

## Determination measure of efficiency using by undesirable inputs of DEA

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**Abstract:** In this paper, the possibility of suitable production is presented, and then a new method is suggested taking into account the existence of some undesirable components in the outputs and inputs of the Decision Making Units (DMUs) in the set.

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### 1. Introduction

When there are no undesirable input and output in the performance of DMUs, models of Data Envelopment Analysis (DEA) to increase efficiency are based on the output increase or input decrease. But many applied problems may consist of inputs whose increase and decrease results in efficiency increase and decrease, respectively, As Koopman (1951) represented. Such reclamation operation needs to increase undesirable inputs in order to increase efficiency or increase and decrease of undesirable outputs decrease and increase efficiency, respectively[7].

Suppose undesirable outputs be factory wastes that should decrease in order to increase efficiency (e.g. Allen, (1999), Smith, (1991))[1,11].

There are direct and indirect methods for consideration and using undesirable outputs in DEA. In indirect methods, undesirable inputs and outputs in every single DMU change into desirable inputs and outputs with a decreasing monotonous function. And then DMUs efficiency is evaluated using standard models of DEA. Koopmans (1951), Golany and Roll (1989) introduced [ADD] and [MLT] methods, respectively, for measuring efficiency with undesirable inputs and outputs[5-6]. In'direct methods, there are some suppositions to Production Possibility Set (PPS), so in evaluation will obtain suitable input and output [2-5].

This paper is structured as follows: section 2 gives definitions of proportionate PPS to undesirable inputs and outputs. The method for measuring efficiency with undesirable inputs and outputs is shown in section 3. Finally, an example with undesirable inputs and outputs, and then the conclusion will be given.

### 2. Production Possibility Set

Suppose we have  $n$  observations on  $n$  DMUs with input and output vectors  $(x_j, y_j)$  for  $j = 1, 2, \dots, n$ . Let

$x_j = (x_1, \dots, x_{mj})^T$  and  $y_j = (y_{1j}, \dots, y_{sj})$ . All

$x_j \in R^m$  and  $y_j \in R^s$  and  $x_j > 0$ ,  $y_j > 0$  for  $j = 1, 2, \dots, n$ . The input matrix  $X$  and output matrix  $Y$  can be represented as

$$X = [x_1, \dots, x_j, \dots, x_n], Y = [y_1, \dots, y_j, \dots, y_n]$$

Where  $X$  is an  $(m \times n)$  matrix and  $Y$  an  $(s \times n)$  matrix.

The production possibility set  $T$  is generally defined as

$$T = \{(x, y) | x \text{ can produce } y\}.$$

(1)

In DEA, the production possibility set under a Variable Return to Scale (VRS) technology is constructed from the observed data  $(x_j, y_j)$  for  $j = 1, 2, \dots, n$  as follows:

$$T = \left\{ (x, y) \left| x \geq \sum_{j=1}^n \lambda_j x_j, y \leq \sum_{j=1}^n \lambda_j y_j, \lambda_j \geq 0, \sum_{j=1}^n \lambda_j = 1, j = 1, \dots, n \right. \right\}. \quad (2)$$

In the absence of undesirable factors when a  $DMU_o$ ,  $o \in \{1, 2, \dots, n\}$ , is under evaluation, we can use the following BCC model:

$$\begin{aligned}
 & \min \theta \\
 & \text{st } \theta x_o - X\lambda \geq 0 \\
 & \quad Y\lambda \geq y_o, \\
 & \quad 1^T \lambda = 1, \\
 & \quad \lambda \geq 0.
 \end{aligned} \tag{3}$$

Corresponding to each output  $y$ ,  $L(y)$  is defined as the following:

$$L(y_j) = \{x \mid (x, y_j) \in T\} \tag{4}$$

In fact,  $L(y_j)$  is a function that  $y_j$  portrays to a subset of inputs so that inputs can produce  $y_j$ .

