Modified Function Projective synchronization of Modified Lü dynamical system

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Abstract: This work investigates modified function projective synchronization between two identical modified Lü system using nonlinear control. The numerical simulations are presented to show the effectiveness of the proposed schemes.
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1. Introduction

Since Pecora and Carrol (Pacora and Carroll, 1990; Carroll and Pacora, 1991) introduced a method to synchronize two identical chaotic systems with different initial conditions, chaos synchronization has gained a lot of attention among scientists due to its important applications in secure communication, chemical and biological systems, human heartbeat regulation, etc. A variety of methods and techniques have been proposed for the synchronization of chaotic systems such as drive-response synchronization (Pacora and Carroll, 1990; Carroll and Pacora, 1991; Wu et al.,2006), linear and nonlinear feedback synchronization (Huang et al.,2004; Wu et al.,2007), coupled synchronization (Lü et al.,2002; Damei et al.,2005), impulsive synchronization (Quansheng and Jianye, 2006; Luo, 2008), adaptive synchronization (Yonghui and Jinde, 2008; Manfeng and Zhenyuan; 2008), generalized synchronization (GS) (Wang and Guan, 2006; Li, 2007, El-Dessoky and Salah, 2011), etc. In 2007, Li considered a new generalized synchronization (GS) method, called modified projective synchronization (MPS), where the responses of the synchronized dynamical states synchronize up to a constant scaling matrix.

Recently, Park (Park, 2007a, 2007b) proposed an AMPS scheme, named adaptive modified projective synchronization, to acquire a general kind of proportional relationship between the drive and response systems with uncertain parameters. More recently, Chen et al. (Chen et al., 2008) extended the modified projective synchronization and raised a new projective synchronization, called function projective synchronization, where the responses of the synchronized dynamical states synchronize up to a scaling function factor.

Projective synchronization is such that the drive and response system could be synchronized upto a scaling factor where as in modified projective synchronization(MPS) the responses of the synchronized dynamical states synchronize up to a constant scaling matrix. Recently a more general form of projective synchronization called function projective synchronization (FPS) in which drive and response systems are synchronized up to a desired scaling function has attracted much attention of scientists and engineers as it provides more secure communication in applications to secure communication. Up to now, there have only been a few papers investigating the FPS method (Chen and Li, 2007; An and Chen, 2008; Du et al., 2008, Luo and Wei, 2009).

In this paper we propose modified function projective synchronization (MFPS) between two identical chaotic systems with known parameters. To illustrate the effectiveness of the proposed MFPS scheme the modified function projective synchronization between two identical modified Lü chaotic is investigated using nonlinear control method. The method is illustrated by applications to continuous chaotic systems and the simulation results demonstrate the effectiveness of the proposed control method.

The organization of this paper is as follows. Sections 2 discusses modified function projective synchronization scheme. Section 3, we present the modified function projective synchronization between two identical modified Lü systems. Section 4, numerical example is given to demonstrate the effectiveness of the proposed method. Finally some concluding remarks are given in Section 5.

2. Modified Function projective synchronization (MFPS) scheme

Consider the following chaotic system:

$$\dot{x} = f(x, t) \qquad (1)$$

$$\dot{y} = g(y, t) + u(x, y, t) \qquad (2)$$

where $x,\ y \in R^n$ are the state vector of the systems (1) and (2), respectively ;

$f,\ g \in R^n \times R \to R^n$ are two continuous nonlinear vector functions, $u(x,\ y,\ t)$ is the vector control input. We define the error dynamical system as

$$e(t) = y - M\,h(t)\,x \qquad (3)$$

where M is a constant diagonal matrix $M = diag\{m_1, m_2, ..., m_n\} \in R^{n\times n}$ and $h(t)$ a continuous differentiable function with $h(t) \neq 0$ for all *t*. The system (1) and (2) are said to be in modified function projective synchronization, if there exists a constant diagonal matrix M and function $h(t)$,such that $\lim_{t\to\infty} |e(t)| = 0$.

It is easy to see that the definition of modified function projective synchronization encompasses function projective synchronization when the scaling matrix M equals I .

