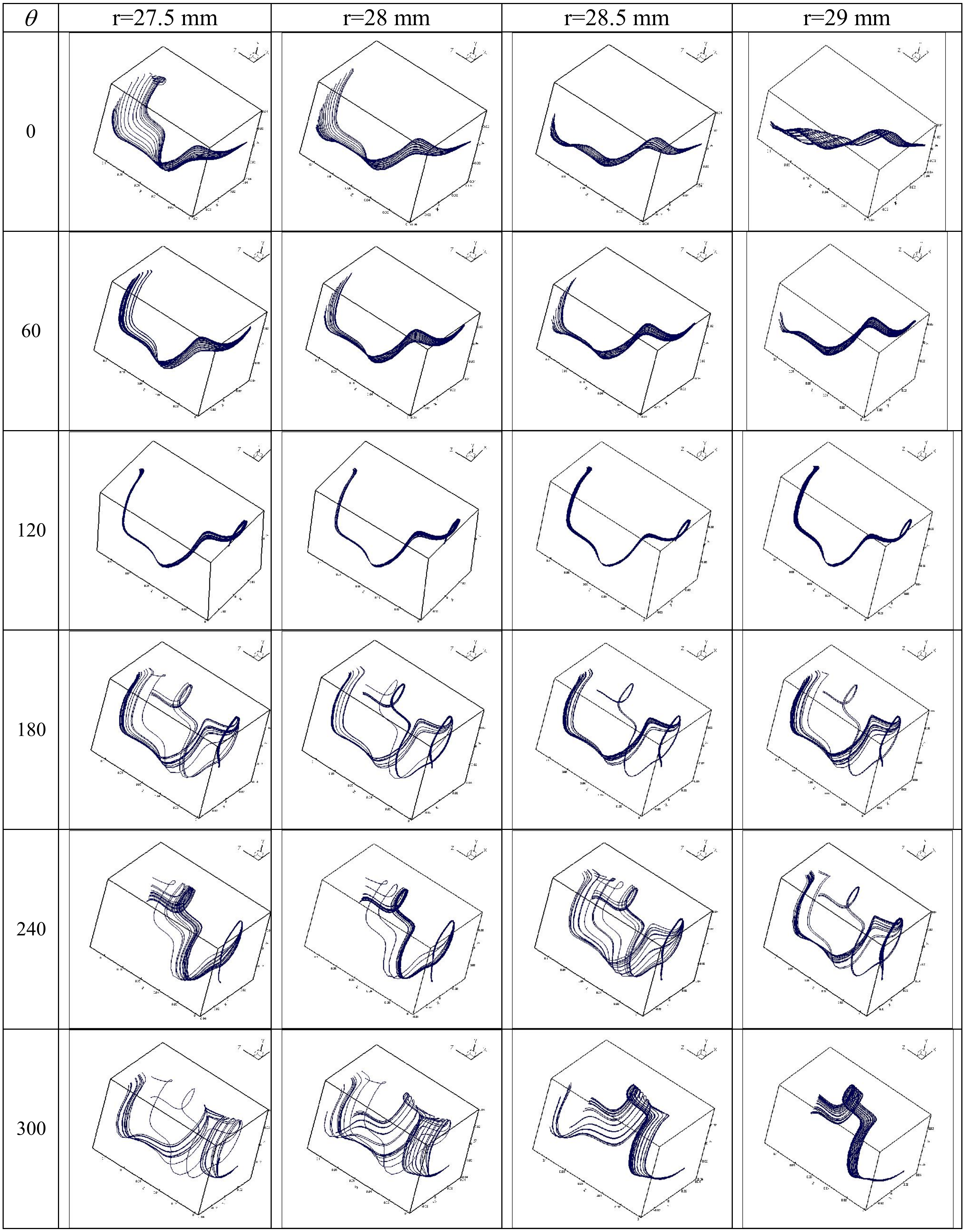


Figure 5. Sensitivity to initial condition of 28 groups located at angle 0º, 60 º, 120 º, 180 º, 240 º, 300

In this analytical study, the Poincare section is obtained using the Lagrangian particle tracing of 121 fluid elements with the initial arrangement of Figure 9 in both chaotic and non-chaotic regions. In order to gain an accurate assessment of the loci of particles in the whole flow domain, superimpositions of these sections for two different entrance regions (130 and 175 degrees) are obtained. Figures 10 and 11 illustrate the results.