Now suppose that some inputs are undesirable so input matrix  $X$  can be represented

as  $X = (X^d, X^u)^T$ , where

$$X^d = (x_{1j}^d, \dots, x_{m_1j}^d), j = 1, \dots, n \text{ and}$$

$$X^u = (x_{1j}^u, \dots, x_{m_2j}^u),$$

$j = 1, \dots, n$  are  $(m_1 \times n)$  and  $(m_2 \times n)$  matrixes that represent desirable (good) and undesirable (bad) inputs, respectively. And similarly, suppose that some outputs are undesirable so outputs. Matrix  $Y$  can be

$$\forall u \in R_+^{m_1}, v \in R_+^{s_1}, (x^d, x^u, y^g, y^b) \in T \Rightarrow (x^d + u, x^u, y^g - v, y^b) \in T$$

This is not necessarily established for undesirable factors, because in this case,  $T$  has no efficient DMU.

represented as  $Y = (Y^g, Y^b)^T$ , where  $Y^g = (y_{1j}^g, \dots, y_{s_1j}^g), j = 1, \dots, n$  and  $Y^b = (y_{1j}^b, \dots, y_{s_2j}^b), j = 1, \dots, n$  are  $(s_1 \times n)$  and  $(s_2 \times n)$  matrixes that represent Desirable (good) and undesirable (bad) inputs, respectively.

**Definition 1:** Let DMU of  $(x_1^d, x_1^u, y_1^g, y_1^b)$  is dominant to DMU of  $(x_2^d, x_2^u, y_2^g, y_2^b)$  if

$x_1^d \leq x_2^d, x_1^u \geq x_2^u, y_1^g \geq y_2^g$  and  $y_1^b \leq y_2^b$  the unequal be strict at least in a component. So that,

$$\begin{pmatrix} -x_1^d \\ x_1^u \\ y_1^g \\ -y_1^b \end{pmatrix} \geq \begin{pmatrix} -x_2^d \\ x_2^u \\ y_2^g \\ -y_2^b \end{pmatrix}$$

**Definition 2:**  $DMU_o$  is efficient if in  $T$  there is no DMU to be dominant over it.

We consider the properties of the Production Possibility Set as the following:

- (1)  $T$  is convex.
- (2)  $T$  is closed.
- (3) The monotony property of desirable inputs and outputs. So that,

We can define the Production Possibility Set  $T$  satisfying (1) through (3) by

$$\begin{aligned}
 T = \left\{ (x^d, x^u, y^b, y^g) \mid x^d \geq \sum_{j=1}^n \lambda_j x_j^d, x^u = \sum_{j=1}^n \lambda_j x_j^u, y^g \leq \sum_{j=1}^n \lambda_j y_j^g, \right. \\
 \left. y^b = \sum_{j=1}^n \lambda_j y_j^b, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j = 1, \dots, n \right\}.
 \end{aligned} \tag{5}$$

### 3. Measures of Efficiency Using Undesirable Factors

In input oriented data, the efficiency of the DMU under evaluation is obtained by decreasing and increasing the desirable and undesirable input, respectively. And similarly, in output oriented data, we increase desirable output and decrease the undesirable output. Farell

(1989) introduced a model to increase and decrease desirable and undesirable output, respectively [12-13]. But there is a problem with his model and it is its nonlinear form.  $[TR\beta]$  Method introduced by Ali and Seiford (1990) simultaneously increase desirable

outputs and decrease undesirable outputs, but measures of efficiency are dependent on the  $\beta$  value[2].

There are some other methods such as [WD] and [MLT] that were introduced by Far (1989) and Galony and Roll (1989) respectively that decrease undesirable outputs only with decreasing desirable outputs[4-5]. We, however, believe that in order to improve efficiency, desirable and undesirable outputs need to be increased and decreased respectively. Suppose

and we consider the subset of  $L(y_o^g, y_o^b)$  as :

$$\partial^s L(y_o^g, y_o^b) = \{(x^d, x^u) \mid \forall (u, v) \geq 0, (u, v) \neq 0 \Rightarrow (x^d - u, x^u + v) \notin L(y_o^g, y_o^b)\} \quad (7)$$

That  $\partial^s L(y_o^g, y_o^b)$  includes all inputs of the efficient DMUs which can produce  $(y_o^g, y_o^b)$ .