3. Modified Function projective synchronization (MFPS) of modified Lü system

The Lü system (Lü and Chen, 2002) is described by:

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= by - xz \\ \dot{z} &= -cz + xy \end{aligned} \qquad (4)$$

Based on Lü system, by adding a cross-product term to the first equation of the Lü system, a new modified Lü (Guangyi et al., 2006) system is attained and given by:

$$\begin{aligned} \dot{x} &= a(y - x + yz) \\ \dot{y} &= by - xz \\ \dot{z} &= -cz + xy \end{aligned} \qquad (5)$$

where $(x,\ y,\ z) \in R^3$ and *a, b* and *c* are real constant parameters. When *a=35, b=14* and *c=5*, system (5) has a chaotic attractor see Fig. 1.

The divergence of the flow (5) is given by:

$$\nabla.F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = -a + b - c < 0,$$

where

$$\begin{aligned} F &= (F_1, F_2, F_3) \\ &= (a(y - x + y\,z), b\,y - x\,z, -c\,z + x\,y). \end{aligned}$$

Hence the system is dissipative under the condition $b < a + c$.

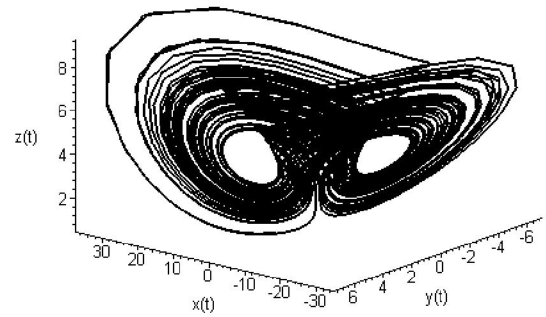

Figure 1: The chaotic attractor of modified Lü system at *a=35, b=14* and *c=5* in 3-dimensional.

In this section, we will study the MFPS of two identical modified Lü chaotic systems. For simplifying the problem, we denote the three state variables of the drive system by the subscript 1, and the response system by the subscript 2. Our aim is to design a controller and make the response system trace the drive system and become ultimately the same. The modified Lü system, as a drive system, is described by the following equations:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1 + y_1 z_1) \\ \dot{y}_1 &= by_1 - x_1 z_1 \\ \dot{z}_1 &= -cz_1 + x_1 y_1 \end{aligned} \qquad (6)$$

and the response system is described by the following equations:

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2 + y_2 z_2) + u_1 \\ \dot{y}_2 &= by_2 - x_2 z_2 + u_2 \\ \dot{z}_2 &= -cz_2 + x_2 y_2 + u_3 \end{aligned} \qquad (7)$$

where $u = [u_1 \quad u_2 \quad u_3]^T$ is the controller function to be designed. The aim of this section is to determine the controller u for the MFPS of the drive and response systems.

According to the MFPS scheme presented in the previous section, the error dynamical system between the drive system (6) and response system (7) can be expressed by

$$e_x = x_2 - m_1\,h(t)\,x_1, e_y = y_2 - m_2\,h(t)\,y_1$$
$$and\; e_z = z_2 - m_3\,h(t)\,z_1$$

thus, the error dynamical system between systems (5) and (6) is

$$\begin{aligned}\dot e_x &= a(e_y - e_x) + a y_2 z_2 - m_1\,h(t) a\,y_1 z_1 - m_1\,x_1\,\dot h(t) + u_1\\ \dot e_y &= b e_y - x_2 z_2 + m_2\,h(t) x_1 z_1 - m_2\,y_1\,\dot h(t) + u_2\\ \dot e_z &= -c e_z + x_2 y_2 - m_3\,h(t) x_1 y_1 - m_3\,z_1\,\dot h(t) + u_3\end{aligned} \quad (8)$$

Referring to the original methods of active control, so we choose the three control functions u_i, (*i=1, 2, 3*) as follows:

$$\begin{aligned}u_1 &= m_1\,h(t) a\,y_1 z_1 + m_1\,x_1\,\dot h(t) - a y_2 z_2 - a e_y\\ u_2 &= x_2 z_2 - m_2\,h(t) x_1 z_1 + m_2\,y_1\,\dot h(t) - 2 b e_y \quad (9)\\ u_3 &= m_3\,h(t) x_1 y_1 + m_3\,z_1\,\dot h(t) - x_2 y_2\end{aligned}$$

then the error dynamical system (8) is described by

$$\begin{aligned}\dot e_x &= -a e_x\\ \dot e_y &= -b e_y \quad (10)\\ \dot e_z &= -c e_z\end{aligned}$$

For this particular choice, the three eigenvalues of the closed loop system (10) are *-a, -b* and *-c*. Based on stability criterion of linear systems, this choice will lead to the error states e_x, e_y *and* e_z converge to zero as time *t* tends to infinity and hence the modified function projective synchronization (MFPS) between two identical modified Lü chaotic systems is achieved.

