An Improved Water Swirl Algorithm for lower order model formulation of Multivariable Linear Time invariant Continuous System

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Abstract: In this paper an Improved Water Swirl Algorithm (IWSA) approach is used for formulation of lower order Multi Input Multi Output (MIMO) model for a given absolutely stable higher order MIMO Continuous system in transfer function form. Water Swirl Algorithm (WSA) is a swarm based optimization technique that mimics the way by which water finds a drain in a sink. It observes the flowing and searching behavior of water for drain and proposes a suitable strength update equations to locate the optimum solution iteratively from the initial randomly generated search space. The strength of a water particle is governed by three components namely, Inertia, Cognitive and Social. In the proposed Improved WSA, the cognitive component of water particle is spitted into good experience component and worst experience component. Due to the inclusion of worst experience component, the particle can bypass the previously visited worst position and try to occupy the best position. A weighted average method is proposed in this paper to reduce the higher order model formulation to lower order form. Integral square error is used as an indicator for selecting the lower order model. An average scheme has been proposed for commonizing the denominators of the individual lower order approximants so that a lower order MIMO model can be declared in the transfer function matrix form. The proposed methodology is illustrated with an example.

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1. Introduction

For many modern control schemes, such as optimal or H∞ based control, it is usually required to perform plant model or controller order reduction prior to or during the process of design. The order reduction problem is the problem of approximating, as closely as possible, the dynamics of a higher order system by a reduced order linear time invariant system model, while retaining the important structural and dynamic properties of the original system. It is usually a multimodal optimization problem in a multidimensional space.

Recently, systems have become complex and the interrelationship of many controlled variables need to be considered in the design process. The computer control systems used to control the fighter aircrafts, fuel injectors and spark timing of automobiles is excellent examples of such multivariable control systems. These MIMO systems are challenging when compared to the analysis of single input-single output (SISO) systems. The exact analysis of higher order MIMO systems are often difficult due to computational considerations, which further emphasizes the importance of formulating lower order models.

During the past three decades, several model reduction methods have been developed by various authors [1-5]. Each of them has its own merits and applications. Extensions of these SISO techniques for MIMO systems are not trivial and most of the approaches yield poor approximants [6-7]. Levy [8] established a complex curve fitting technique to minimize quadratic errors of a single input and single output transfer function. This was later formalized by Elliott and Wolovich [9] and extended to MIMO systems [10]. Recently, mixed methods [11-13] are achieving great attention in the model reduction of MIMO systems. In these methods, the common denominator characteristic equation of the transfer function of the reduced model is fixed by using a stability preserving algebraic method, while the numerators are obtained using one of the available model reduction methods.

Most problems in this area are heuristics in nature which can find good solution in a reasonable amount of time. Heuristics are rules to search for optimal or near-optimal solutions. Some of the heuristic tools include Simulated Annealing (SA), Ant Colony Optimization, Evolutionary Computation methods like Genetic Algorithm, Fuzzy Logic, Neural Networks etc., [14-15] and Swarm Intelligence techniques like Particle swarm Optimization, Water Swirl Algorithm etc., In this paper, novel attempt is made to simulate the motion of water in the sink is presented. This behaviour of water is studied by Menser and Hereford [16] as Water Swirl Algorithm based on the method of fluid flow around a drain. In the proposed Improved Water

Swirl Algorithm, the search behaviour of water for drains and proposes suitable update equation to locate the optimum solution iteratively from the initial randomly generated search space.

This paper presents an average scheme for the formulation of initial second order MIMO model for a given absolutely stable linear time invariant higher order MIMO continuous system. An IWSA for modifying the characteristics of the initially formulated lower order model has been proposed based on the Transient Gain (TG) and Steady State Gain (SSG). This methodology results in a lower order model that closely matches the characteristics of the given higher order systems. The integral square error is used as an indicator for this purpose. The algorithm presented in this paper requires the higher order MIMO system to be represented in the form of a transfer function matrix. Leverrier algorithm [17] can be used to find the transfer function matrix, if the given system is in the state-space form.

The rest of the paper is organized as follows: Section 2 gives the problem definition; Section 3 deals an overview of WSA, Section 4 on proposed IWSA and our proposed methodology for MIMO system in Section 5. Numerical example is presented in Section 6, which is followed by discussion of the results in Section 7. Section 8 presents a conclusion.

