Darboux Helices in Minkowski space R_1^3

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Abstract: In the present study, we give the conditions for a curve in the Minkowski space to be a Darboux helix. We show that α is a Darboux helix if there exists a fixed direction d in R_1^3 such that the function $\langle W(s), d\rangle$ is constant. We give the relation between slant helice and Darboux helice. As a particular case, if we take $\|w\| = \text{constant},$ the curves are constant precession. Some more particular cases of constant precession curves are studied.

[Şenol A, Zıplar E, Yayli Y, Ghadami R. **Darboux Helices in Minkowski space** R_1^3**.** *Life Sci J* 2012;9(4):5905-5910] (ISSN:1097-8135). http://www.lifesciencesite.com. 884

Keywords: Minkowski 3-space, Frenet equations, Helices, Slant helices, Curves of constant precession, Darboux vector

1. Introduction

Let R_1^3 denote the 3-dimensional Lorentz space (Minkowski 3-space), i.e. the Euclidean 3-space R^3 with Lorentzian inner product defined by
$\langle x, y\rangle = x_1y_1 + x_2y_2 - x_3y_3$ where $x, y \in R^3$.

A vector $x = (x_1, x_2, x_3) \in R_1^3$ is called space-like if $\langle x, x\rangle > 0$ or $x = 0$, time-like if $\langle x, x\rangle < 0$ and null if $\langle x, x\rangle = 0$ for non-zero x.

A curve $\alpha : I \subset R \to R_1^3$ is said to be spacelike, timelike and null if all of its velocity vectors $\alpha'(t)$ are spacelike, timelike and null. When $|\alpha'(s)| = 1$, α is arc-lenght parameterized or unit speed curve.

Let α be a unit speed curve in Minkowski space R_1^3. Then, it is possible to define a Frenet frame $\{T(s), N(s), B(s)\}$ at every point s [5,6,8]. Here T, N and B are the tangent, normal and binormal vector field, respectively. The geometry of the curve α can be described by the differentiation of the Frenet frame, which leads to the corresponding Frenet equations.

The norm of vector x is defined as

$$\|x\| = \sqrt{|\langle x, x\rangle|}$$

The Lorentzian sphere and hyperbolic sphere of radius 1 in R_1^3 are given by

$$S_1^2 = \{x = (x_1, x_2, x_3) \in R_1^3 \ / \ \langle x, x\rangle = 1\}$$

and

$$H_0^2 = \{x = (x_1, x_2, x_3) \in R_1^3 \ / \ \langle x, x\rangle = -1\}$$

respectively. In differential geometry, a curve of constant slope or general helix in Euclidean 3-space R^3 is defined by the property that the tangent makes a constant angle with a fixed straight line (the axis of general helix). Helices are characterized by the fact that the ratio $\frac{\tau}{\kappa}$ is constant along the curve, where τ and $\kappa \neq 0$ denote the torsion and curvature, respectively [2] · In Minkowski space R_1^3, one defines a helix in a similar fashion. Several authors introduce different types of helices and investigate their properties. For instance, Izumiya and Takeuchi define in [3] slant helices by the property that the principal normal makes a constant angle with a fixed direction. Moreover, they have characterized that α is a slant helix if and only if the function $\frac{\kappa^2}{(\kappa^2+\tau^2)^{3/2}}\left(\frac{\tau}{\kappa}\right)'$ is constant. Kula &Yayl investigate spherical images of tangent indicatrix of binormal indicatrix of slant helix and they have shown that spherical images are spherical helix [7].

