Application of the New Modified Decomposition Method to the Regularized Long-Wave Equation

H.O. Bakodah, F.A. Hendi and N.A. Al-Zaid

Department of Mathematics, Science Faculty for Girls, King Abdulaziz University, Saudi Arabia
hbakodah@kau.edu.sa, falhendi@kau.edu.sa, nalzaid@kau.edu.sa

Abstract: In this paper a new modified Adomian decomposition method (ADM) is applied to the regularized long-wave (RLW) equation which is the one of important soliton equations. The performance and the accuracy of the method are illustrated by solving some test examples. The obtained results are compared with the exact solutions.
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1. Introduction

Since Korteweg de Vries first obtained their equation(KdV) in 1885 to describe nonlinear dispersive long-wave, many other partial differential equations have been derived to model wave phenomena in diverse nonlinear systems. The (KdV) equation

$$u_t + u\,u_x - u_{xxx} = 0$$

describes propagation of shallow-water dispersive waves in the limit of small amplitudes and long waves. The (KdV) equation have been studied intensively over the last twenty years and shown to possess remarkable mathematical properties. They have a denumerable infinity of conservation laws and admit multisolitons solutions. These solutions describe the "elastic" collision of solitary waves- the so-called "soliton" property where any pair of unequal solitary remain unaltered after nonlinear interaction. For water waves, the underlying assumptions that led to the (KdV) equation, equally well justify the nonlinear wave equation

$$u_t + u_x + auu_x - bu_{xxt} = 0$$

This equation- known as the regularized long wave(RLW) or Benjamin Bona Mahony (BBM) equation- has been proposed as an alternative model governing the evolution of long waves in nonlinear dispersive system Parker, 1995 [8]. The (RLW) equation, at first, proposed by peregrine[9]. This equation plays a major role in the study of nonlinear dispersive waves Bona, 1985[3] because of its description to a larger number of important physical phenomena, such as shallow water waves and ion acoustic plasma waves. The aim goal of this paper is to solve the RLW equation by applying new modified of ADM. The ADM has been widely applied in solving nonlinear partial differential equations which represent various phenomena in engineering and physics. This method is easy to program and can be provide analytical solutions to the problems by utilizing the initial conditions. Since it was first presented in 1980's [1], ADM has led to several modifications on the method made by various researchers in an attempt to improve the accuracy or expand the application of the original method. One of the important modifications was proposed by Wazwaz[10], he divided the original function into two parts, then Wazwaz and El-Sayed, 2001 [11] presented new type of modification to ADM, the purpose of this new approach was to overcome the difficulties that arise when applying the standard ADM. Several other researchers have developed some modifications to the ADM [7,2,5]. In this work, a new modified ADM is used to solve the RLW. A comparative study between the modified ADM and the classical ADM will be presented.

2. Adomian Decomposition Method

For the purpose of illustration of the (ADM), we begin by consider Eq.(1) in the operator form

$$Lu + u_x - bR(u) + aN(u) = 0 \qquad (2)$$

Where ($L = \frac{\partial}{\partial t}$) is a linear operator and ***R*** its remainder of the linear operator. The nonlinear term is represented by ***N(u)***. Thus we get

$$Lu = -u_x + bR(u) - aN(u) = 0 \qquad (3)$$

Assuming the inverse operator L^{-1} exists and it can be taken as the definite integral with respect to ***t*** from t_0 to ***t***, i.e.

$$L^{-1} = \int_{t_0}^{t} (.)dt \qquad (4)$$

Then applying the inverse operator L^{-1} on both sides of equation (3) yields

$$u = f(x) + L^{-1}[u_x + bR(u) - aN(u)] \qquad (5)$$

Where $f(x)$ is the solution of the homogeneous equation $Lu = 0$, involving the constant of integration. The integration constants that involved in the solution of homogeneous equation are to be determined by the initial condition $u(x,0) = u_0 = f(x)$.