2. Problem Definition

Consider an nth order linear time invariant dynamic multivariable system with q inputs and r outputs described in time domain by state space equations as,

$$\dot{x} = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \tag{1}$$

where $\dot{x}$ is n dimensional state vector, u is q dimensional control vector and y is r dimensional output vector with $q \leq n$ and $r \leq n$. Also, A in $n \times n$ system matrix, B is $n \times q$ input matrix and C is $r \times n$ output matrix.

Alternatively, equation (1) can be described in frequency domain by the transfer function matrix of order $r \times q$ as,

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{n-1} A_i s^i}{\sum_{i=0}^{n} a_i s^i} \tag{2}$$

where $N(s)$ is the numerator matrix polynomial and $D(s)$ is the common denominator polynomial of the higher order system. Also, A_i and a_i are the constant matrices of numerator and denominator polynomial respectively.

Irrespective of the form represented in equation (1) or (2) of the original system *G(s)*, the problem is to find a mth order reduced model *Rm(s)*, where $m < n$ in the following form represented by equation (3), such that the reduced model retains the important characteristics of the original system and approximates its response as closely as possible for the same type of inputs with minimum integral square error.

$$R^m(s) = \frac{N^m(s)}{D^m(s)} = \frac{\sum_{i=0}^{m-1} B_i s^i}{\sum_{i=0}^{m} b_i s^i} \tag{3}$$

where, *Nm(s)* and *Dm(s)* are the numerator matrix polynomial and common denominator of the reduced order model respectively. Also, B_i and b_i are the constant matrices of numerator and denominator polynomial of the same order respectively.

Mathematically, the integral square error [18] can be expressed as,

$$E = \sum_{t=o}^{\tau} (Y_t - y_t)^2 \tag{4}$$

where, Y_t is the unit step time response of the given higher order system at the tth instant in the time interval $0 \leq t \leq \tau$, where τ is to be chosen and yt is the unit step time response of the lower order system at the tth time instant.

The objective is to find *Rm(s)*, which closely approximates *G(s)* for a specified set of inputs.

3.Overview Of Water Swirl Algorithm

Water Swirl Algorithm is a swarm based optimization technique that mimics the way by which water finds a drain in a sink. This algorithm considers a certain number of water particles that represents the number of possible solutions for a variable in the search space. The sink holding the water particle represents the boundary conditions limiting the search space.

Water is an inevitable substance in the nature for all the livings. Water has peculiar characteristics that when it is poured or contained in a sink, it tries to find the drain (hole) in the sink continuously to leave out. The answer for this excellent natural behaviour of water can be found in the field of Fluid Dynamics [19] that deals with the nature of fluid flow.

When the drain of the sink is opened, a swirling motion is started in the water mass near the drain leading to the release of water though the drain. The swirl motion of water leads to an important phenomenon called vortex formation as depicted in Figure 1. [20].

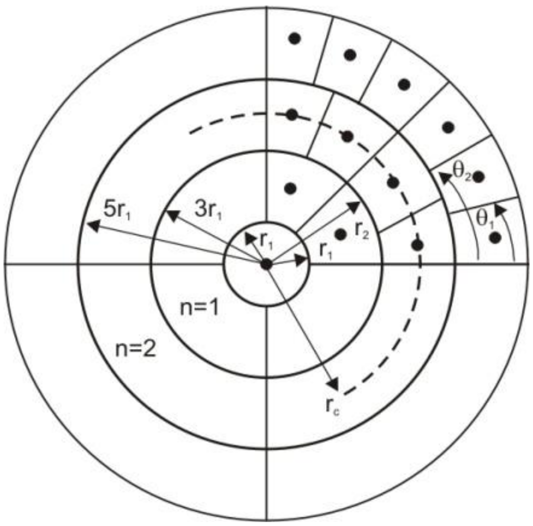

Figure 1: Illustration of Vortex Ring

During swirling motion, the vortex is not stationary, but rather continually moves towards the highest value in the search space. Since the particles are drawn towards the vortex, the search is concentrated in areas that have previously yielded good results (Previous Best). The drain would signify the best among the previous best known as the Global Best. Water particles exist for a certain amount of time, or iterations, and update the previous and global best position until an overall best solution is found. In view of this, strength update equation (5) and position update equation (6) are modified as below:

$$\alpha_{new} = \alpha_{old} + \alpha_{q,ref} \times \left(x_{prevBest} - x_{q,ref}\right) + \alpha_{q,ref} \times \left(x_{gBest} - x_{p,old}\right) \quad (5)$$

$$x_{p,new} = \alpha_{new} + \alpha_{q,ref} \times \left(x_{p,old} - x_{q,ref}\right) \quad (6)$$

