Trigonometric based time domain constellation shaping technique for peak-to-average power ratio reduction in OFDM systems

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Abstract: In this paper, a modified technique based on constellation shaping is proposed to reduce the peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems. To accomplish significant reduction in PAPR, the energy compaction property of trigonometric transforms such as discrete cosine transform (DCT) is utilized in combination with the time domain constellation shaping technique. The application of trigonometric transforms results in OFDM signal with few energy components, which is then transformed by constellation shaping in time domain. This combined technique with extremely low computational complexity does not involve transmission of side information and is independent of OFDM input. Closed form expression for the complementary cumulative distribution function (CCDF) of PAPR of the proposed technique is derived and confirmed by simulation results. Performance of the proposed algorithm is evaluated and compared for even information bits in case of square QAM, and for odd information bits in case of cross QAM and rectangular QAM. Comparative analysis with the time domain constellation shaping and trigonometric based transform OFDM techniques shows better performance of the proposed scheme with no increase in terms of average power.
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1. Introduction

OFDM has been exploited for various high speed wireless communication and broadcasting systems. OFDM is a multi-carrier modulation scheme that allows the transmission of high bit rate data by using many subcarriers for parallel transmission. The main implementation disadvantage of OFDM signals is high peak-to-average power ratio, which tends to increase the implementation complexity of the Analog-to-Digital and the Digital-to-Analog converters, and reduce the power efficiency of high power amplifier. Large peaks in transmitted OFDM signal are clipped by the power amplifier creating non-linear distortions which degrades BER performance.

Several techniques have been proposed in the literature to mitigate the PAPR problem. Clipping reduces PAPR by limiting the peak amplitudes in the signal to desired threshold level, at the cost of nonlinear distortions and spectral spreading which can be reduced to some extent by filtering with reduced implementation complexity (Wang et al., 2005). Clipping and filtering combined with precoding method can give better results in PAPR reduction (Namitha et al., 2010). Multiple signal representation techniques such as Selective Mapping (SLM) (Han et al., 2004) and Partial Transmit Sequence (PTS) (Jiang et al., 2007) require the transmission of side information and experience an increase in the computational complexity. Several techniques have been introduced to overcome the issue of computational complexity (Lim et al., 2005; Lim et al., 2006). SLM and PTS techniques combined with error correction codes can eliminate the need for side information but causes reduction in transmission rate (Yang et al., 2003; Chen et al., 2007). The loss of data rate and transmission of side information can be avoided by employing constellation shaping techniques which reduce the PAPR by modifying the transmitted data symbols. Constellation shaping technique based on symbol predistortion extends the outer constellation points using decision metric without affecting the minimum distance and degrading performance (Sezginer et al., 2007). The main drawback is an increase in average power and dependence on the modulation format. Time domain constellation shaping technique uses a simple transformation to reduce PAPR (Dalakas et al., 2009) . This technique is independent of OFDM input and avoids the power increase by power normalization but it influences the BER performance. Recently, new techniques based on trigonometric transforms such as discrete cosine transform (DCT) have been developed to solve the PAPR problem in OFDM making use of their energy compaction property with a slight increase in complexity (Shanlin et al., 2008; Ahmad et al., 2007) . These techniques can accomplish significant PAPR reduction, but there is a need for much reduction in the PAPR of transmitted OFDM signals along with reduced implementation complexity.

In this paper a modified scheme based on constellation shaping is presented and analyzed. This new technique combines the energy compaction of trigonometric transforms with low computational complexity of time domain constellation shaping proposed in (Dalakas et al., 2009) and attains considerable PAPR reduction. The character of DCT concentrates the input data of IFFT in a few components. The application of time domain constellation shaping after IFFT further reduces the PAPR and makes this technique independent of modulation scheme. Power normalization is utilized to keep the average power of the signal unchanged.

Analytical closed form expression for the PAPR distribution of the proposed technique is derived and verified by simulations. Performance of the proposed technique is compared for square, cross and rectangular QAM for different number of bits per symbol. The effect of varying the constellation parameters like, minimum distance between constellation points and constellation energy is also explored. CCDF and bit error rate (BER) performance of the proposed technique is compared with time domain constellation shaping and trigonometric based OFDM.