In [9], we give that α is Darboux helix with $\frac{\tau}{\kappa} \neq 0$ if its Darboux vector makes a constant angle with a fixed direction d. That means $\langle W, d\rangle$ is constant along the curve, where d is a unit vector field in R_1^3, $W = \tau T + \kappa B$ and the direction of the vector d is the axis of the Darboux helix. We have characterized that a curve is a Darboux helix if and only if the function $\frac{\left(\tau^2+\kappa^2\right)^{\frac{3}{2}}}{\kappa^2}\frac{1}{\left(\frac{\tau}{\kappa}\right)'}$ is constant · A unit speed curve α is called a slant helix if there exist a non-zero constant vector field U in R_1^3 such that the function $\langle N(s),U\rangle$ is constant [1]. On the other hand, Ali&Lopez give the following characterization of slant helices.

Theorem 1. Let α be a unit speed time-like curve in R_1^3. Then α is a slant helix if and only if either one the next two functions

$$\frac{\kappa^2}{\left(\tau^2-\kappa^2\right)^{3/2}}\left(\frac{\tau}{\kappa}\right)' \text{ or } \frac{\kappa^2}{\left(\kappa^2-\tau^2\right)^{3/2}}\left(\frac{\tau}{\kappa}\right)'$$

is constant everywhere $\tau^2-\kappa^2$ does not vanish [1].

Theorem 2. Let α be a unit speed space-like curve in R_1^3.

i) if the normal vector of α is space-like, then α is a slant helix if and only if either one the next two functions

$$\frac{\kappa^2}{\left(\tau^2-\kappa^2\right)^{\frac{3}{2}}}\left(\frac{\tau}{\kappa}\right)' \text{ or } \frac{\kappa^2}{\left(\kappa^2-\tau^2\right)^{\frac{3}{2}}}\left(\frac{\tau}{\kappa}\right)'$$

is constant everywhere $\tau^2-\kappa^2$ does not vanish.

ii) If the normal vector of α is time-like, then α is a slant helix if and only if the function

$$\frac{\kappa^2}{\left(\tau^2+\kappa^2\right)^{3/2}}\left(\frac{\tau}{\kappa}\right)'$$

is constant [1].

The purpose of the present paper is to give a similar characterization for Darboux helices in Minkowski 3-space. As a byproduct, we show that a curve in R_1^3 is a slant helix if and only if it is a Darboux helix.

2. Time-Like Darboux helices

Let α be a unit speed timelike curve in R_1^3. The Frenet frame $\{T,N,B\}$ of α is given by

$$T(s)=\alpha'(s),\quad N(s)=\frac{\alpha''(s)}{\|\alpha''(s)\|},\quad B(s)=T(s)\Lambda N(s)$$

where Λ is the Lorentzian cross product. In this trihedro, T is timelike vector, N and B are spacelike vectors. For these vectors, we have $T\Lambda N=B$, $N\Lambda B=-T$, $B\Lambda T=N$. Then we will use the Frenet equations

$$T'(s)=\kappa(s).N(s)$$
$$N'(s)=\kappa(s).T(s)+\tau(s).B(s)$$
$$B'(s)=-\tau(s).N(s)$$

where κ and τ stand for the curvature and torsion of the curve, respectively. When the curve α is timelike, we define the Darboux vector W as $W=\tau T+\kappa B$ when $\langle W,W\rangle=\kappa^2-\tau^2$. If we take the norm of the Darboux vector, we find $\|W\|=\sqrt{\left|\kappa^2-\tau^2\right|}$ satisfying $W\Lambda T=T', W\Lambda N=N', W\Lambda B=B'$.

Case 1. We assume that W is spacelike then, $\kappa^2-\tau^2>0$. Now we write the unit Darboux vector W_0

$$W_0=\frac{W}{\|W\|}=\frac{\tau}{\sqrt{\kappa^2-\tau^2}}T+\frac{\kappa}{\sqrt{\kappa^2-\tau^2}}B$$

$$W_0=\mathrm{sh}\phi T+\mathrm{ch}\phi B$$

Since $\langle W_0,W_0\rangle>0$, W_0 is spacelike vector. If we take W_0 as unit space-like vector, then it defines a curve on the Lorentzian unit sphere S_1^2.