Here $x_{q,ref}$ is randomly generated using the range given for the solution variable. $\alpha_{q,ref}$ is a random number generated between zero and one. α_{old} and α_{new} are the strength vectors of water particles at ith and (i+1)th iterations respectively. Similarly, $x_{p,old}$ and $x_{p,new}$ are the positions of water particles at ith and (i+1)th iterations. $x_{q,ref}$, $x_{prevBest}$ and x_{gBest} denotes the reference position, previous best position and global best position of the water particle respectively.

The solution search equation (5) for updating particle strength has well balanced exploration and exploitation abilities. Even though it resembles the velocity updating equation of PSO, the use of reference position $x_{q,ref}$ in second term performs very well in exploration. The generation of strength vector 'α' within the actual range of the variable avoids the problem of velocity clamping. There is no weight 'w' and parameters 'c1' and 'c2' in this search equation and it is completely parameter free.

While updating strength of a particle using (6), different variables have different value that provides larger search space and makes WSA always perform better due to independent updating of each variable. The position update equation given in (6) performs a local selection task by using new strength vector and reference position $x_{q,ref}$ for determining the next adjacent position around the vortex ring.

4. Proposed Improved Water Swirl Algorithm

In the new proposed Improved water swirl algorithm, the strength update equation is splitted into three components namely inertia, cognitive and social component. The cognitive component is splitted into two components. The first component is good experience component which refers to the previously visited good position of the water particle and second component is worst experience component which refers to the previously visited worst position of the water particle. Due to the inclusion of worst experience component in the strength update equation, the particle can bypass the previously visited worst position and try to occupy the best position.

The modified strength update equation (7) is given by

$$\alpha_{new} = \alpha_{old} + \alpha_{q,ref} * (x_{prevBest} - x_{q,ref}) + \alpha_{q,ref} * (x_{gBest} - x_{p,old}) + \alpha_{q,ref} * (x_{prevWorst} - x_{q,ref}) \quad (7)$$

Here $x_{q,ref}$ is randomly generated using the range given for the solution variable. $\alpha_{q,ref}$ is a random number generated between zero and one. α_{old} and α_{new} are the strength vectors of water particles at ith and (i+1)th iterations respectively. Similarly, $x_{p,old}$ and $x_{p,new}$ are the positions of water particles at ith and (i+1)th iterations. $x_{q,ref}$, $x_{prevBest}$, x_{gBest} $x_{worstBest}$ denotes the reference position, previous best position, global best position of the water particle and previous worst position respectively.

The algorithmic steps for the proposed Improved WSA are as follows:

Step1: Select the number of water particles, range of water particle and maximum iterations

Step2: Init ialize the particle position and strength.

Step3: For each water particle (x_p), evaluate fitness function using equation (4)

Step4: Select the particle global best value and the particles individual worst value.

Step 5: Compare the fitness value of (x_p) with fitness value of ($x_{prevBest}$). If it is greater value, then set $x_{prevBest}$ as xp. Otherwise, go to Step 3 for evaluating fitness value.

Step6: Update the particle individual best ($X_{prevBest}$), global best (X_{gBest}) and particle worst ($X_{worstBest}$) in the strength update equation (7) and obtain the position of the particle.

Step 7: The optimal solution is obtained when the integral square error is minimum.

Step 8: Stop

The proposed Improved WSA starts with the initialization of control parameters like Number of Water Particles ('*N*'), Boundary or range of each water particle ('*B*') and the maximum iterations ('*I*'). Then the initial position, reference position and the strength of the water particle are randomly generated within the actual range of each particle. Fitness of each water particle is evaluated.