The ADM assumes that the unknown function $u(x,t)$ can be expressed by a sum of components defined by the decomposition series of the form

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \qquad (6)$$

with u_0 defined as $u(x,0)$ where $u(x,t)$ will be determined recursively. The nonlinear operator $N(u)$ in eq. (3) can also be decomposed by an infinite series of polynomials given by

$$N(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n) \qquad (7)$$

where A_n are the appropriate Adomian's polynomials. These polynomials are defined as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0} \qquad (8)$$

Approximate component $u_n(x,t)$ for $n \geq 1$, is computed as follows

$$u(x,0) = f(x)$$

$$u_{n+1}(x,t) = L^{-1}((u_n)_x + bRu_n - aA_n), \ n \geq 0 \qquad (9)$$

Applying the relations to (RLW) we get

$$u_0(x,t) = f(x)$$

$$u_{n+1}(x,t) = -L^{-1}(aA_n + (u_n)_x - b(u_n)_{xxt}), \ n \geq 0 \qquad (10)$$

Where A_n are given by

$$A_0 = u_{0x}u_0$$
$$A_1 = u_{0x}u_1 + u_{1x}u_0$$
$$A_2 = u_{0x}u_2 + u_{1x}u_1 + u_{2x}u_0$$

The scheme in eq. (10) can easily determine the components $u_n(x,t), n \geq 0$ and the first few components of $u_n(x,t)$ follows immediately upon setting

$$u_0(x,0) = f(x)$$
$$u_1(x,t) = -L^{-1}(aA_0 + (u_0)_x - b(u_0)_{xxt}$$
$$u_2(x,t) = -L^{-1}(aA_1 + (u_1)_x - b(u_1)_{xxt}$$
$$u_3(x,t) = -L^{-1}(aA_2 + (u_2)_x - b(u_2)_{xxt}$$

.

Consequently, one can recursively determine each individual term of the series $\sum_{n=0}^{\infty} u_n(x,t)$, and hence the solution $u(x,t)$ is readily obtained in a series form.

1. **Modified Adomian Decomposition Method**

(i) Reliable Modification

Wazwaz,1999 [10] proposed reliable modification form based on the assumption that the function $f(x)$ in (5)can be divided into two parts, i.e.

$$f(x) = f_0(x) + f_1(x)$$

Accordingly, a slight variation was proposed only on the components u_0 and u_1.

The suggestion was that only the parts f_0 be assigned to the component u_0 ,whereas the remaining part f_1 be combined with other terms given in (5)to define u_1. Consequently, the recursive relation,

$$u_0 = f_0$$

$$u_1(x,t) = f_1 - L^{-1}(aA_0 + (u_0)_x - b(u_0)_{xxt} \ ) \qquad (11)$$

$$u_{n+2}(x,t) = -L^{-1}(aA_{n+1} + (u_{n+1})_x - b(u_{n+1})_{xxt} \ )$$

Although this variation in the formation of u_0 and u_1 is slight, however it plays a major role in accelerating the convergence of the solution and in minimizing the size of calculations.

Furthermore, there is no need sometimes to evaluate the so-called Adomian polynomials required for nonlinear operators. Two important remarks related to the modified method were made by Wazwaz,1999 [10]. First, by proper selection of the function f_0 and

f_1 , the exact solution u may be obtained by using very few iterations, and sometimes by evaluating only two components. The success of this modification depends only on the choice of f_0 and f_1, and this can be made through trials, that are the only criteria which can be applied so far.

Second, if f consists of one term only, the standard decomposition method should be employed in this case.

(ii) The New Modification

As indicated earlier, although the modified decomposition method may provide the exact solution by using two iterations only, and sometimes without any need for Adomian polynomials, but its effectiveness depends on the proper choice of f_0 and f_1. In the new modification[11], Wazwaz replaces the process of dividing g into two components by a series of infinite components. He suggests that f be expressed in Taylor series

$$f = \sum_{n=0}^{\infty} f_n \qquad (12)$$

Moreover, he suggest a new recursive relationship expressed in the form

$$u_0 = f_0$$

$$u_{n+1}(x,t) = f_{n+1} - L^{-1}(aA_n + (u_n)_x - b(u_n)_{xxt}), n \geq 0 \qquad (13)$$

It is important to note that if f consists of one term only, then scheme (13) reduces to relation (10). Moreover, if f consists of two terms, then relation (13) reduces to the modified relation (11). It is easily observed that algorithm (13) reduces the number of terms involved in each component, and hence the size of calculations is minimized compared to the standard Adomian decomposition method only. Moreover, this reduction of terms in each component facilitates the construction of Adomian polynomials for nonlinear operators.