If we called the spherical image as β, $\beta(s_c)=W_0(s)=\mathrm{sh}\phi T+\mathrm{ch}\phi B$ where s_c is the arc parameter of β .

$$\frac{d\beta}{ds_c}=\frac{d\beta}{ds}\frac{ds}{ds_c}$$

By taking the derivative on both sides with respect to s, we can write:

$$\frac{d\beta}{ds_c}=\left(\phi'\mathrm{ch}\phi T+\phi'\mathrm{sh}\phi B+\kappa\mathrm{sh}\phi N-\tau\mathrm{ch}\phi N\right)\frac{ds}{ds_c}$$

$$\beta_{s_c}=\frac{d\beta}{ds_c}=\left(\phi'\mathrm{ch}\phi T+\phi'\mathrm{sh}\phi B\right)\frac{ds}{ds_c}$$

And by taking the norm β_{s_c}, we

$$\left\|\beta_{s_c}\right\|=\left\|\left(\phi'\mathrm{ch}\,\phi T+\phi'\mathrm{sh}\,\phi B\right)\frac{ds}{ds_c}\right\|$$

$$\frac{ds}{ds_c}=\frac{1}{\phi'}$$

$$\beta_{s_c}=\mathrm{ch}\phi T+\mathrm{sh}\phi' B \qquad (2.1)$$

so, β_{s_c} is a timelike curve since $\left\langle\beta_{s_c},\beta_{s_c}\right\rangle=-ch^2\phi+sh^2\phi=-1$, Hence since the tangent $T_c=\beta_{s_c}$ of the indicatrix curve β is timelike, the curve β is timelikle. Now, we will find the curvature κ_β of the curve $\beta(s_c)$:

$$\kappa_\beta=\|\beta''\|=\left\|\beta'_{s_c}\right\|$$

$$\beta'_{s_c}=\frac{d\beta'}{dS_c}=\frac{d\beta'}{dS_c}\frac{ds}{ds_c}$$

$$\beta'_{s_c} = \left(\phi'\text{sh}\phi T + \phi'\text{ch}\phi B + \kappa\text{ch}\phi N - \tau\text{sh}\phi' N\right)\frac{1}{\phi'}$$

$$\beta'_{s_c} = \left(\text{sh}\phi T + \text{ch}\phi B + \frac{\|w\|}{\phi} N\right) \tag{2.2}$$

$$\kappa_\beta = \left\|\beta'_{s_c}\right\| = \sqrt{1 + \left(\frac{\|w\|}{\phi'}\right)^2} \tag{2.3}$$

For the curve β , $V_2 = \frac{\beta'_{s_c}}{\left\|\beta'_{s_c}\right\|} = \frac{\beta'_{s_c}}{\kappa_\beta}$ and so, we get:

$$\langle V_2, V_2 \rangle = \left\langle \frac{\beta'_{s_c}}{\kappa_\beta}, \frac{\beta'_{s_c}}{\kappa_\beta} \right\rangle = \frac{1}{\kappa_\beta^2}\left\langle \beta'_{s_c}, \beta'_{s_c} \right\rangle$$

$$= \frac{1}{\kappa_\beta^2}\left[1 + \left(\frac{\|w\|}{\phi'}\right)^2\right]$$

Since $\langle V_2, V_2 \rangle \geq 0$, V_2 must be spacelike. And so,

$$\kappa_\beta^2 = 1 + \left(\frac{\|w\|}{\phi'}\right)^2$$

Curvatures of curve on surface satisfy the following relation

$$\kappa_\beta^2 = \kappa_g^2 + 1$$

Then

$$\kappa_g = \frac{\|w\|}{\phi'} \tag{2.4}$$

On the other hand, taking the derivative of th

$$\phi = \frac{\tau}{\kappa},\ \phi'.\left(1 - \text{th}^2\phi\right) = \left(\frac{\tau}{\kappa}\right)',$$

$$\phi' = \left(\frac{\kappa^2}{\kappa^2 - \tau^2}\right)\left(\frac{\tau}{\kappa}\right)' \tag{2.5}$$