Until all the water particle exhaust, update the previous best (*prevBest*), previous worst (*worstBest*) and set the best of *prevBest* as global best (*gBest*). Then the strength and position of the water particle are updated using equation (7) and (6) respectively until the maximum iteration is reached. At last, the final *gBest* value is returned as an optimal solution.

5. Proposed Methodology

The various steps involved in the proposed scheme are as follows:

Step 1: Consider the given transfer function matrix of the multivariable system as,

$$G(s)=\begin{bmatrix} G_{11}(s) & G_{12}(s) & \dots & G_{1r}(s) \\ G_{21}(s) & G_{22}(s) & \dots & G_{2r}(s) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ G_{q1}(s) & G_{q2}(s) & \dots & G_{qr}(s) \end{bmatrix} \quad (8)$$

where, $G(s)=\dfrac{N_{ij}(s)}{D_{ij}(s)}=\dfrac{\sum_{k=0}^{n-1} A_k(i,j)s^k}{\sum_{k=0}^{n} a_k(i,j)s^k}$ with

$i = 1,2,\dots, q$ and $j = 1,2,\dots,r$.

Take common denominator $D(s)$ for the given $G(s)$, then the transfer function of the system can be represented in the matrix form as:

$$G_{ij}(s)=\frac{\begin{bmatrix} N_{11}(s) & N_{12}(s) & \dots & N_{1r}(s) \\ N_{21}(s) & N_{22}(s) & \dots & N_{2r}(s) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ N_{q1}(s) & N_{q2}(s) & \dots & N_{qr}(s) \end{bmatrix}}{D(s)} \quad (9)$$

where, $N_{ij}(s)=\sum_{k=0}^{n-1} A_k(i,j)s^k$ and

$$D(s)=a_0+a_1s+a_2s^2+\dots+a_{n-1}s^{n-1}+a_ns^n$$

Step 2: For each $G_{ij}(s)$ do the following,

Step 2.1: From equation (9) we can write,

$$G_{ij}(s)=\frac{N_{ij}(s)}{D(s)}=\frac{A_0(i,j)+A_1(i,j)s+A_2(i,j)s^2+\dots+A_{n-1}(i,j)s^{n-1}}{a_0+a_1s+a_2s^2+\dots+a_{n-1}s^{n-1}+a_ns^n} \quad (10)$$

Step 2.2: Compute the transient gain (TG_{ij}) and steady state gain (SSG_{ij}) of $G_{ij}(s)$ in equation (10) as follows:

$$TG_{ij}=\frac{A_{n-1}(i,j)}{a_n} \quad (11)$$

$$SSG_{ij}=\frac{A_0(i,j)}{a_0} \quad (12)$$

Step 2.3: Apply average scheme [Appendix I] to obtain an approximate lower order model of order m using the coefficients of numerator polynomial of each $G_{ij}(s)$ and common denominator polynomial $D(s)$. For each G_{ij}(s) of order n, lower order models of order 2, 3, (n-1) can be generated.

For simplicity and without loss of generality, the approximate lower order model to be formulated is assumed to be of order 2, (*i.e.*, m=2) in equation (13). Thus the transfer function matrix of approximate second order model using auxiliary scheme is given by,

$$R_{ij}^m(s)=\frac{B_1(i,j)s+B_0(i,j)}{b_2s^2+b_1s+b_0} \quad (13)$$

Step 2.4: Scaling equation (13), we get,

$$R_{ij}^m(s)=\frac{s+\left(\dfrac{B_0(i,j)}{B_1(i,j)}\right)}{s^2+\left(\dfrac{b_1}{b_2}\right)s+\left(\dfrac{b_0}{b_2}\right)} \quad (14)$$

Step 2.5: Tuning equation (14) to maintain the transient gain and steady state gain obtained in equations (11) and (12), we get, the approximated second order model as,

$$R_{ij}^m(s)=\frac{(TG_{ij})s+\left[(SSG_{ij})\times\left(\dfrac{b_0}{b_2}\right)\right]}{s^2+\left(\dfrac{b_1}{b_2}\right)s+\left(\dfrac{b_0}{b_2}\right)} \quad (15)$$