The model to evaluate the efficiency of DMUo with the most decrease of  $x_o^d$  and the most increase of  $x_o^u$  is as follows:

$$d_o^d = x_o^d$$

And :

$$d_o^u = x_o^u - x_{\max}^u$$

So that :

$$(x_{\max}^u)_i = \text{Max}_j \{x_{ij}^u\}$$

Therefor in defined inefficiency as follows:

$$\theta_o^* = \text{Max} \quad \theta_o$$

st.

$$\sum_{j=1}^n \lambda_j x_j^d + s^- = x_o^d - \theta_o d_o^d$$

$$\sum_{j=1}^n \lambda_j x_j^u = x_o^u - \theta d_o^u$$

$$\sum_{j=1}^n \lambda_j y_j^g - s^+ = y_o^g$$

$$\sum_{j=1}^n \lambda_j y_j^b = y_o^b$$

$$\sum_{j=1}^n \lambda_j = 1$$

$$\lambda_j \geq 0 \quad \text{for all } j = 1, \dots, n$$

DMUo =  $(x_o^d, x_o^u, y_o^g, y_o^b)$  be unit under evaluation, corresponding to the output  $y_o = (y_o^g, y_o^b)$  and using

(2)  $L(y_o^g, y_o^b)$  in defined as follows:

$$L(y_o^g, y_o^b) = \{(x^d, x^u) \mid (x^d, x^u, y_o^g, y_o^b) \in T\} \quad (6)$$

**Theorem 1:** The DMUo ill model (8) is efficient if and only if

- 1)  $\theta_o^* = 1$
- 2) All slacks are zero for all optimal solutions.

**Theorem 2:** If all optimal solution of model (8) be  $(\theta^*, s^-)$ , then

$$(x^d - \theta^* d^d - s^-, x^u - \theta^* d^u) \in \partial^s L(y_o^b, y_o^g)$$

#### 4. Numerical example

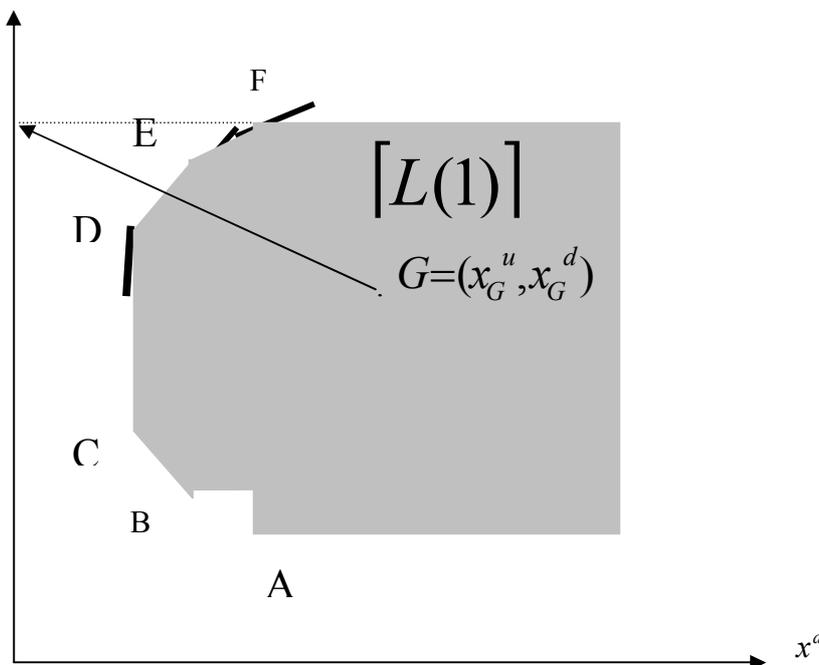
As an example, consider seven DMUs with one desirable input, one undesirable input and one desirable output.

Regarding Table 1 and Figure 1, it can be seen that DMUs D, E, and F are efficient and they are on the  $\partial^s L(y_G^g)$ . On the other hand, efficiency of other DMUs have been examined through their image on  $\partial^s L(y_G^g)$ . (Efficient Frontiers)

**Table 1.** The inputs and outputs data for 7 DMUs.

<i>DMU's</i>	$x^d$	$x^u$	$y^g$	$1 - \theta^*$
<i>A</i>	3	1	1	0.33
<i>B</i>	2	2	1	0.5
<i>C</i>	1	3	1	1
<i>D</i>	1	5	1	1
<i>E</i>	2	6	1	1
<i>F</i>	3	7	1	1
<i>G</i>	4	4	1	0.43

Similar discussion can be presented for the output oriented.



**Figure 1:** The graph of the  $L(y_G)$

**5. Conclusion**

Throughout this paper, a new model is defined for the evaluation of efficiency where some inputs and outputs may be undesirable. Also, this model assures that the DMUs under evaluation will be compared with a corresponding unit of  $\partial^s L(y_G^s)$ . (Efficient Frontiers)

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