Results show that the Poincare section of the chaotic region, preferably regarded as an effective mixing, is well-scattered which indicates that the particles freely move over the mixer cross-section. On the other hand, particles in the Poincare section of the non-chaotic region, preferably regarded as a poor mixing, are remaining close to each other.

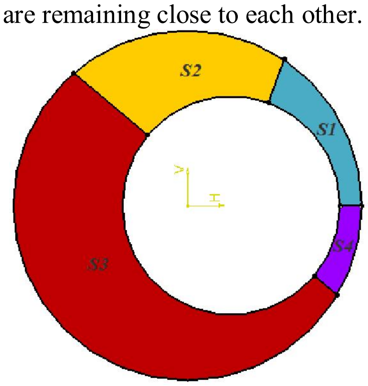


Figure 6. Distribution of initial conditions which yield to different mixing behavior.

Table 1. Regions with different sensitivity to initial condition.

Region Name	Area Specification
S1	$0 < \theta < 70$
S2	$70 < \theta < 140$
S3	$140 < \theta < 320$
S4	$320 < \theta < 360$

7. Line stretching

Understanding of a more significant characterization of mixing is possible by studying the local stretching of different material elements (Kokini et al., 2007). Therefore, the effectiveness of a dough mixer can be evaluated by examining the ability of the novel mixer to stretch and distribute ingredients through the dough.

Length stretching of fluid elements is a cooperative action between velocity gradients and orientation which can be easily computed using tracer particle tracking. The calculation of the average specific stretching rate and efficiency requires the knowledge of the instantaneous values of the orientation and length stretch. These can be obtained by integrating the following equation (Kokini et al., 2007):

$$\begin{cases} \dfrac{d1_x}{dt} = \dfrac{\partial u}{\partial x}1_x + \dfrac{\partial u}{\partial y}1_y + \dfrac{\partial u}{\partial z}1_z \\ \dfrac{d1_y}{dt} = \dfrac{\partial v}{\partial x}1_x + \dfrac{\partial v}{\partial y}1_y + \dfrac{\partial v}{\partial z}1_z \\ \dfrac{d1_z}{dt} = \dfrac{\partial w}{\partial x}1_x + \dfrac{\partial w}{\partial y}1_y + \dfrac{\partial w}{\partial z}1_z \end{cases} \quad (5)$$

in which, at each time lx, ly and lz are the vector components considered to be placed at an arbitrary position in the flow and stretches along it. If the initial vector is considered to be unit length, the quantity of the stretching is obtained by:

$$Stretching \equiv |1_t| \big/ |1_0| = |1_t| \big/ 1 = |1_t| \quad (6)$$

Where l_0 is the length of initial vector. The mean value of stretching is given by:

$$\overline{str_t} = \frac{1}{N}\sum_{n=1}^{N} str_t \quad (7)$$

The line stretching defined above is a technique to quantitatively evaluate the length stretching and consequently, mixing efficiency. Motivated by this work, 25 particles, shown in Figure 12, were radially aligned at two specific regions (130 and 175 degrees) at the entrance section of computed flow domain discussed above. The mean values of stretching of these elements in terms of the time were calculated. The results are depicted in Figure 13. The interaction surface of fluid elements can be stretched differently depending on initial location which can lead them either to a chaotic zone or non-chaotic zone. Obviously, length stretching of fluid elements in the region referring to the 175 degrees section evolves exponentially while advancing in the axial direction and is much higher than that referring to the 130 degrees section. This is due to the exponential evolution of the distance between two adjacent tracers in the chaotic zone which can lead to greatly enhanced length stretching and cause local mixing more efficient. Since this exponential length stretch is a clear symptom of chaotic advection, results apparently confirm the chaotic advection mechanism.