Comparing equations (13) and (15), it can be noted,

$$B_1(i,j)=TG_{ij}$$

$$B_0(i,j)=SSG_{ij}\times\left(\frac{b_0}{b_2}\right) \quad (16)$$

Step 2.6: The coefficients of the approximated second order model R_{ij}(s) in equation (15) are fed as input to proposed improved water swirl algorithm process. The main aim of proposed IWSA is to minimize the objective function integral square error. The integral square error is computed as in equation (4). The proposed IWSA algorithm [Section IV] is invoked to search the better values of $\left(\frac{b_0}{b_2}\right)$ and

$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ in equation (15), so that the characteristics of the formulated second order model matches the given higher order system. The proposed IWSA is carried out within the constraint of maintaining the transient and steady state gain of the second order model in accord with that of the given higher order system calculated in equations (11) and (12).

Step 2.7: The transfer function of the lower second order model corresponding to the minimum integral square error using proposed IWSA of each $G_{ij}(s)$ is given by,

$$R_{ij}^m(s) = \frac{N_{ij}^m(s)}{D_{ij}^m(s)} = \frac{C_1(i,j)s + C_0(i,j)}{s^2 + d_1(i,j)s + d_0(i,j)} \quad (17)$$

where i=1,2,…, q and j=1,2,…, r.

Step 3: Obtain the common denominator $D_m(s)$ of the lower order model as the average of the corresponding coefficients of each $D_{ij}^m(s)$ from equation (17), which is mathematically expressed as,

$$D^m(s) = s^2 + d_1 s + d_0 \quad (18)$$

where,

$$d_k = \frac{\sum_{i=1}^{q}\sum_{j=1}^{r} d_{ij}(k)}{\text{Total number of } d_{ij}(k)} \quad (19)$$

k=0, 1 with reference to equation (18).

Step 4: Reconstruct the numerators of each $R_{ij}(s)$ with,

$$N_{ij}^m(s) = (TG_{ij}\, s) + (SSG_{ij} \times d_0) \quad (20)$$

so that the characteristics of the original higher order system are maintained in the formulated lower order model.

Comparing equations (17) and (20), it can be noted,

$$C_1(i,j) = (TG_{ij})$$
$$C_0(i,j) = (SSG_{ij} \times d_0) \quad (21)$$

Step 5: The transfer function matrix of the lower order system of order m can now be represented as,

$$G_{ij}^m(s) = \frac{\begin{bmatrix} N_{11}^m(s) & N_{12}^m(s) & \dots & N_{1r}^m(s) \\ N_{21}^m(s) & N_{22}^m(s) & \dots & N_{2r}^m(s) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ N_{q1}^m(s) & N_{q2}^m(s) & \dots & N_{qr}^m(s) \end{bmatrix}}{D^m(s)} \quad (22)$$

Step 6: Calculate the integral square error for the obtained lower order model in equation (22) and the given higher order system in (8) and tabulate it.

The proposed methodology is illustrated with a numerical example in the forthcoming section.

6. Numerical Example

The proposed procedure is applied to linear time invariant multivariable continuous system as follows.

Step 1: Consider the given sixth order system [21] described by the transfer function matrix,

$$G(s) = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix} \quad (23)$$

The common denominator $D(s)$ for the given $G(s)$ in equation (23) is,

$$D(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$
$$= s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000 \quad (24)$$

Step 2: For each $G(s)$ do the following,

Step 2.1: Now $G(s)$ can be represented as,

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \quad (25)$$

where,

$$G_{11}(s) = \frac{2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000}{D(s)} \quad (26)$$

$$G_{12}(s) = \frac{s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400}{D(s)} \quad (27)$$

$$G_{21}(s) = \frac{s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000}{D(s)} \quad (28)$$

$$G_{22}(s) = \frac{s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000}{D(s)} \quad (29)$$

Step 2.2: Table 1 gives the transient gain ratio (TG_{ij}) and steady state gain ratio (SSG_{ij}) computed for $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$ and $G_{22}(s)$ represented in equations (26) through (29).

