Modified FOC-based Root-MUSIC algorithm for DOA estimation of coherent signal groups

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Abstract: Conventional multiple signal classification (MUSIC) and Root-MUSIC algorithms cannot resolve coherent sources. In this paper, a novel Root-MUSIC algorithm using the idea of modified MUSIC is proposed which overcomes shortcoming of conventional Root-MUSIC algorithm. It is based on fourth order cumulant (FOC) for direction of arrival (DOA) estimation of coherent signal groups. Several simulations are made to evaluate the performance of proposed algorithm. Two groups of coherent signals are considered that each group contains two signals. Numerical results show that the proposed algorithm can resolve sources with different QAM modulation sizes (4 and 16). It also shows that in wide range of signal to noise ratios, angular root mean square error (RMSE) performance metric of proposed algorithm is lower than modified MUSIC algorithm.

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1. Introduction

The problem of estimating direction of arrival (DOA) of coherent signals is an important subject in array signal processing. High resolution methods, such as multiple signal classification (MUSIC), fail to resolve coherent sources, because the source covariance matrix does not satisfy the full rank condition. Thereby, to decorrelate coherent sources, some preprocessing techniques are proposed [1]. In smoothing preprocessing scheme, such as forward backward spatial smoothing (FBSS), original array is divided into several overlapping arrays. Then, covariance matrix is computed for each subarray. Finally, the average of covariance matrices associated with high resolution methods is used to resolve sources [2]. However, the number of sensors must be more than the number of signals [1]. In deflation approach, first the effect of uncorrelated sources is eliminated and then it constructs Toeplitz matrix for DOA estimation of coherent signals [3]. However, its performance degrades due to its significant loss of array aperture [4].

Many other methods have also been proposed for DOA estimation of coherent signals, such as the methods based on fourth order cumulants. Method in [5] first, estimate steering vectors blindly. Then, for each steering vector performs modified forward-backward linear prediction (MFBLP) technique. Combination of joint approximate diagonalization of eigenmatrices (JADE) algorithm and MFBLP algorithm is utilized in [6]. However the methods in [5] and [6] require the large number of snapshots. In [7], JADE and MUSIC algorithms are combined. Modified MUSIC based on fourth order cumulant (FOC) is presented in [8]. However, computational complexity due to spatial spectrum calculation and peak searching of proposed methods in references [7] and [8] is very high.

The Root-MUSIC algorithm [9] is similar to MUSIC algorithm in many aspects except that the DOA is computed by roots of a polynomial formed by the noise subspace which are closer to unit circle. It other words, it does not need to peak searching in power spectrum. However, the Root-MUSIC algorithm same as MUSIC algorithm fails to resolve coherent signals. Thereby, in this paper, we utilize method in modified MUSIC algorithm [8] to overcome this shortcoming. Modified MUSIC algorithm utilizes switching matrix to construct a new fourth order cumulants matrix. Then, it uses eigen-decomposition of sum of two cumulant matrices to estimate DOAs.

The notation $(.)^T$, $(.)^*$, $E(.)$ and $(.)^H$ denote transpose, conjugate, expectation value and conjugate transpose, respectively.This paper is organized as follows. In section II, we describe signal model. Root-MUSIC algorithm is presented in section III. In section IV, new proposed method is introduced. Monte-Carlo simulations and conclusion are presented in section V and VI, respectively.

2. Signal Model

To begin, let us assume that N signals, $s_i(k);\ i = 1,2,...,N$ impinge on an M-element antenna array from direction θ_i. Then, the received signal is

$$\mathbf{X}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{n}(k), \qquad k=1,2,?\ N_s$$

where N_s is the number of snapshots and $\mathbf{A}$ is the array response (manifold) matrix.

$$\mathbf{A} = [\mathbf{a}(\theta_1)\mathbf{a}(\theta_2)...\mathbf{a}(\theta_N)] \tag{2}$$

A typical steering vector is

$$\mathbf{a}(\theta_k) = [1\ e^{j\omega_k} e^{j2\omega_k}...e^{j(M-1)\omega_k}]^T \tag{3}$$

where

$$\omega_k = \frac{2\pi f d \sin\theta_k}{c} \tag{4}$$

$\mathbf{s}$ is a vector of signals

$$\mathbf{s}(k) = [s_1(k)s_2(k)...s_N(k)]^T \tag{5}$$

and $\mathbf{n}$ is a vector of additive Gaussian noise

$$\mathbf{n}(k) = [n_1(k)n_2(k)....n_M(k)]^T \tag{6}$$

We assume signals are statistically independent and they are non-Gaussian zero-mean complex random processes with symmetric distribution.
It is due to this fact that the fourth-order cumulants of Gaussian signals are zero [10,11].