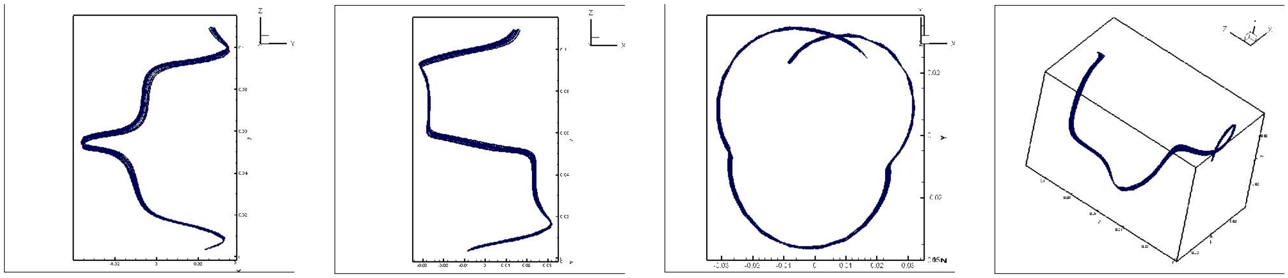


Figure 7. Significant example of non-chaotic zone (130 degrees) in the proposed mixer.

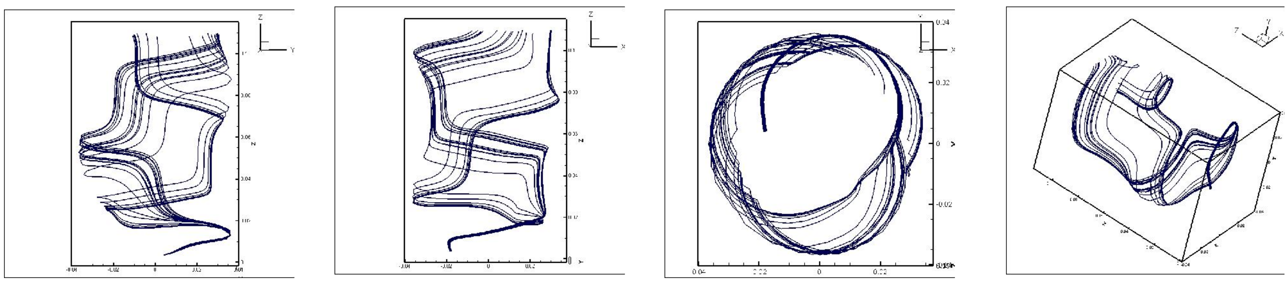


Figure 8. Significant example of chaotic zone (175 degrees) in the proposed mixer.

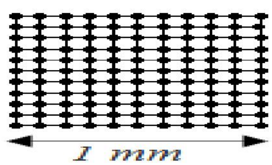
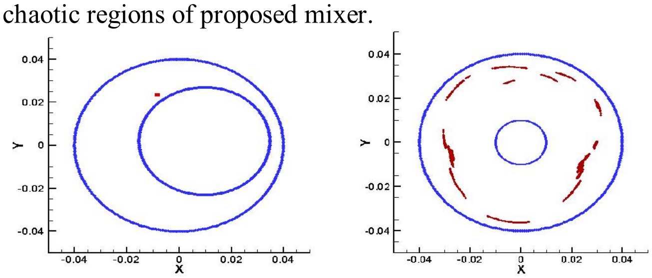
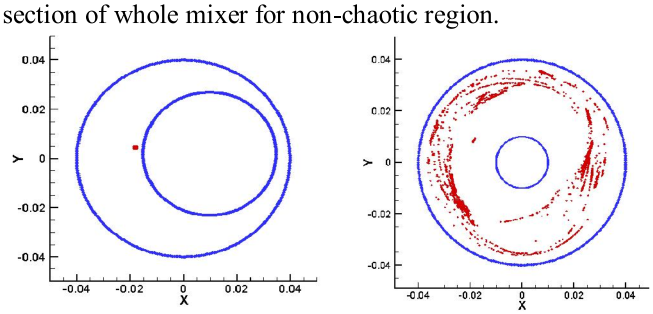