2. System model and PAPR definition

OFDM is a digital multicarrier technique which divides the available bandwidth or spectrum into multiple parallel subcarriers and each is modulated with low rate data streams. An OFDM system with N subcarriers can be implemented by taking the N-point inverse discrete Fourier transform (IDFT) of a block of information data symbols followed by digital-to-analog converter. The information bits are mapped to a block of N complex data samples $\{x_n, n = 1, \ldots, N\}$, taken from an appropriate signal constellation such as quadrature amplitude modulation (QAM) or phase shift keying (PSK). The IFFT of the data block gives the discrete time domain samples $\{X_k, k= 1, \ldots, N\}$ of the OFDM given by

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp\left(j2\pi \frac{kn}{N} \right) \qquad (1)$$

Superposition of a large number of independently modulated subcarriers with same phase in OFDM signal can give large peaks. These peaks in the signal degrade the performance of OFDM system due to amplitude limiting caused by nonlinear components in transmission path. A measure of signal degradation is peak-to-average power ratio defined as

$$PAPR = \frac{\max |x_n|^2}{E\left[|x_n|^2 \right] / N} \qquad (2)$$

where $E[.]$ denotes expectation. Reducing $\max|x_n|$ is the main goal of PAPR reduction schemes. The continuous-time PAPR is obtained by taking LN point IDFT of the zero-padded input data sequence of length LN, where L is the oversampling factor (Sezginer et al., 2007). In this paper, evaluations are done for the case $L = 1$.

Complementary cumulative distribution function is a measure of PAPR efficiency of an OFDM system and is expressed as:

$$CCDF = 1 - [1 - \exp(-x^2)]^N \qquad (3)$$

3. Time domain constellation shaping

To reduce PAPR, time domain OFDM symbol $\mathbf{x}$ obtained after IFFT operation is translated into another symbol $\mathbf{y}$ by adding a peak cancelling vector which involves the multiplication of a positive real constant with phase information of OFDM symbol (Dalakas et al., 2009).

$$\mathbf{y} = \mathbf{x} + \alpha \mathbf{u} \qquad (4)$$

where α is a positive real constant and $\mathbf{u}$ is the phase information vector of OFDM symbol $\mathbf{x}$. The nth element of $\mathbf{u}$ is defined as:

$$u_n = \exp(j\theta_n), \quad 0 \le n \le N-1 \qquad (5)$$

where θ_n is the phase of x_n in radians.

PAPR due to the addition of cancelling vector should be less than the PAPR of the original OFDM symbol $\mathbf{x}$ in order to have reduced PAPR of resulting symbol y (Dalakas et al., 2009). The OFDM symbol $\mathbf{y}$ obtained after the transformation process results in reduced PAPR with an increase in the total average power. This increase in power can be avoided by normalization, keeping the PAPR reduction unchanged (Dalakas et al., 2009).

$$\mathbf{x}_{norm} = \lambda \mathbf{y} \qquad (6)$$

where λ is the normalizing factor given by

$$\lambda = \frac{P_{avg}^{x_{norm}}}{P_{av}^{y_n}} = \frac{1}{1 + \alpha^2 + \alpha\sqrt{\pi}} \qquad (7)$$

Normalization restores the total average power but it results in minimum distance reduction which degrades the BER performance. The simple transformation makes the implementation complexity of this algorithm quite low and easily reversible at the receiver.

3.1 Performance Analysis of time domain constellation shaping

The analytical expression for the PAPR of the proposed technique in terms of CCDF is expressed in

(Dalakas et al., 2009). Consider the time domain OFDM samples x_n obtained by the IFFT operation. For large value of subcarriers, both real and imaginary parts of x_n follow Gaussian distribution with mean of zero and variance of 0.5. The amplitude of x_n normalized by average power is given by

$$s_n = \frac{|x_n|}{\sqrt{P_{avg}^{x}}} \tag{8}$$

The magnitude of x_n is Rayleigh distributed with CDF as follows

$$F_{s_n}(s) = 1 - \exp(-s^2) \tag{9}$$

The CCDF distribution of PAPR is expressed (Dalakas et al., 2009) as follows

$$CCDF(s) = 1 - \left[1 - \exp\left(-\left(\frac{s-t}{r}\right)^2\right)\right]^N \tag{10}$$

where $r = \frac{\sqrt{P_{avg}^{x_n}}}{\sqrt{P_{avg}^{y_n}}}$ and $t = \frac{\alpha}{\sqrt{P_{avg}^{y_n}}}$

4. Trigonometric transforms based OFDM system

The energy packing capability of trigonometric transforms, like discrete cosine transform (DCT) can be utilized in OFDM systems for providing high PAPR reduction with low implementation cost. DCT transform of a sequence of N real data points is defined as:

$$X_k = \sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N}\left(n + \frac{1}{2}\right)k\right], \quad 0 \le k \le N-1 \tag{11}$$