Hence, by using the equations (2.4) and (2.5), we get:

$$\kappa_g = \frac{\sqrt{\kappa^2 - \tau^2}}{\left(\frac{\kappa^2}{\kappa^2 - \tau^2}\right)\left(\frac{\tau}{\kappa}\right)'},\ \kappa_g = \frac{\left(\kappa^2 - \tau^2\right)^{3/2}}{\kappa^2}\frac{1}{\left(\frac{\tau}{\kappa}\right)'},$$

where $\|w\| = \sqrt{\kappa^2 - \tau^2}$.

If the spherical indicatrix of the darboux vector W is a Lorentzian circle or a part of Lorentzian circle, then the curve α is a darboux helis.

Case 2. We assume that W is **timelike** then $\kappa^2 - \tau^2 < 0$**.** Now we write the unit Darboux vector W_0 :

$$W_0 = \frac{W}{\|W\|} = \frac{\tau}{\sqrt{\tau^2 - \kappa^2}} T + \frac{\kappa}{\sqrt{\tau^2 - \kappa^2}} B$$

$$W_0 = \text{ch}\phi T + \text{sh}\phi B$$

Since $\langle W_0, W_0 \rangle < 0$, W_0 is time-like vector. If we take W_0 as unit time-like vector, then it defines a curve on the hyperbolic unit sphere H_0^2. If we called the spherical image as β,

$$\beta(s_c) = W_0(s) = \text{ch}\phi T + \text{sh}\phi B$$

where s_c is the arc parameter of β .

$$\frac{d\beta}{ds_c} = \frac{d\beta}{ds}\frac{ds}{ds_c}$$

By taking the derivative on both sides with respect to s , we can write:

$$\frac{d\beta}{ds_c} = \left(\phi'\text{sh}\phi T + \phi'\text{ch}\phi B + \kappa\text{ch}\phi N - \tau\text{sh}\phi N\right)\frac{ds}{ds_c}$$

$\beta_{s_c} = \frac{d\beta}{ds_c} = \left(\phi'\text{sh}\phi T + \phi'\text{ch}\phi' B\right)\frac{ds}{ds_c}$ and by taking the norm β_{s_c}, we have,

$$\left\|\beta_{s_c}\right\| = \left\|\left(\phi'\text{ch}\phi T - \phi'\text{sh}\phi B\right)\frac{ds}{ds_c}\right\|$$

$$\frac{ds}{ds_c} = \frac{1}{\phi'},$$

$$\beta_{s_c} = \text{sh}\phi T + \text{ch}\phi B \tag{2.6}$$

so, β_{s_c} is a spacelike curve since $\left\langle \beta_{s_c}, \beta_{s_c} \right\rangle = -sh^2\phi + ch^2\phi = 1$, Hence since the tangent $T_c = \beta_{s_c}$ of the indicatrix curve β is spacelike, the curve β is spacelike. Now, we will find the curvature κ_β of the curve $\beta(s_c)$:

$$\kappa_\beta = \left\|\beta'_{s_c}\right\|$$

$$\beta'_{s_c} = \frac{d\beta}{ds_c} = \frac{d\beta}{ds}\frac{ds}{ds_c}$$

$$\beta'_{s_c} = \left(\phi'\text{ch}\phi T + \phi'\text{sh}\phi B + \kappa\text{sh}\phi N - \tau\text{ch}\phi' N\right)\frac{1}{\phi'}$$

$$\beta'_{s_c} = \left(\text{ch}\phi T + \text{sh}\Phi B - \frac{\|w\|}{\phi} N\right) \tag{2.7}$$

$$\kappa_\beta = \left\|\beta'_{s_c}\right\| = \sqrt{-1 + \left(\frac{\|w\|}{\phi'}\right)^2} \tag{2.8}$$