3. Application and Numerical Results

In this section, the new modified ADM described earlier will be demonstrated on illustrative examples and we compare the approximate solution obtained for our RLW equation with known exact solutions. We define u_m to be m-term approximate solution, i.e.

$$u_m = \sum_{i=0}^{m} u_i(x,t) \qquad (13)$$

u_e the exact solution and e_m the absolute error between the exact solution and the approximate solution

$$e_m = |u_e - u_m| \qquad (14)$$

Example (1)

Consider the RLW equation(1) with the initial condition

$$u(x,0) = 3c\, sech^2\big(k(x - x_0)\big) \qquad (15)$$

Where k is an arbitrary constant and

$$k = \tfrac{1}{2}\left(\frac{c}{1+c}\right)^{\frac{1}{2}}.$$

The single solitary-wave solution of RLW equation is given by

$$u(x,t) = 3c\, sech^2(k(x - (1+c)t))$$

where $c > 0$ is constant.

We use the new modified ADM to solve this equation. Tables (1-4) show the numerical results for u_m in comparison with the analytical solution when $c = 0.05$ *and* $k = 0.109$.

Table(1): Absolute errors for example(1) with c=0.05 , k=0.109 and m=4 .

$x_i \backslash t_i$	0.01	0.03	0.05
0.1	5.33982×10^{-8}	5.33982×10^{-8}	4.32171×10^{-7}
0.2	4.10852×10^{-7}	4.10852×10^{-7}	5.56933×10^{-7}
0.3	7.63759×10^{-7}	7.63759×10^{-7}	6.83084×10^{-7}
0.4	1.11084×10^{-7}	1.11084×10^{-6}	8.11842×10^{-7}
0.5	1.45080×10^{-7}	1.45076×10^{-6}	9.44397×10^{-7}

Table(2): Absolute errors for example(1) with c=0.05 , k=0.109 and m=6 .

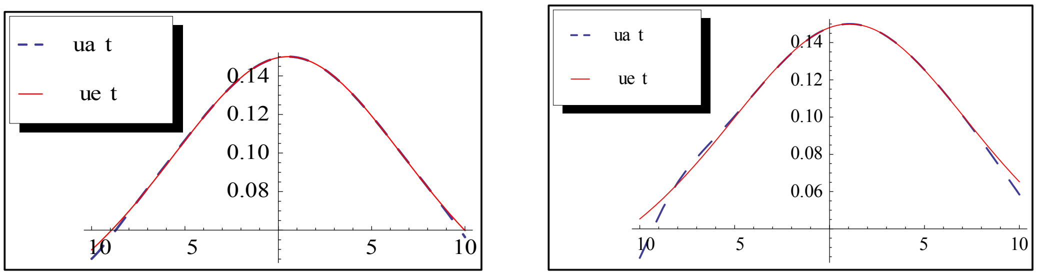
$x_i \backslash t_i$	0.01	0.03	0.05
0.1	3.74187 ×10-8	1.85816 ×10-7	4.32171 ×10-7
0.2	6.27457 ×10-8	2.61290 ×10-7	5.56933 ×10-7
0.3	8.85283 ×10-8	3.37863 ×10-7	6.83084 ×10-7
0.4	1.15012 ×10-7	4.16270 ×10-7	8.11842 ×10-7
0.5	1.42439 ×10-7	4.97230 ×10-7	9.44397 ×10-7

Table(3): Absolute errors for example(1) with c=0.05 , k=0.109 and m=10 .

t	$m=4$	$m=6$	$m=10$
0.2	2.38564 ×10-5	7.52448 ×10-6	6.29652 ×10-7
0.4	7.60588 ×10-5	2.32311 ×10-5	3.11536 ×10-6
0.6	1.56281 ×10-4	4.71745 ×10-5	5.85957 ×10-6
0.8	2.64570 ×10-4	7.99948 ×10-5	6.80166 ×10-6
1.0	4.01613 ×10-4	1.23004 ×10-4	6.64784 ×10-6