Discrete sinusoidal transforms are applied in the frequency domain of OFDM to focus the energy of the signal in few samples corresponding to low special frequencies by redistributing the signal power on every subcarrier. As a result of this redistribution some subcarriers become concentrated with signal energy while others remain close to zero, keeping the signal energy after transformation unaffected (Shanlin et al., 2008; Wang et al., 2010). This decrease in signal components is the main cause of PAPR reduction in Sinusoidal transforms based OFDM system. To reduce PAPR, DCT is applied to constellation symbols. The transformed OFDM signal is then modulated by IFFT for transmission. At the receiver, inverse DCT is applied to the received symbols after FFT.

4.1 CCDF of trigonometric transforms

The complex envelope of DCT is given as

$$x_c = x_{ci} + jx_{cq} \tag{12}$$

DCT signals converge to Gaussian distributed variable for large number of subcarriers. Central chi-squared distribution with one degree of freedom is followed by power distribution (Ahmad et al., 2007). The amplitudes follow the Rayleigh distribution. CDF of DCT signal is given as

$$F_{s_c}(s) = 1 - \exp\left(-\frac{s^2}{2}\right) \tag{13}$$

CCDF for DCT signal is

$$CCDF^{(c)} = \left[1 - \exp\left(-\frac{s^2}{2}\right)\right]^N \tag{14}$$

5. The novel PAPR reduction technique

The proposed technique is an efficient PAPR reducing scheme with low implementation cost and requires no transmission of side information to the receiver. The main aim of the proposed scheme is to obtain high level PAPR reduction by combining the time domain constellation shaping with the energy focusing trigonometric transforms, as illustrated in Figure 1.

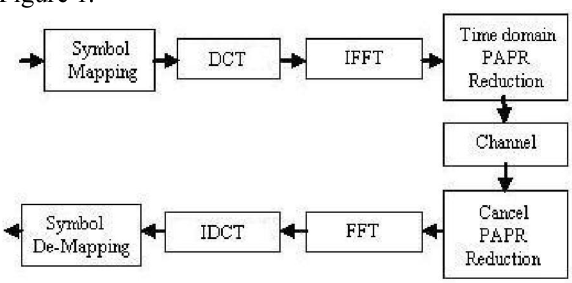

Figure 1: Block Diagram of the proposed PAPR Reduction Scheme

In the proposed technique, OFDM signal is first transformed by the sinusoidal transforms in frequency domain. DCT is a scalar transformation so in order to obtain the transformed OFDM signal DCT is applied on both the real and imaginary components of mapping symbols separately. The rearrangement of energy results in most of the subcarriers equal to zero. The application of trigonometric transforms does not influence the orthogonality of OFDM (Shanlin et al., 2008). The transformed OFDM signal is then modulated by IFFT operation to get the time domain signal. Due to the less energy components obtained by DCT operation, PAPR of the modulated OFDM signal is minimized. The time domain constellation shaping is then applied on the modulated OFDM to further reduce the PAPR.

The proposed algorithm is summarized as follows,

1. Obtain DCT transform of the input data block using (11).
2. Compute IFFT of the transformed signal using (1).
3. Choose value for α according to PAPR reduction required.
4. Compute phase vector using (5)
5. Transform the time domain OFDM signal using (4).
6. Calculate λ using (7).
7. Perform normalization using (6) to obtain the

transmitted signal.

6. Performance analysis of proposed technique

The analytical expression for the PAPR of the proposed technique in terms of CCDF is derived and confirmed by simulations. Central limit theorem is used to approximate the input data block by Gaussian distribution for large number of subcarriers.

6.1 CCDF of proposed technique

The proposed algorithm is the combination of sinusoidal transform and time domain constellation shaping. Since time domain constellation shaping takes place after modulating the transformed DCT signal, CCDF of PAPR for the proposed technique can be obtained by combining the CCDF for constellation shaping from (10) and CCDF for DCT signal from (14).

$$CCDF = 1 - \left[1 - \exp\left(-\left(\left(\frac{s-t}{r}\right) + \left(\frac{s}{2}\right)\right)^2\right)\right]^N \quad (15)$$

where $r = \frac{\sqrt{P_{avg}^{x_n}}}{\sqrt{P_{avg}^{y_n}}}$ and $t = \frac{\alpha}{\sqrt{P_{avg}^{y_n}}}$

Closed form expression for the CCDF of PAPR of the proposed technique derived in (15) is confirmed by simulation results for varying values of α. Figure 2 shows the CCDF performance of the proposed scheme with DCT transform for 16-QAM with α varying from 0 to 1.5 with step 0.5. Results verified the analytical expression derived for proposed technique since the theoretical and simulated results overlap to great extend. Simulation results also show reduction in PAPR with the increase in the value of α.

