Practical Approach To Solve The Nonlinear Programming Problems

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Abstract: In this research paper we introduce the some solutions of the applications to non-linear programming problems.
[Rohit Sarin, Mayank Pawar, S. Rajan. **Practical Approach To Solve The Nonlinear Programming Problems.** *Life Sci J* 2013;10(2):2967-1970] (ISSN:1097-8135). http://www.lifesciencesite.com.

Key-words: Optimization, Constrained optimization, Unconstrained optimization, Nonlinear programming.

Introduction:

Any optimization problem essentially consists of an objective function. Depending upon the nature of objective function, there is a need to either maximize or minimize it. For example,
(*a*) Maximize the profit
(*b*) Maximize the reliability of equipment
(*c*) Minimize the cost
(*d*) Minimize the weight of an engineering component or structure, etc.

If certain constraints are imposed, then it is referred to as constrained optimization problem. In the absence of any constraint, it is an unconstrained problem. Linear programming (LP) methods are useful for the situation when objective functions as well as constraints are the linear functions. Such problems can be solved using the simplex algorithm.

Nonlinear programming (NLP) is referred to the followings:
(*a*) Nonlinear objective function and linear constraints
(*b*) Nonlinear objective function and nonlinear constraints
(*c*) Unconstrained nonlinear objective function

The method of optimization for constraint problems which involves the addition of unknown multiples became known by the name of its inventors Lagrange. Cauchy made the first application of the steepest descent method to solve unconstrained minimization problems. Despite these early contributions, very little progress was made until the middle of the twentieth century. When high speed digital computers made implementation of the optimization procedures possible and stimulated further research on new methods, spectacular advances followed, producing a massive literature on optimization techniques. This advancement also resulted in the emergence of several well defined new areas in optimization theory.

Constrained/Unconstrained Optimization:

Unconstrained optimization problems arise directly in many practical applications. If there are natural constraints on the variables, it is sometimes safe to disregard them and to assume that they have no effect on the optimal solution. Unconstrained problems arise also as reformulations of constrained optimization problems, in which the constraints are replaced by penalization terms in the objective function that have the effect of discouraging constraint violations.

Constrained optimization problems arise from models that include explicit constraints on the variables. These constraints may be simple bounds such as $0 \le x_i \le 100$, more general linear constraints such as $\sum x_i \le 1$,or nonlinear inequalities that represent complex relationships among the variables.

Newton Methods

More than three hundred years have passed since a procedure for solving an algebraic equation was proposed by Newton in 1669 and later by Raphson in 1690 **N. Bicanic, K.H. Johnson [5].** The method is now called Newton's method or the Newton {Raphson method and is still a central technique for solving nonlinear equations. Many topics related to Newton's method still attract attention from researchers. For example, the construction of globally convergent effective iterative methods for solving non differentiable equations in R_n or C_n is an important research area in the fields of numerical analysis and optimization.

Let X and Y be Banach spaces and $F : D \subseteq X \to Y$ be an operator where D is a domain of F. If F is differentiable in an open convex set $D_0 \subseteq D$, then Newton's method for solving the equation

$F(x) = 0$ (1.1)

with a solution x^* is defined by the following:

(1) Let x_k be an approximation to x^*;

(2) Solve the linear equation

$$F(x_k) + F'(x_k)h = 0 \quad (1.2)$$

with respect to h, provided that $F'(x_k)$ is nonsingular;

(3) Set $x_{k+1} = x_k + h$ expecting for it to be an improvement to x_k, where $k = 0,1,2,3,...$

Since

$$0 = F(x^*) = F(x_k + h) = F(x_k) + F'(x_k)h + \xi$$

with $\xi = o(|h|)(\xi) = o(|h|^2)$ if F' satisfies a Lipschitz condition), (1.2) is a linearization procedure for the operator F around x_k. The procedure first employed by Newton in 1669 for the cubic equation $3x^3 - 2x - 5 = 0$ is different from the (1), but it is easily verified that both are mathematically equivalent. The procedure (1) can also be written

$$x_{k+1} = x_k - F'(x_k)^{-1} F(x_k); \quad k = 0,1,2,3,... \quad (1.3)$$

Since Raphson described in 1690 the formula (1.3) for a general cubic equation $x_3 - bx = c$, the procedure (1.3) is also called the Newton-Raphson method.