For the curve β , $V_2 = \frac{\beta'_{s_c}}{\left\|\beta'_{s_c}\right\|} = \frac{\beta'_{s_c}}{\kappa_\beta}$ and so, we get:

$$\langle V_2,V_2\rangle = \left\langle \frac{\beta'_{s_c}}{\kappa_\beta},\frac{\beta'_{s_c}}{\kappa_\beta}\right\rangle = \frac{1}{\kappa_\beta^2}\left\langle \beta'_{s_c},\beta'_{s_c}\right\rangle$$

$$= \frac{1}{\kappa_\beta^2}\left[-1+\left(\frac{\|w\|}{\phi'}\right)^2\right]$$

Assume that V_2 is a spacelike,

$$\kappa_\beta^2 = -1+\left(\frac{\|w\|}{\phi'}\right)^2$$

Curvatures of curve on surface satisfy the following relation

$$\kappa_\beta^2 = \kappa_g^2 - 1$$

Then

$$\kappa_g = \frac{\|w\|}{\phi'} \tag{2.9}$$

On the other hand, taking the derivative of $\coth\phi = \frac{\tau}{\kappa}$,

$$\phi'.\left(1-\coth^2\phi\right) = \left(\frac{\tau}{\kappa}\right)'$$

$$\phi' = \left(\frac{\kappa^2}{\kappa^2-\tau^2}\right)\left(\frac{\tau}{\kappa}\right)' \tag{2.10}$$

Hence, by using the equations (2.9) and (2.10), we get:

$$\kappa_g = \frac{\sqrt{\tau^2-\kappa^2}}{\left|\left(\frac{\kappa^2}{\kappa^2-\tau^2}\right)\left(\frac{\tau}{\kappa}\right)'\right|},\ \kappa_g = \frac{\left(\tau^2-\kappa^2\right)^{3/2}}{\kappa^2}\frac{1}{\left(\frac{\tau}{\kappa}\right)'},$$

where $\|w\| = \sqrt{\tau^2-\kappa^2}$.

If the spherical indicatrix of the darboux vector W is a Lorentzian circle or a part of Lorentzian circle, then the curve α is a darboux helis.

Theorem 3. Let α be a unit speed **time-like** curve in R_1^3. Then α is a Darboux helix if and only if either one the next two functions

$$\frac{\left(\kappa^2-\tau^2\right)^{3/2}}{\kappa^2}\frac{1}{\left(\frac{\tau}{\kappa}\right)'} \text{ or } \frac{\left(\tau^2-\kappa^2\right)^{3/2}}{\kappa^2}\frac{1}{\left(\frac{\tau}{\kappa}\right)'} \text{ is constant,}$$

with $\frac{\tau}{\kappa} \neq 0$.

Similarly, when α is a space-like , the following results can be obtained easily.

Theorem 4. Let α be a unit speed **space-like** curve in R_1^3.

i) if the normal vector of α is **space-like**, then α is a Darboux helix if and only if either one the next two functions

$$\frac{\left(\tau^2-\kappa^2\right)^{3/2}}{\kappa^2}\frac{1}{\left(\frac{\tau}{\kappa}\right)'} \text{ or } \frac{\left(\kappa^2-\tau^2\right)^{3/2}}{\kappa^2}\frac{1}{\left(\frac{\tau}{\kappa}\right)'} \text{ is constant,}$$

with $\frac{\tau}{\kappa} \neq 0$.

ii) If the normal vector of α is **time-like**, then α is a Darboux helix if and only if the function $\frac{\left(\tau^2+\kappa^2\right)^{3/2}}{\kappa^2}\frac{1}{\left(\frac{\tau}{\kappa}\right)'}$ is constant, with $\frac{\tau}{\kappa} \neq 0$.

As a cosequence of our main results together with the characterization of slant helices given in [1] , we easily obtain the following resuts.