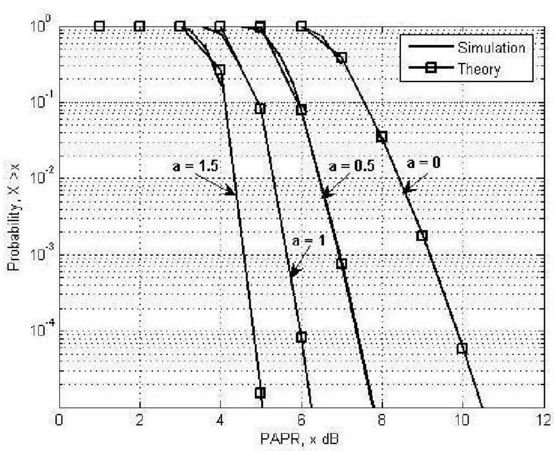

Figure 2: CCDF of DCT-Time Domain Constellation Shaping

7 Performance evaluation of proposed technique

In this section numerical results for the proposed technique are presented for even and odd number of bits per symbol. Simulations are carried out for different number of bits per symbol and comparative analysis is performed for same minimum distance and same energy. The main parameters that comprise any constellation are:

a) Number of bits per symbol to be transmitted.
b) Minimum distance between constellation points.
c) Constellation energy.

Results have been developed for the proposed technique by taking into account the effect of varying the above mentioned parameters. Comparison of the proposed scheme with time domain constellation shaping and trigonometric based OFDM is performed.

7.1 Performance evaluation for different QAM schemes

Square QAM is utilized for transmitting even number of bits per symbol while in case of odd number of bits two constellation formats namely cross QAM and rectangular QAM are used. The main difference between the two is that rectangular QAM requires more energy to transmit symbols as compared to the cross QAM. CCDF performance of DCT based proposed technique for different modulation schemes (8-QAM, 16-QAM, 32-QAM, 64-QAM, 256-QAM) with N=256 and α=1 is shown in Figure 3. The results show that PAPR of the proposed technique increases with the increase in signal constellation points.

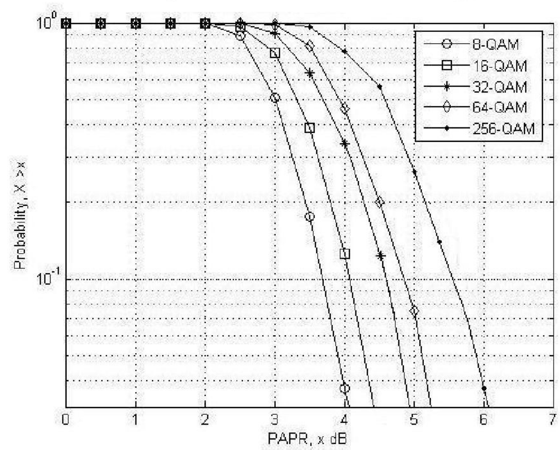

Figure 3: CCDF of the PAPR for Various QAM Constellations

7.2 Performance evaluation for varying constellation parameters

As mentioned earlier, for transmitting odd number of bits either cross QAM or rectangular QAM can be employed. Cross QAM is a preferred QAM signal constellation when the number of bits is odd since both the peak and average power can be reduced by using a cross QAM constellation compared to the rectangular QAM.

Rectangular QAM offers more constellation energy compared to cross QAM, keeping the minimum distance between the constellation points same for both the odd constellation formats (cross QAM and rectangular QAM). Comparison of cross and rectangular QAM for odd number of bits (32-QAM, 128-QAM, and 512-QAM) in terms of same minimum distance between constellation points for DCT based proposed technique is presented in Figure 4. It is observed that PAPR reduction in case of cross QAM is more as compared to the rectangular QAM due to high initial energy of rectangular QAM.