Quasi-Newton Methods:

The goal of quasi-Newton methods, which are also called variable metric method, is not different from the goal of conjugate gradient method: to accumulate information from successive line minimizations so that N such line minimizations lead to the exact minimum of a quadratically convergent for more general smooth functions. Both quasi-Newton and conjugate gradient methods require that you are able to compute your function's gradient, or first partial derivatives, at arbitrary points. The quasi-Newton method approach differs from the conjugate gradient in the way that it stores and updates the information that is accumulated. Instead of requiring intermediate storage on the order of N, the number of dimensions, it requires a matrix of size $N \times N$, Generally, for any moderate N, this hardly matters. On the other hand, there is not, as far as we know, any overwhelming advantage that the quasi- Newton method hold over the conjugate gradient techniques, except perhaps a historical one. Developed somewhat earlier, and more widely propagated, the quasi-Newton methods have by now developed a wider constituency of satisfied users.

The basic idea of quasi-newton method is to build up, iteratively, a good approximation to the inverse Hessian matrix A^{-1},i.e. to construct a sequence of matrices H_i with the property:

$$\lim_{i \to \infty} H_i = A^{-1}$$

Even better if the limit is achieved after N iterations instead of ∞.

The Newton-like methods are generally defined by the recursion

$$x_{k+1} = x_k - (M(x_k))^{-1} F(x_k) \quad k = 0,1,2,3,...$$

where $M(x_k)$ is usually an approximation to $F'(x^*)$, These methods formally include the quasi-Newton and inexact Newton methods, as well as.

Constrained Optimization:

This discussion focuses on the constrained optimization problem and looks into different methods for solving it. Constrained optimization is approached somewhat differently from unconstrained optimization because the goal is not to find the global optima. Often, constrained optimization methods use unconstrained optimization as a sub-step.

The standard form of the constrained optimization problem is as follows:

$$\begin{aligned} &Min \ \ f(x) \\ &Subject\ \ to\ \ g_j(x) \le 0;\ j = 1,2...p \\ &\qquad\qquad h_i(x) = 0;\ i = 1,2...q \end{aligned} \quad (1.4)$$

Where $f(x)$ is the objective function to be minimized, $g_j(x)$ are a set of inequality constraints, and $h_i(x)$ are a set of equality constraints

Karush Kuhn-Tucker Conditions:

The Karush-Kuhn-Tucker (KKT) condition are definitely among the most important results in modern optimization theory. The Karush-Kuhn-Tucker conditions provide the conditions under which the Lagrange multipliers associated with the gradient of the objective function to be positive. The KKT conditions were originally known as **Kuhn-Tucker condition (4)**. **Kuhn-Tucker condition (4)** first presented their result in 1951. It was detected later on that W. Karush had presented a similar result way back in 1939. Thus the Kuhn-Tucker conditions are now known as Karush-Kuhn-Tucker (KKT) conditions

Min f(x)

subject to $g_j(x) \le 0,\ j = 1, 2, ..., m$

where f and each $g_j,\ j = 1, 2, ..., m$ are real-valued differentiable function on R^n. Let us assume that x_0 is a solution of (MP). Moreover assume that $\{\nabla g_1(x_0), ..., \nabla g_m(x_0)\}$ is a linearly independent set of vectors. Then there exists non-negative scalars $\lambda_1, ..., \lambda_m$ such that

(i) $\nabla f(x_0) + \sum_{j=1}^{m} \lambda_j \nabla g_j(x_0) = 0$

(ii) $\lambda_j\ g_j(x_0) = 0,\ j = 1, 2, ..., m$

Successive Quadratic Programming (SQP):