Theorem 5. Let $\alpha : I \to R_1^3$ be a curve such that $\frac{\kappa}{\tau}$ is not constant, where κ and τ are curvature of α. Then, α is a slant helice if and only if α is a Darboux helice From the previous theorem, firstly we are going to find the axis of the slant helices since a slant helice is also a darboux helice.

3. The axis of the Darboux helice (Time-Like)

Let α be a unit speed timelike curve in R_1^3. The Frenet frame $\{T,N,B\}$ of α is given by

$$T(s) = \alpha'(s),\quad N(s) = \frac{\alpha''(s)}{\|\alpha''(s)\|},\quad B(s) = T(s)\Lambda N(s)$$

where Λ is the Lorentzian cross product.satisfying $T\Lambda N = B$, $N\Lambda B = -T$, $B\Lambda T = N$. Then we will use the Frenet equations

$$T'(s) = \kappa(s).N(s)$$
$$N'(s) = \kappa(s).T(s) + \tau(s).B(s)$$
$$B'(s) = -\tau(s).N(s)$$

where κ and τ stand for the curvature and torsion of the curve, respectively. We first assume that α is a slant helix. Let d be the vector field such that the function $\langle N,d\rangle = c$ is constant. There exists a_1 and a_3 such that

$$d = a_1T + a_3B + cN \tag{3.1}$$

Then, if we take the derivative of the equation (3.1) and by using Frenet equation, we have:

$$d' = (a_1' + c.\kappa)T + (a_1\kappa - \tau a_3)N + (a_3' + c.\tau)B.$$

Since the system $\{T,N,B\}$ is linear independent, we get:

$$a_1 + c\kappa = 0$$
$$a_1\kappa - \tau a_3 = 0 \tag{3.2}$$

$$a_3' + c\tau = 0 \tag{3.3}$$

and from (3.2) and (3.1), respectively

$$a_1 = \left(\frac{\tau}{\kappa}\right).a_3 \tag{3.4}$$

$$\langle d, d\rangle = -a_1^2 + a_3^2 + \mathrm{c}^2 = \text{constant} \tag{3.5}$$

By using the equalities (3.4) and (3.5), we obtain:

$$-\left(\frac{\tau}{\kappa}\right)^2 a_3^2 + a_3^2 = \text{constant - c}^2 \tag{3.6}$$

and from the equation (3.6) we have

$$\left(\left(\frac{\tau}{\kappa}\right)^2 - 1\right) a_3^2 = m^2$$

where m^2 is constant. So,

$$a_3 = \frac{m}{\sqrt{\left(\frac{\tau}{\kappa}\right)^2 - 1}} \tag{3.7}$$

Taking the derivative in each part of the equation (3.7) and by using (3.5), we get:

$$\frac{\kappa^2}{\left(\tau^2 - \kappa^2\right)^{\frac{3}{2}}} . \left(\frac{\tau}{\kappa}\right)' = \text{constant} \tag{3.8}$$

We deduce from that the curve α is slant helice when we have d . Conversely, assume that the condition (3.8) is satisfied. In order to simplify the computations, we assume that the function (19) is constant. Define

$$d = \frac{\tau}{\sqrt{\tau^2 - \kappa^2}} T + \frac{\kappa}{\sqrt{\tau^2 - \kappa^2}} B + \mathrm{c}.N \tag{3.9}$$

A differentiation of (3.9) together the frenet equations gives $d' = 0$, that is, d is a constant vector. It can easily be seen that $d' = 0$, that is d is a constant. On the other hand, $\langle N, d\rangle =$ c and this means that α is a slant helix.

The constant direction d is the axis of both the slant helice α and the darboux helice α. These axises coincide.

Similarly, when α is spacelike , the following results can be obtained easily for axes.