Figure 9. Initial arrangement of 121 particles for Poincare section calculations for chaotic and non-chaotic regions of proposed mixer.



a) b)

Figure 10. (a) initial position of 121 fluid elements and (b) the superimposed Poincare sections in cross section of whole mixer for non-chaotic region.



a) b)

Figure 11. (a) initial position of 121 fluid elements and (b) the superimposed Poincare sections in cross section of whole mixer for chaotic region.

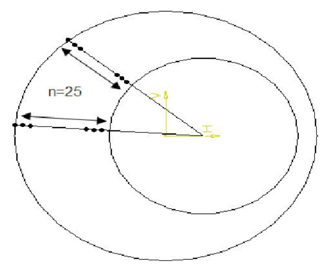


Figure 12. Initial arrangement of 25 particles for stretching calculations for chaotic and non-chaotic regions of proposed mixer.

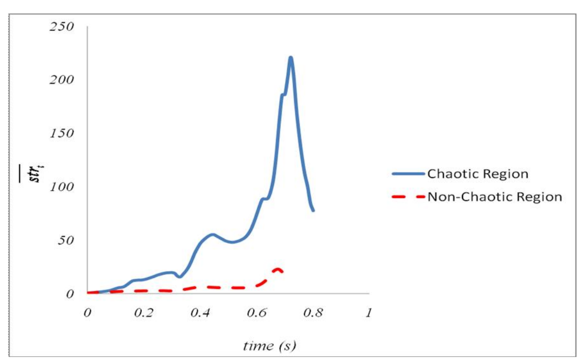


Figure 13. Changes of average stretching of 25 elements in two different regions of proposed mixer.

8. Horseshoe map

The horseshoe maps occupy a central position in dynamical systems, and as we shall see they are very relevant to mixing. The existence of such a map indicates that the system is chaotic; in fact they can be regarded as the archetypical chaotic map. In fact horseshoe maps are qualitatively tools used for identification of presence of chaotic advection in a flow field.

Figure 14 shows that the non-chaotic region of proposed mixer possesses no horseshoe map at all while Figure 15 provides an obvious evident of occurring horseshoe map in the chaotic region.

9. Conclusions

This paper examines the advantages of chaotic advection in order to make some progress in complicated mixing mechanisms of high viscous materials in food industries, especially in dough preparation process. This study implements a proposed dough mixer which possesses chaotic advection with high mixing performance in some regions.

In order to identify the presence of chaotic regimes in the flow field, the sensitivity to initial conditions was examined using Lagrangian particle tracing. The results indicate that the flow filed is a combination of both effective and poor mixing due to different flow patterns. Therefore, additional computations were focused on comparing the mixing performance between two typical examples refer to the chaotic zone, namely as effective mixing, and the non-chaotic zone, namely as poor mixing.

The comparison via line-stretching and Lyapunov exponent quantitatively showed a distinct difference in the mixing performance of these two regions. The well-scattered Poincare section of the chaotic region illustrates the free movement of particles over the mixer cross-section, while in the Poincare section of the non-chaotic region, particles remain close to each other. In the chaotic zones, the particle trajectories can diverge rapidly. In fact, chaotic advection causes fluid particles to follow chaotic trajectories that make them 'visit' a large number of transverse positions in the mixer cross-section. Also, the rate of stretching is exponential in the chaotic region resulting in a much faster rate for non-chaotic zone. All these results indicate that chaotic advection is a robust technique for increasing mixing performance.