Table(4): L_2 errors for example(1) with m=4,6 and 10

t	$m=4$	$m=6$	$m=10$
0.2	2.38564 ×10-5	7.52448 ×10-6	6.29652 ×10-7
0.4	7.60588 ×10-5	2.32311 ×10-5	3.11536 ×10-6
0.6	1.56281 ×10-4	4.71745 ×10-5	5.85957 ×10-6
0.8	2.64570 ×10-4	7.99948 ×10-5	6.80166 ×10-6
1.0	4.01613 ×10-4	1.23004 ×10-4	6.64784 ×10-6



(a) (b)

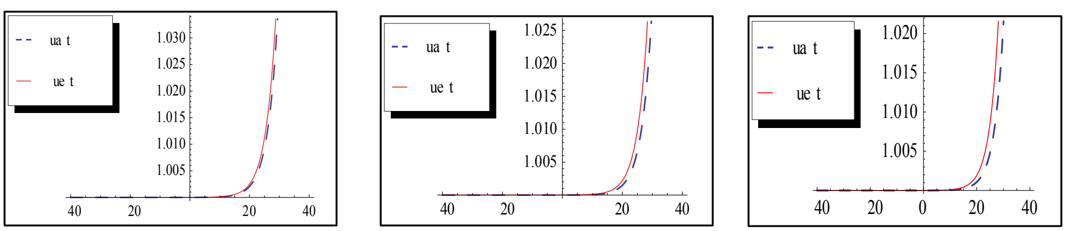
Fig.1. The exact solution and numerical solution with new modified ADM for Ex.1 , (a) for $t=0.5$ and (b) for $t=1.0$.

Table(5): Absolute errors for example(2) with c=0.1 and m=7 .

$x_i \setminus t_i$	0.5	1.0	1.5
0	1.07848 ×10-6	1.71574 ×10-6	2.05345 ×10-6
20	4.44170 ×10-4	7.08273 ×10-4	8.49419 ×10-4
40	4.06848 ×10-3	1.70003 ×10-2	4.09707 ×10-2
60	7.02596 ×10-4	1.76923 ×10-3	3.33876 ×10-3
80	1.72020 ×10-6	4.35845 ×10-6	8.28389 ×10-6

Table(6): L_2 errors for example(2) with c=0.1 and d=1.

$m \setminus t_i$	0.5	1.0	1.5
7	1.28255 ×10-6	2.04041 ×10-6	2.44203 ×10-6
5	1.28250 ×10-6	2.03736 ×10-6	2.47395 ×10-6
3	1.44136 ×10-6	3.01559 ×10-6	4.70583 ×10-6



(a) (b) (c)

Fig.2. The exact solution and numerical solution with new modified ADM for Ex.2 , (a) for $t=0.5$, (b) for $t=1.0$ and (c) for $t=1.5$.

5. Concluding remarks

In this paper, we presented the application of new modified ADM for solving RLW equation. An advantage of the method is that it displays a fast convergence of the solution. The numerical results obtained by this method have illustrated a high degree of accuracy compared with the exact solution. Tables (1-3) show the difference of analytical solution and numerical solution of the error of RLW equation. It is to be noted that numerical comparison were used in evaluating the approximate solutions of RLW equation with u_4, u_6 and u_{10} . We achieved a good approximation with the exact solution of equation(1) with initial condition, also the results are found to be in good agreement with Kaya and El-sayed, 2003 [6]. Table (5) shows the absolute error for m=7 in example 2. From the numerical results we conclude that the method gives remarkable accuracy in comparison with the analytical solution especially for small values of time t . the results are found to be in good agreement with El-Danaf et.al., 2005 [4].

Corresponding author
H.O. Bakodah
Department of Mathematics, Science Faculty for Girls, King Abdulaziz University, Saudi Arabia
hbakodah@kau.edu.sa, falhendi@kau.edu.sa, nalzaid@kau.edu.sa

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