Conclusion 1. i) If the normal vector of α is timelike, the axis of α is

$$d = \frac{\tau}{\sqrt{\tau^2 + \kappa^2}} T + \frac{\kappa}{\sqrt{\tau^2 + \kappa^2}} B + \mathrm{c}.N$$

ii) If the normal vector of α is spacelike:, the axis of α is

$$d = \frac{-\tau}{\sqrt{\tau^2 - \kappa^2}} T + \frac{\kappa}{\sqrt{\tau^2 - \kappa^2}} B + \mathrm{c}.N$$

4. Curves of constant precession

Letr α be a spacelike curve (The normal vector of α is timelike). Then we use the following frenet equations

$$T'(s) = \kappa(s).N(s)$$
$$N'(s) = \kappa(s).T(s) + \tau(s).B(s)$$
$$B'(s) = \tau(s).N(s)$$

where κ and τ stand for the curvature and torsion of the curve, respectively. Since $W = \tau T - \kappa B$ and $\langle W, W\rangle = \tau^2 + \kappa^2$, then W is a spacelike vector and $\|W\| = \sqrt{\tau^2 + \kappa^2}$. Recall that the centrode axis of the Frenet frame is given by

$$d = \frac{\tau}{\sqrt{\tau^2 + \kappa^2}} T + \frac{\kappa}{\sqrt{\tau^2 + \kappa^2}} B + \mathrm{c}.N$$

and

$$d = \frac{W}{\|W\|} + \mathrm{c}.N \tag{4.1}$$

where $W = \tau T + \kappa B$. From (4.1),

$$\|W\|.d = W + \|W\|\mathrm{c}.N$$

By taking $\varpi = \|W\| = \sqrt{\tau^2 + \kappa^2}$, $\varpi.d = A$ and $\varpi.$ c $= \mu$, we get $A = W + \mu.N$

If we take $\|W\|$ =constant, the darboux helice α are constant precession. We deduce from that [4] is true.

A unit speed curve of constant precession is defined by the property that its (Frenet) Darboux vector revolves about a fixed line in space with angle and constant speed. A curve of constant precession is characterized by having

$$\kappa(s) = \varpi \sin(\mu(s),$$
$$\tau(s) = \varpi \cos(\mu(s)),$$

where $\varpi \rangle 0$, μ and are constant [10].

Similarly, the following results can be obtained easily.

Conclusion 2. i) Let α be a unit speed spacelike curve (the normal vector of α is spacelike) in R_1^3 . A curve of constant precession is characterized by having

$$\kappa(s) = -\varpi \mathrm{sh}(\mu(s),$$
$$\tau(s) = \varpi \mathrm{ch}(\mu(s)),$$

where $\varpi \rangle 0$, μ and are constant.

ii) Let α be a unit speed timelike curve. A curve of constant precession is characterized by having

$$\kappa(s) = \varpi \mathrm{ch}(\mu(s),$$
$$\tau(s) = -\varpi \mathrm{sh}(\mu(s)),$$

where $\varpi \rangle 0$, μ and are constant.

Example 1. Let $\alpha(s)$ be a spacelike curve (the normal vector of α is timelike) parametrized by the vector function:

$$\alpha(s)=\left(\frac{9}{400}\sin(25s)+\frac{25}{144}\sin(9s),\frac{-9}{400}\cos(25s)+\frac{25}{144}\cos(9s),\frac{15}{136}\sin(17s)\right)$$

where $s\in[0,2\pi]$.

The spacelike curve α is rendered in the following figure 1.

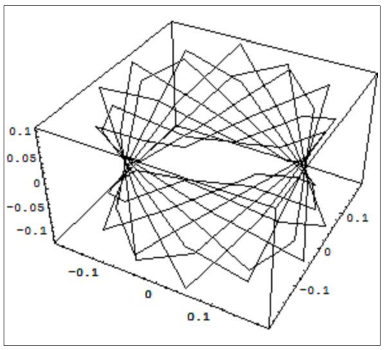

Fig 1. The spacelike curve α(s)

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12/29/2012

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