Using the numerical simulation of the dough flow through the proposed novel chaotic mixer, it will be possible to assess mixing performance during the process and to predict the quality of the processed material. This approach will be developed in the near future by studying and developing chaotic zones in such mixers of very high viscous materials and also by designing new chaotic mixers to take the advantages of high mixing performance induced by chaotic advection.

d : Distance between two adjacent trajectories

e : Eccentricity between rotor's axis and stator's axis

V : Velocity

N : Total number of particles for stretching calculations

S : Mixing Region

t : Time

u : X component of velocity

v : Y component of velocity

w : Z component of velocity

x : X component of position

y : Y component of position

z : Z component of position

dt : Time step

Pt : Pitch of the rotor

Re : Reynolds number

St : Strouhal number

d_0 : Initial circle radius

$\bar{d}_n$: Averaged distance from the center particle

D_h : Rotor's hydraulic diameter

l_x : X component of fluid elements length

l_y : Y component of fluid elements length

l_z : Z component of fluid elements length

l_z : Initial length of fluid elements

Str_t : Line stretching of fluid element

$\overline{Str_t}$: Mean value of stretching

r_i : Radius of rotor's cross section area

r_o : Radius of stator

α : Constant material coefficient

λ : Lyapunov exponent

ρ : Density

τ : Stress tensor

ω : Rotational speed

$\dot{\gamma}$: Shear rate

η_∞ : Infinitive shear viscosity

η_0 : Zero shear viscosity

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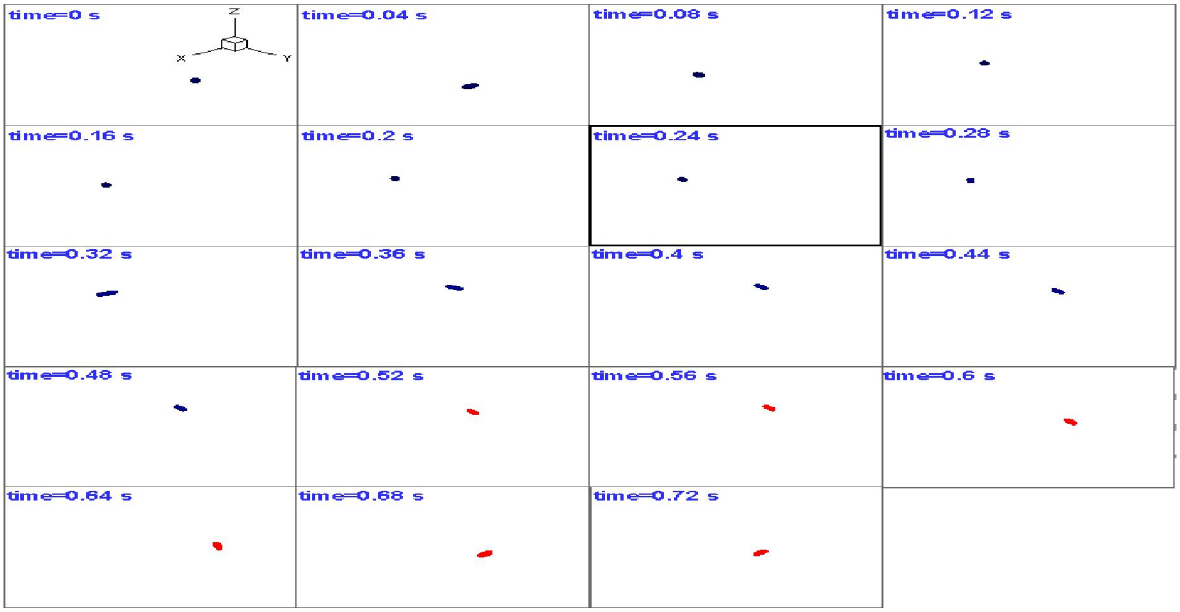

Figure 14. The non-chaotic region of proposed mixer possesses no horseshoe map at all.