Sequential (or Successive) Quadratic Programming (SQP) is a technique for the solution of Nonlinear Programming (NLP) problems. A nonlinear programming problem is the minimization of a nonlinear objective function $f(x)$, $x \in R^n$, of n variables, subject to equation and/or inequality constraints involving a vector of nonlinear functions c(x). A basic statement of the problem, useful for didactic purposes is

$$\textit{Min}\ \ f(x)$$

$$\textit{Subject to}\ \ c_j(x) \le 0;\ j = 1, 2 ... p$$

In this formulation, equation constraints must be encoded as two opposed inequality constraints, that is $c(x) = 0$ is replaced by $c_j(x) \le 0$ and $-c_j(x) \le 0$, which is usually not convenient. Thus in practice a more detailed formulation is appropriate, admitting also equations, linear constraints and simple bounds. One way to do this is to add slack variables to the constraints, which together with simple bounds on the natural variables, gives rise to

$$\textit{Min}\ \ f(x)$$

$$\textit{Subject to}\ \ A^T x = b, c(x) = 0$$

Optimality in Non-linear Programming

Optimality conditions are very important because they lead to the identification of optimal solutions. The classical approach to constrained optimization was developed mainly by Lagrange in the 18^{th} century. Lagrange developed a novel method by converting the constrained problem into an unconstrained one and then using rules of unconstrained minimization to compute the required minimum. This method became famous as the method of undetermined multipliers and later on came to be known as the method of Lagrange multipliers. In the last century after the second World War, it became apparent that there are many problems of applications which involve constraints not in the form of equalities but in the form of inequalities. Thus there was the need to develop new mathematical ideas so that one might be able to develop a method of undetermined multipliers in the case of inequality constraints. This led to the development of convex analysis which still continues to be at the heart of the subject. But the mathematics one uses for proving the existence of Lagrange multipliers in the case of inequality constrained programs turned out to be simpler than the techniques one needs for the equality constrained case. Though nowadays many economists, engineers, scientists and mathematicians use the KKT conditions on a regular basis. Fritz John, who was the first to develop a Lagrange multiplier rule for constrained optimization in 1948. Fritz John gave necessary optimality criteria for a non-linear programming problem without imposing any constraint qualification. They have proved that if x^* is an optimal solution of (P), then there exists

$$r_0^* \in R,\ r^* \in R^m$$

s.t. $r_0^*\ \nabla f(x^*) + r^{*T}\ \nabla g(x^*) = 0$

$$r^{*T}\ g(x^*)\ =\ 0$$

$$(r_0^*, r^*)\ \ge\ 0$$

There is no guarantee that $r_0^* > 0$. In case $r_0^* = 0$, the objective functions f disappears from the Fritz-John conditions and we have a degenerate case. In order to exclude such cases, **Kuhn-Tucker (4)** introduced restrictions on the constraints. These restrictions are nothing but the constraint qualifications mentioned earlier. The Kuhn-Tucker necessary optimality conditions state that if x^* is an optimal solution of (P) and the function g satisfies certain constraint qualification then there exists

$$r^* \subset R^n$$

s.t. $\nabla f(x^*) + r^{*T}\ \nabla g(x^*) = 0$

$$r^{*T}\ g(x^*) = 0$$

$$r^*\ \ge\ 0$$

Kuhn-Tucker (4) also obtained sufficient optimality conditions by assuming the functions to be convex. Optimality conditions involving generalized convex functions have been studied by several authors, for example **Singh (2)**, **Bector and Bector (1)**, **Chandra and Bector, Durga(3)**, etc.

Conclusions:

Generally it can be concluded that, mathematical or numerical optimization techniques can be very effective and also very efficient in nonlinear programming problems. These optimization methods are easy to use and more applicable in comparison with some other optimization methods.

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6/20/2013