3. Root- MUSIC Algorithm

The Root-MUSIC algorithm is based on MUSIC algorithm. The MUSIC spectrum is wrote as follows

$$P_{MUSIC1}^{-1} = \sum_{k=0}^{N-1}\sum_{p=0}^{N-1} e^{-j2\pi d \sin\theta} C_{kp} e^{j2\pi k d \sin\theta} \tag{7}$$

or

$$P_{MUSIC}^{-1} = \sum_{p-k=cons\tan t=l} C_l \ e^{-j2\pi(p-k)d\sin\theta} \tag{8}$$

where

$$\mathbf{C} = \mathbf{E}_n\mathbf{E}_n^H \tag{9}$$

Sum of the lth diagonal of the matrix $\mathbf{C}$ is named C_l. $\mathbf{E}_n$ is noise subspace of covariance matrix $\mathbf{R} = E\{\mathbf{X}(t)\mathbf{X}^H(t)\}$. A polynomial equivalent to P_{MUSIC}^{-1} is defined as follows

$$D(z) = \sum_{l=-N+1}^{N+1} C_l z^{-l} \tag{10}$$

The N zeros of $D(z)$ that lie on unit circle, i.e., the zeros with magnitude 1, determine the unknown frequencies ω_k in (4). Then, DOAs will be computed as follows [9]

$$\theta_k = \sin^{-1}\frac{\varpi_k c}{2\pi f d} \tag{11}$$

4. Proposed Modified Root-Music Algorithm

As mentioned above, Root-MUSIC algorithm cannot resolve coherent sources. So, the method in [8] is used to decorrelate the coherent signals. Using this idea, covariance matrix will become Toeplitz. So, proposed modified Root-MUSIC algorithm can resolve coherent sources. We now present the steps of proposed algorithm.

1) Form fourth order cumulants of $\mathbf{X}(t)$ as follows.

$$\mathbf{Cum}(x_{k_1}, x_{k_2}, x_{l_1}, x_{l_2}) = E\{x_{k_1}x_{k_2}x_{l_1}^*x_{l_2}^*\} - E\{x_{k_1}x_{l_1}^*\}E\{x_{k_2}x_{l_2}^*\} - E\{x_{k_1}x_{l_2}^*\}E\{x_{k_2}x_{l_1}^*\} \tag{12}$$

where x_{k_1} is the k_1 element in the vector $\mathbf{X}$.

2) Compute $\mathbf{Cum}_{xx}$ as follows

$$\mathbf{Cum}_{xx} = \begin{bmatrix} \mathbf{Cum}_{1,1} & \mathbf{Cum}_{1,M+1} & \cdots & \mathbf{Cum}_{1,M^2-M+1} \\ \mathbf{Cum}_{1,2} & \mathbf{Cum}_{1,M+2} & \cdots & \mathbf{Cum}_{1,M^2-M+2} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{Cum}_{1,M} & \mathbf{Cum}_{1,2M} & \cdots & \mathbf{Cum}_{1,M^2} \end{bmatrix} \tag{13}$$

where $\mathbf{Cum}_{xx}$ is $M \times M$ cumulant matrix. $\mathbf{Cum}_{1,j}$ is the $(1,j)$ element of the cumulant matrix $\mathbf{Cum}$ [5].

3) Compute matrix $\mathbf{Cum}_{new}$

$$\mathbf{Cum}_{new} = \mathbf{Cum}_{xx} + \mathbf{Cum}_{yy} \tag{14}$$

where

$$\mathbf{Cum}_{yy} = \mathbf{J}_M \mathbf{Cum}_{xx}^* \mathbf{J}_M \tag{15}$$

$\mathbf{J}_M$ is anti-diagonal matrix that all the entries on the diagonal going from lower left corner to the upper right corner are one.

4) Compute noise subspace of $\mathbf{Cum}_{new}$ matrix. then form $\mathbf{C}$ matrix with (9).

5) Form $D(z)$ with (10). Then compute θ_k with (11).

5. Simulation Results

This section presents simulation results to demonstrate the effectiveness of the proposed method. It is assumed that two coherent signal groups impinging the 8 element uniform linear array. The inter-element spacing of ULA is $\frac{\lambda}{2}$, where λ the wavelength is. Each group contains two coherent signals. The fading coefficients of the coherent signals are $[1,-0.6425+0.7266j]$ and $[1,-0.8677+0.0632j]$, respectively [7]. Noise is considered to be additive white Gaussian noise (AWGN). Simulations are organized in two sections. In both sections, new approach is compared with modified MUSIC algorithm [8].

In section A, mean value and standard deviation of estimated DOAs are used as performance measures in different quadrature amplitude modulation (QAM) sizes.

In section B, root mean square error (RMSE) is utilized as performance metric.