TABLE I.TRANSIENT GAIN AND STEADY STATE GAIN FOR $G_{11}(S)$, $G_{12}(S)$, $G_{21}(S)$ AND $G_{22}(S)$

Transfer function	Transient Gain Ratio TG_{ij}	Steady State Gain Ratio SSG_{ij}
$G_{11}(s)$	$\frac{2}{1} = 2$	$\frac{6000}{6000} = 1$
$G_{12}(s)$	$\frac{1}{1} = 1$	$\frac{2400}{6000} = 0.4$
$G_{21}(s)$	$\frac{1}{1} = 1$	$\frac{3000}{6000} = 0.5$
$G_{22}(s)$	$\frac{1}{1} = 1$	$\frac{6000}{6000} = 1$

Step 2.3: Applying Average scheme to obtain the initial lower second order models $R_{11}(s)$, $R_{12}(s)$, $R_{21}(s)$ and $R_{22}(s)$ for the corresponding $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$ and $G_{22}(s)$ represented in equations (26) through (29), we get,

$$R_{11}(s) = \frac{7700s + 6000}{10060s^2 + 13100s + 6000} \quad (30)$$

$$R_{12}(s)=\frac{4160s+2400}{10060s^2+13100s+6000} \quad (31)$$

$$R_{21}(s)=\frac{3700s+3000}{10060s^2+13100s+6000} \quad (32)$$

$$R_{22}(s)=\frac{9100s+6000}{10060s^2+13100s+6000} \quad (33)$$

Step 2.4: Scaling equations (30) through (33) we get,

$$R_{11}(s)=\frac{s+0.7792}{s^2+1.3022s+0.5964} \quad (34)$$

$$R_{12}(s)=\frac{s+0.5769}{s^2+1.3022s+0.5964} \quad (35)$$

$$R_{21}(s)=\frac{s+0.8108}{s^2+1.3022s+0.5964} \quad (36)$$

$$R_{22}(s)=\frac{s+0.6593}{s^2+1.3022s+0.5964} \quad (37)$$

Step 2.5: Tuning the equation (34) through (37) to maintain the transient gain ratio and steady state gain ratio of $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$ and $G_{22}(s)$ as shown in Table 1, we get the second order model as,

$$R_{11}(s)=\frac{2s+0.5964}{s^2+1.3022s+0.5964} \quad (38)$$

$$R_{12}(s)=\frac{s+0.2386}{s^2+1.3022s+0.5964} \quad (39)$$

$$R_{21}(s)=\frac{s+0.2982}{s^2+1.3022s+0.5964} \quad (40)$$

$$R_{22}(s)=\frac{s+0.5964}{s^2+1.3022s+0.5964} \quad (41)$$

Step 2.6: The proposed improved water swirl algorithm is now invoked to search the values of s-term (1.3022) and constant term (0.5964) of the denominator in $R_{11}(s)$, $R_{12}(s)$, $R_{21}(s)$ and $R_{22}(s)$ represented by equation (38) through (41), so that the characteristics of second order model matches the given higher order system given by equations (26) through (29). The proposed IWSA is performed within the constraint of maintaining the transient and steady state gain of the lower second order model in accord with that of the given higher order system as shown in Table 1. Proposed IWSA determines a better-reduced second order model for which the integral square error is minimal, using its algorithm in Section IV.

Step 2.7: The transfer function of the second order models obtained using proposed IWSA searching process are given by,

$$R_{11}(s)=\frac{N_{11}(s)}{D_{11}(s)}=\frac{2s+10.0648}{s^2+11.0537s+10.0648} \quad (42)$$

$$R_{12}(s)=\frac{N_{12}(s)}{D_{12}(s)}=\frac{s+2.5651}{s^2+5.3858s+6.4128} \quad (43)$$

$$R_{21}(s)=\frac{N_{21}(s)}{D_{21}(s)}=\frac{s+9.8146}{s^2+20.6362s+19.6293} \quad (44)$$

$$R_{22}(s)=\frac{N_{22}(s)}{D_{22}(s)}=\frac{s+5.9849}{s^2+4.9928s+5.9849} \quad (45)$$

The IWSA searching process for second order model formulation is carried out maintaining the transient gain and steady state gain ratio of given higher order system.