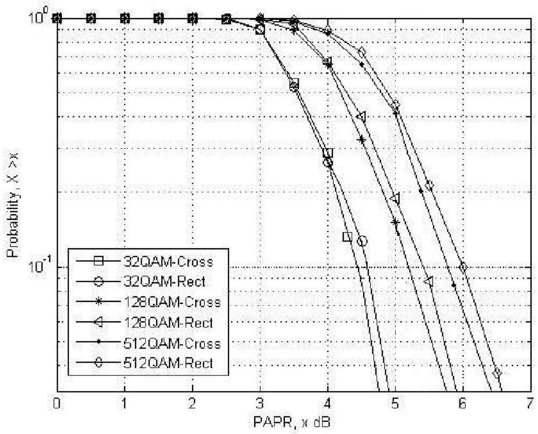

Figure 4: CCDF of the PAPR for Cross & Rectangular QAM Constellations

Results of proposed technique can be evaluated for cross and rectangular QAMs by making constellation energy of rectangular QAM equal to that offered by corresponding cross QAM. This is done by reducing the minimum distance of rectangular QAM to make its energy equal to the corresponding cross QAM. Figure 5 shows the comparison of cross and rectangular QAMs (32-QAM, 128-QAM, and 512-QAM) for same energy with DCT based proposed technique. For same energy, both cross and rectangular QAMs show almost equal reduction in PAPR by the application of proposed scheme.

7.3 Performance comparison with competitive techniques

Comparative analysis of the proposed scheme (New Technique) with time domain constellation shaping (TDCS) and trigonometric based OFDM (DCT OFDM) is performed for 16 QAM with N=256 and α=1. Figure 6 presents the comparison of DCT based systems for 16-QAM. OFDM signal performance without PAPR reduction is also shown as a reference. The results show the better performance of the proposed new scheme in PAPR reduction compared with other recent schemes. It is observed that for the proposed technique improvement is on the order of 4.3dB for α=1.

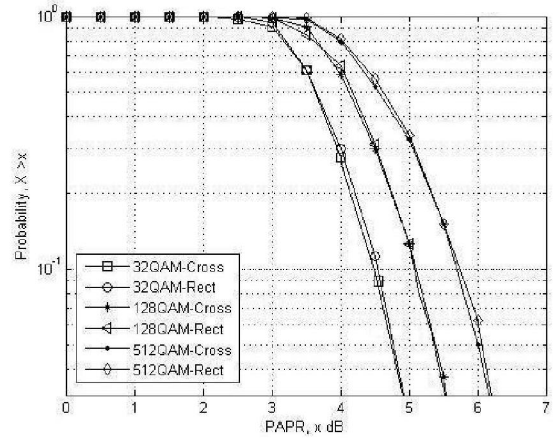

Figure 5: CCDF of the PAPR for Cross & Rectangular QAM Constellations with Equal Transmitted Energy

The proposed scheme is also compared with the dynamic constellation shaping technique (DCS) (Sezginer et al., 2005). In dynamic constellation shaping symbols are pre-distorted based on a cost function. Comparison of the proposed technique with dynamic constellation shaping scheme for 16-QAM, is shown in Figure 7. It is observed that in case of dynamic constellation shaping technique (DCS) improvement in performance is on the order of 2.3dB while for the proposed technique (TDCS) performance gain is approximately 4.3dB.

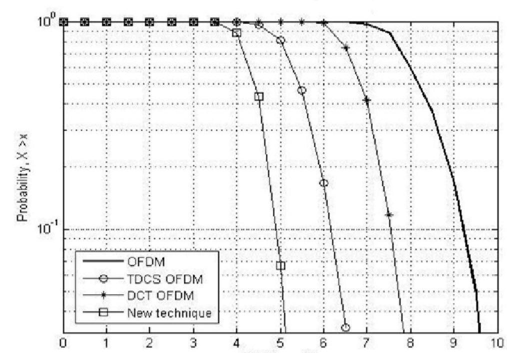

Figure 6: CCDF Performance Comparison of the Proposed Scheme with OFDM, TDCS based OFDM and DCT based OFDM for 16-QAM Constellation

In this section the BER performance of the proposed technique is explored. Figure 8 presents the BER performance of the proposed technique for 16-QAM, N=256 and α varying from 0 to 1.5 with step 0.5. It can be seen that BER performance of the proposed technique degrades with the increase in value of α. BER performance of OFDM signal without PAPR reduction is given by α=0.