4. Numerical Results

By using Maple 12, we select the parameters of the modified Lü as *a=35, b=14* and *c=5*. The initial values of the drive system and response system are taken as $x_1(0) = 1.2, y_1(0) = 2.4, z_1(0) = 4,$ $x_2(0) = 3, y_2(0) = 1\,and\; z_2(0) = -1$ respectively. If we take $m_1 = 2, m_2 = 3, m_3 = 4$ and the scaling function $h_1 = 10 + 3\sin(0.2\pi t)$. Then the modified function projective synchronization (MFPS) between two identical modified Lü system are shown in Figure 2.

If we take $m_1 = m_2 = m_3 = 1$ and the scaling function $h_1 = 10 + 3\sin(0.2\pi t)$. Then the function projective synchronization (FPS) between two identical modified Lü system are shown in Figure 3.

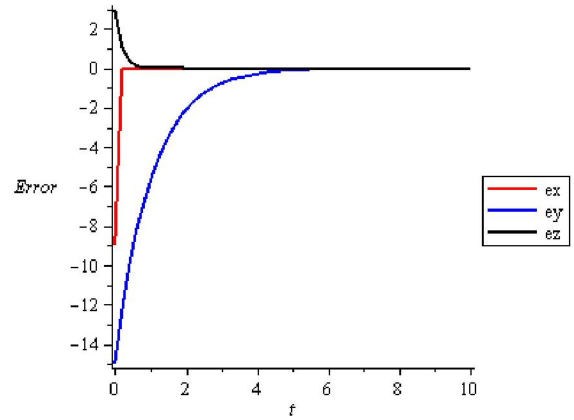

Figure 2: The behaviour of the trajectories e_x, e_y *and* e_z of the error system tends to zero for the modified function projective synchronization (MFPS).

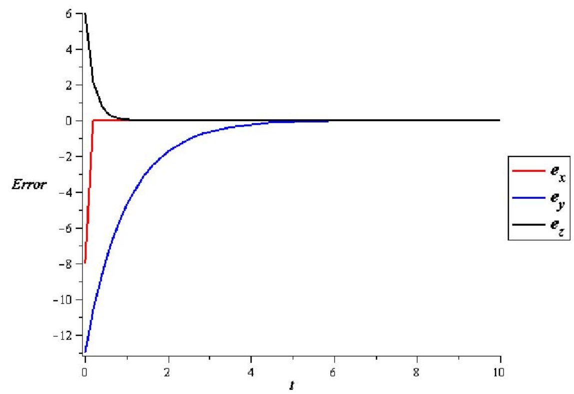

Figure 3: The behaviour of the trajectories e_x, e_y *and* e_z of the error system tends to zero for the function projective synchronization (FPS).

If We take $m_1 = m_2 = m_3 = -1$ and the scaling function is $h_1 = 10 + 3\sin(0.2\pi t)$. Then the function projective anti-synchronization between two identical modified Lü system are shown in Figure 4.

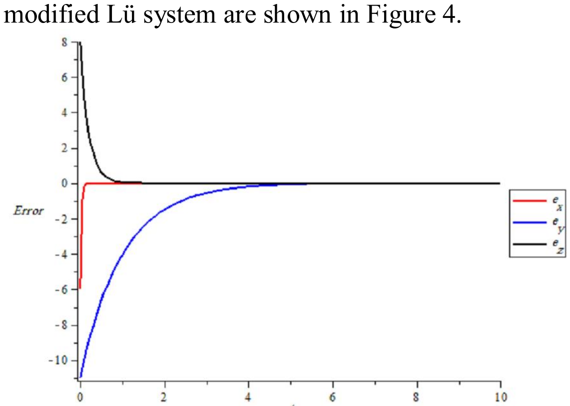

Figure 4: The behaviour of the trajectories e_x, e_y *and* e_z of the error system tends to zero for the function projective anti - synchronization (FPS).

5. Conclusion

In this paper a modified function projective synchronization between two identical chaotic systems with known parameters is demonstrated. The proposed scheme is successful in achieving modified function projective synchronization of modified Lü chaotic dynamical system and can be applied to similar chaotic systems. Numerical simulations are used to verify the effectiveness of the proposed control techniques.

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