Step 3: Obtaining the common denominator $D_2(s)$ of the lower second order model as the average of the corresponding coefficients of $D_{11}(s)$, $D_{12}(s)$, $D_{21}(s)$ and $D_{22}(s)$, we get,

$$D^2(s)=\left(\frac{4}{4}\right)s^2+\left(\frac{11.0537+5.3858+20.6362+4.9928}{4}\right)s+\left(\frac{10.0648+6.4128+19.6293+5.9849}{4}\right)$$

$$D^2(s)=s^2+10.5171s+10.5229 \quad (46)$$

Step 4: Using the transient gain ratio and steady state gain ratio of $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$ and $G_{22}(s)$ shown in Table 1 and $D_2(s)$, the transfer function represented in equations (42) through (45) can be reconstructed as,

$$R_{11}(s)=\frac{2s+10.5319}{s^2+10.5171s+10.5229} \quad (47)$$

$$R_{12}(s)=\frac{s+4.1096}{s^2+10.5171s+10.5229} \quad (48)$$

$$R_{21}(s)=\frac{s+5.1034}{s^2+10.5171s+10.5229} \quad (49)$$

$$R_{22}(s)=\frac{s+10.0193}{s^2+10.5171s+10.5229} \quad (50)$$

Step 5: The second order MIMO model in transfer function matrix is,

$$G^2(s)=\begin{bmatrix} R_{11}(s) & R_{12}(s) \\ R_{21}(s) & R_{22}(s) \end{bmatrix} = \frac{1}{D^2(s)}\begin{bmatrix} 2s+10.5319 & s+4.1096 \\ s+5.1034 & s+10.0193 \end{bmatrix} \quad (51)$$

where $D^2(s)=s^2+10.5171s+10.5229$

Step 6: The unit step time responses of the given higher order system, the proposed second order system and that of the second order systems obtained using other known methods are shown in Figure 2 – Figure 5. The integral square errors computed are shown in Table 2.