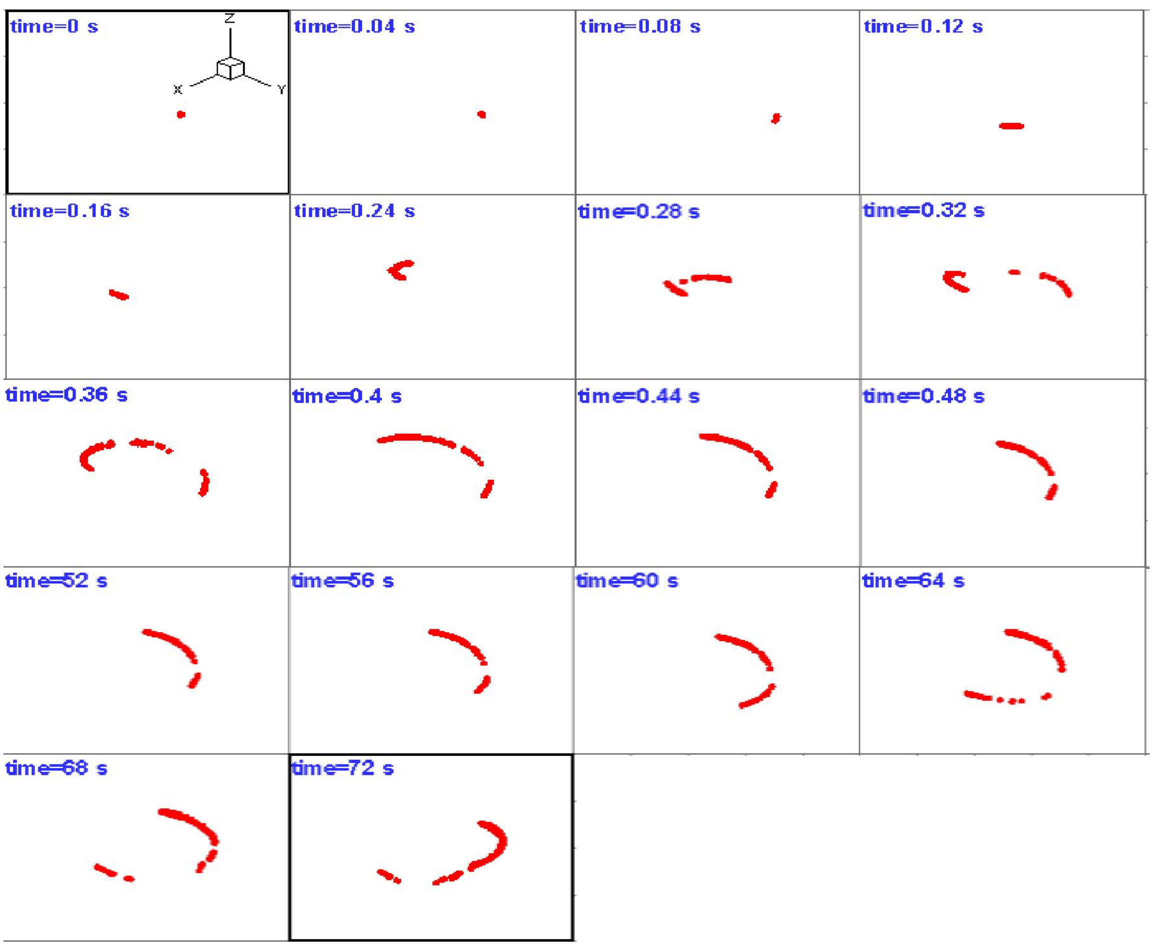

Figure 15. Horseshoe map in the non-chaotic region of proposed mixer at interval time of 0.4s.

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