5.1. Performance of proposed modified Root MUSIC algorithm in different QAM modulation sizes

In examples 1-5, we consider two groups of coherent signals, $N_s = 2000$, $SNR = 10dB$ and $M = 8$

Table 1. Mean value and standard deviation of estimated DOAs of proposed modified Root-MUSIC algorithm for different QAM modulation sizes in AWGN

$$N_s = 2000,\ SNR = 10dB,\ L = 200 \text{ and } M = 8$$

Ex.	Original DOAs Group1	Original DOAs Group2	Mean value of estimated DOAs Group1	Mean value of estimated DOAs Group2	Standard deviation of estimated DOAs Group1	Standard deviation of estimated DOAs Group2	QAM Sizes Group1	QAM Sizes Group2
1	0 7	20 28	-0.06 7.00	20.01 28.00	0.59 0.45	0.13 0.08	4	4
2	0 7	20 28	-0.02 7.02	20.00 28.00	0.13 0.23	0.04 0.04	4	16
3	0 7	20 28	-0.00 7.00	19.99 27.99	0.07 0.12	0.13 0.08	16	4
4	0 7	20 28	-0.10 6.96	20.00 28.00	1.39 0.50	0.05 0.04	16	16
5	-3 9	14 25	-3.18 8.96	13.99 25.00	2.53 0.88	0.26 0.08	4	4

In each example, QAM modulation size is changed for each group and 200 Monte-Carlo runs is made. Simulation results of proposed algorithm are presented in table 1.

Table 2 shows simulation results of modified MUSIC algorithm in [8].

Tables 1 and 2 show that standard deviation of estimated DOAs depends on QAM modulation size. As table I shows, in all examples, proposed algorithm can resolve sources.

Table 2. Mean value and standard deviation of estimated DOAs of modified MUSIC algorithm [8] for different QAM modulation sizes in AWGN

$$N_s = 2000,\ SNR = 10dB,\ L = 200 \text{ and } M = 8$$

Ex.	Original DOAs Group1	Original DOAs Group2	Mean value of estimated DOAs Group1	Mean value of estimated DOAs Group2	Standard deviation of estimated DOAs Group1	Standard deviation of estimated DOAs Group2	QAM Sizes Group1	QAM Sizes Group2
1	0 7	20 28	-0.01 7.07	20.01 27.99	4.92 0.74	0.13 0.11	4	4
2	0 7	20 28	-10.71 9.20	21.84 29.99	22.38 7.57	3.83 8.92	4	16
3	0 7	20 28	-0.00 6.99	20.01 28.02	0.17 0.27	0.21 0.16	16	4
4	0 7	20 28	0.05 7.08	20.03 28.24	0.55 1.17	0.57 3.54	16	16
5	-3 9	14 25	-4.40 8.77	14.20 25.46	7.69 2.42	1.63 4.67	4	4

Table II shows that modified MUSIC algorithm [8] cannot resolve sources in examples 2. These tables also show that standard deviation of estimated DOAs of proposed modified Root-MUSIC algorithm is lower than standard deviation of estimated DOAs of modified MUSIC algorithm.

In other words, performance of proposed modified Root-MUSIC algorithm is higher than modified MUSIC algorithm.

5.2. Performance of proposed algorithm considering RMSE metric

Assume two coherent signal groups from $[-3^{\circ},9^{\circ}]$ and $[14^{\circ},25^{\circ}]$ impinging the 8 element uniform linear array. Method in [8] is selected to be comparative method. RMSE is used as performance measure. RMSE is defined as

$$RMSE = \sqrt{\frac{1}{LN}\sum_{k=1}^{L}\sum_{i=1}^{N}(\hat{\theta}_i(k)-\theta_i)^2} \qquad (17)$$

where $\hat{\theta}_i(k)$ is the estimate of θ_i for the kth Monte Carlo trial. L is the number of Monte Carlo trials. In this simulation, 200 Monte-Carlo runs with $N_s = 2000$ are made. RMSE versus SNR is shown in Fig. 1.

As shown in Fig. 1, in wide range of SNRs, proposed modified Root-MUSIC algorithm demonstrates smaller RMSE with respect to modified MUSIC.

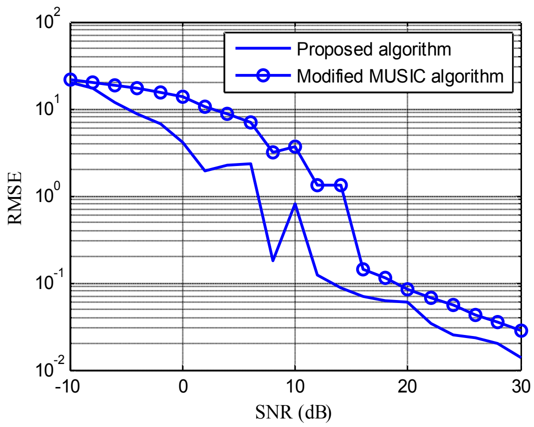

Figure 1. RMSE versus SNR

6. Conclusion

In this investigation, a new Root-MUSIC algorithm based on FOC is proposed. Idea of modified MUSIC is utilized to resolve coherent sources. We have made several simulations to investigate the performance of proposed algorithm. In all simulations, two groups of coherent signals are considered that each group contains two signals. Simulation results show that proposed algorithm can resolve sources with different QAM modulation sizes (4 and 16). It also shows that in wide range of SNRs, performance of proposed modified Root-MUSIC algorithm is better than modified MUSIC algorithm. It should be noted that besides higher performance of proposed modified Root-MUSIC algorithm, it needs lower computational complexity with respect to modified MUSIC.

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