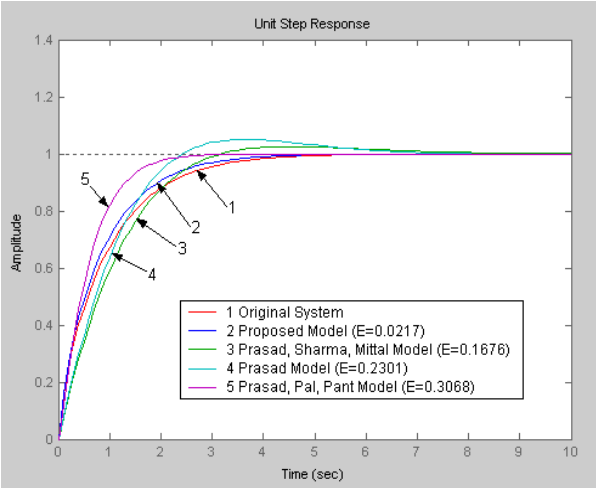

Figure 2: Unit step response curves for $G_{11}(s)$

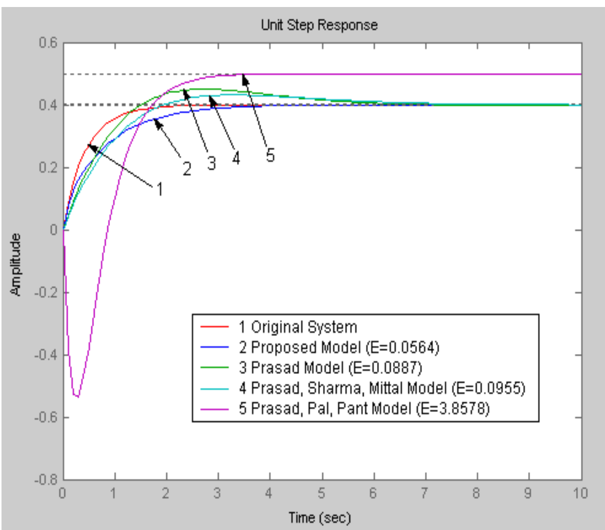

Figure 3: Unit step response curves for $G_{12}(s)$

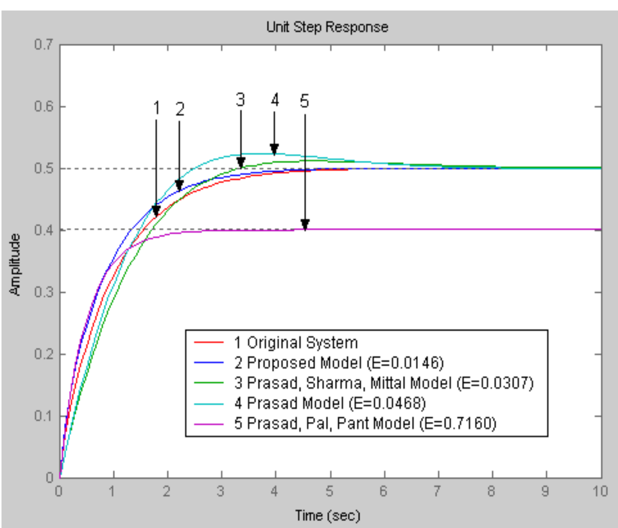

Figure 4: Unit step response curves for $G_{21}(s)$

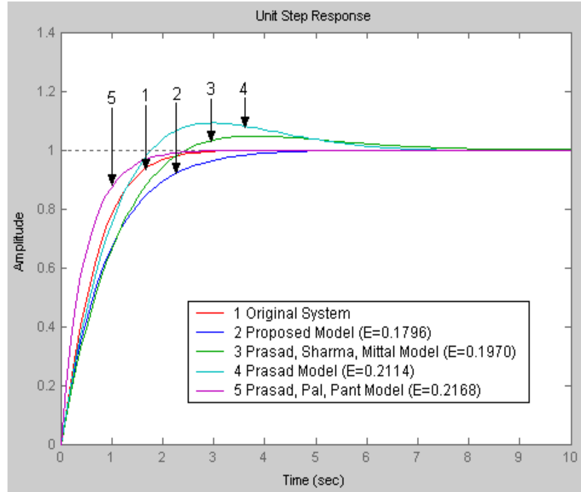

Figure 5: Unit step response curves for $G_{22}(s)$

TABLE II
COMPARISON OF INTEGRAL SQUARE ERROR FOR ILLUSTRATION

Model Reduction Method	Integral Square Error E for 10 seconds			
	$G_{11}(s)$	$G_{12}(s)$	$G_{21}(s)$	$G_{22}(s)$
R.Prasad, J.Pal& A.K.pant [22]	0.3068	3.8578	0.7170	0.2168
R.Prasad [23]	0.2301	0.0887	0.0468	0.2114
R.Prasad, S.P.Sharma& A.K.Mittal [24]	0.1676	0.0955	0.0307	0.1970
Proposed Method	0.0117	0.0424	0.0125	0.1621

It is observed from Table 2 that the proposed scheme yields better value for integral square error with respect to other methods considered for comparison.

7. Discussion

The salient points noted in the illustration are discussed here. The given illustration deals with a sixth order linear time invariant multivariable continuous system. The analysis is carried out in s-domain and the lower order MIMO model for the individual transfer functions of the MIMO transfer function matrix are formulated using the proposed auxiliary scheme. Then proposed IWSA is invoked to search for the better lower order model minimizing the integral square error with the constraint of maintaining the transient and steady state gain of the given higher order system. Further, an average procedure is used to obtain the common denominator for the lower order transfer function matrix. The illustration indicates that the characteristics of the formulated lower second order model closely follow that of the given higher order system. Table 2 reveals that the proposed scheme yields better value for integral square error in comparison with other techniques [21-23]. Figure 2-5 depicts that the unit step time response of the proposed lower second order system maintains the original characteristics of the given sixth order system.

The proposed IWSA algorithm is coded in Intel Pentium Processor 4.0, 2.8 GHz, 256 MB RAM and it took 16 seconds by the CPU for the complete simulation of 120 generations.

8. Conclusion

It has been established that the proposed approach is suitable for formulating a lower order MIMO model that retains the characteristic features of a given absolutely stable higher order linear time invariant MIMO continuous system. The lower order MIMO model for the individual transfer functions of the MIMO transfer function matrix are formulated using the proposed average approach and the parameters transient gain and steady state gain. The proposed IWSA systematically searches for the global optimum solution for the parameters of lower order model. The IWSA approach helps in minimizing the integral square error of the formulated lower order model with respect to the given higher order system. The advantage of this approach is that it guarantees an absolutely stable lower order model provided the given original higher order system is absolutely stable. This approach can also be extended for well-known discrete system problems. The obtained lower order model can be further used for designing suitable controllers and state space observers for the given higher order system.

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