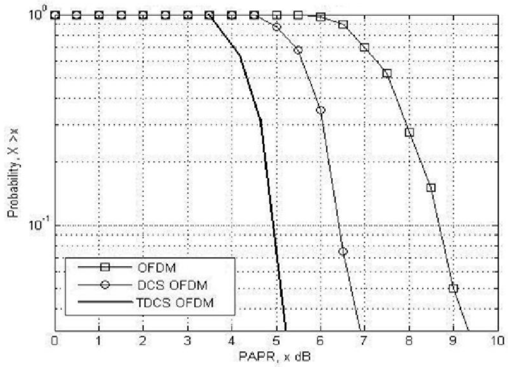

Figure 7: CCDF Performance Comparison of the Proposed Scheme with OFDM and DCS based OFDM**8 BER Performance of proposed technique**

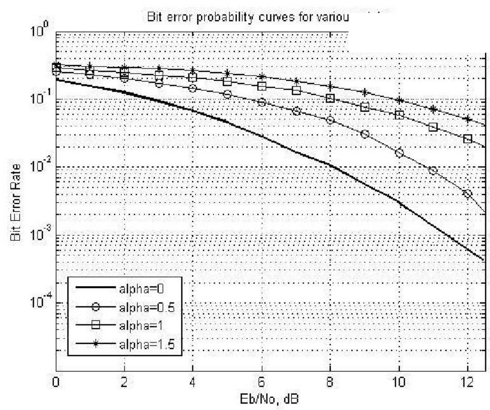

Figure 8: BER Performance of the Proposed Technique for Various Values of Alpha

8.1 BER Performance comparison with competitive techniques

BER performance of the proposed scheme is the same as that of original time domain constellation shaping technique. Figure 9 shows the BER comparison of the proposed DCT transform based scheme (New Technique DCT) with time domain constellation shaping (TDCS) and trigonometric based OFDM (DCT OFDM) for 16-QAM with N=256 and α=0.5. OFDM signal performance without PAPR reduction is also shown as a reference. BER in case of proposed scheme is slightly degraded compared to the original OFDM signal reference. It is observed that BER performance of the proposed scheme overlaps that of original time domain constellation shaping technique. It is also observed that for the proposed technique BER degradation is on the order of 1.5dB for α=0.5.

Figure 10 shows the BER comparison of the proposed DCT transform based scheme with competitive techniques for 16-QAM with N=256 and α=1.

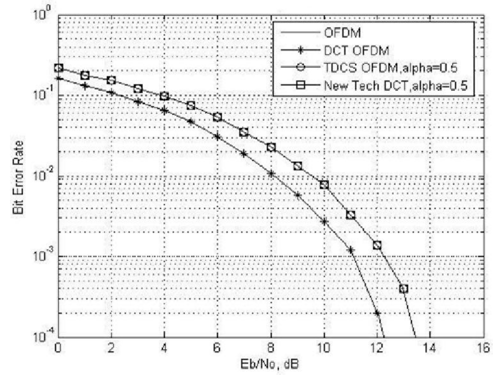

Figure 9: BER Performance Comparison of the Proposed Technique with Existing Techniques

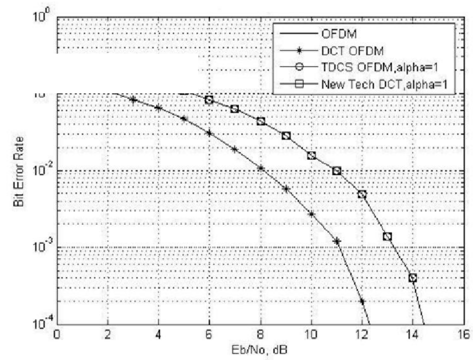

Figure 10: BER Performance Comparison of the Proposed Technique with Existing Techniques with alpha = 1

It can be seen that the only degradation that occurs is due the time domain constellation shaping technique (TDCS). The integration of DCT with time domain constellation shaping technique in the proposed scheme does not further degrades BER. It is observed that for the proposed technique BER degradation is on the order of 2dB for α=1.

It can be concluded that for a 4.3 dB gain in the CCDF performance there is approximately a 2 dB loss for 16-QAM and α=1 in case of proposed technique.

9 Conclusion

We have proposed a constellation shaping technique which achieves a considerable PAPR reduction in OFDM system by employing the energy packing capability of sinusoidal transforms to generate an OFDM signal for modification in time domain by a simple transformation. Closed form expression for the CCDF performance of PAPR distribution is derived and verified by the simulation results. Performance evaluation of the proposed technique with other schemes shows better performance in terms of PAPR reduction without increasing the signal power at the cost of slight degradation in BER performance. Comparison between the cross and rectangular QAM for odd number of bits per symbol is analyzed for same minimum distance and same energy. The proposed technique outperforms other recently proposed PAPR reduction schemes without much increase in system complexity.

10 References

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