Using Common Weight Of Anti Ideal Decision Making Unit In Trade Off Models Of Data Envelopment Analysis For Computing Expanded Malmquist Index

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Abstract: The Malmquist Index the prominent Index for measuring the productivity change of Decision Making Units (DMUs) in multiple time periods that use Data Envelopment Analysis (DEA) models with Variable Return to Scale (VRS) and Constant Return to Scale (CRS) technology. The Trade Offs (TO) approach is an advanced tool for the improvement of the discrimination of DEA model. In this paper, we compute the Expanded Malmquist Index based on Common weights by using anti ideal DMU evaluation in Trade Off models in DEA, and by using this method we can rank DMUs by logical criteria.
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1. Introduction

Data Envelopment Analysis (DEA) is a mathematical programming technique that measures the relative efficiency of Decision Making Units (DMUs) with multiple inputs and outputs. Charnes and et al.(1978) first proposed DEA as an evaluation tool to measure and compare the relative efficiency of DMUs. Their model assumed Constant Returns to Scale (CRS, the CCR model), the model with Variable Return to Scale (VRS, the BCC model) was developed by Banker and et al. (1984).

The Malmquist Index is the most important Index for measuring the relative productivity change of DMUs in multiple time periods. For the first time, the Malmquist Index was introduced by Caves and et al. (1982); later DEA was used by Fare, Gross Kopf, Lindgren and Ross (FGLR, Fare et al, 1992), and (FGNZ, Fare etal.1994) for measuring the Malmquist Index. They used DEA model (CRS) and VRS for computing Malmquist Index.

Podinovski suggests the incorporation of production Trade Offs in to DEA models, under this circumstance (Podinovski 2004), when we use Trade Offs in our models, the original technology expands to include the new area, Podinovski and et. al (2004) show that the production possibility set (PPS), generated by the traditional DEA axioms, may not include all the producible production points, the PPS generated by the DEA models is only the subset of the PPS with Trade Offs. Podinovski also describes the theatrical development of Trade Offs and demonstrated that Trade Offs can improve the traditional meaning of efficiency as a radial impronment factor for input or outputs (Podinovski, 2007a, 2007b).

The rest of the paper is organized as follows: In sections 2 describe Data Envelopment Analysis (DEA). In section 3 we explain computing of common weight. Section 4 shows computing of efficiency by using common weight in different period. In section 5 we compute Malmquist Index based on common weight. The last section summarizes and concludes.

2. Data Envelopment Analysis (DEA)

Assuming that there are n DMUs each with m inputs and s outputs, the relative Efficiency of a particular DMUo $\left(o\in\{1,2,\ldots,n\}\right)$ is obtained by solving the following fractional programming problem:

$$\theta_o = \max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$\begin{aligned} &\textit{subject to}: \\ &\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad j=1,2,\ldots,n \\ &u_r \ge 0 \qquad r=1,2,\ldots,s \\ &v_i \ge 0 \qquad i=1,2,\ldots,m \end{aligned} \tag{1}$$

Where j is the DMU index $j=1,2,\ldots,n,r$ the output index, $r=1,2,\ldots,s$ and i the input index $i=1,2,\ldots,m,$ y_{ri} the value of the rth output for the jth DMU, x_{ij} the value of the i input for the jth DMU,

u_r the weight given to the rth output, v_i the weight given to the i input. DMU_O is efficient if and only if w_o =1.

DMUO selects weights that maximize its output to input ratio, subject to the constraints. A relative efficiency score of 1 indicates that the DMU under consideration is efficient, whereas a score less than 1 imply that it is inefficient. This fractional program can be converted into a linear programming problem where the optimal value of the objective function indicates the relative efficiency of DMUO. The reformulated linear programming problem, also known as the Linear CCR model, is as follows:

$\theta_0^* = w_0 = max \sum_{r=1}^{s} u_r\, y_{ro}$

Subject to:

$\sum_{i=1}^{m} v_i\, v_{io}$

$\sum_{r=1}^{s} u_r\, y_{ro} - \sum_{i=1}^{m} v_i\, v_{io} \leq 0$ j=1, 2… n

$u_r \geq 0$ r= 1, 2… s

$v_i \geq 0$ i=1, 2… m

3. Trade Offs Model in Data Envelopment Analysis

Considering the observed output vector as $Y_j \in R^s$ and the input vector as $X_j \in R^m$, we assume that the inputs and outputs are nonnegative and $X_j \neq 0$, $Y_j \neq 0$ for DMU_j, j=1,2,…,n.

A Trade Off is a judment of possible variation in some input and or output levels, with which DMU can work without changing the other inputs and or outputs. For example, in the case of two inputs and a single output, the trade-off (P, Q) = (2,−1, 0) indicates that the DMU can work by increasing the first input by two and decreasing the second input one without changing its output (for more details, see Podinovski, 2004).

Now, suppose we have k Trade Offs. We shall represent the Trade Offs in the following form: (P_r, Q_r), where r = 1, 2 ,…, k. Also, the vector $P_r \in R^m$ and $Q_r \in R^s$ modify the inputs and outputs, respectively.For using Trade Offs in DEA models, Podinovski makes some assumptions and extends the axioms of PPS in the following manner:

Assumption:

1-All the DMUs should accept the Trade Offs.

2- Each Trade Off can be used repetitively by the DMUs.

Extended axioms:

1- (Nonempty). The observed $(X_j, Y_j) \in$ T; j = 1, 2, ..., n.

2- (Proportionality). If (X,Y) ∈ T , then (λX, λY) ∈ T for all $\lambda \geq 0$.

3- (Convexity). The set T is convex.

4- (Free disposability). If (X, Y) ∈ T, $\bar{X} \geq X, \bar{Y} \geq Y$,then $(\bar{X}, \bar{Y}) \in$ T.

5- (Feasibility of Trade Offs). Let (X, Y) ∈T. Then for any Trade Off r in the form of (P_r, Q_r) ∈T and any $\pi_r \geq 0$, the unit $(X + \pi_r P_r, Y + \pi_r Q_r) \in$ T, provided that $X + \pi_r P_r \geq 0$ and $Y + \pi_r Q_r \geq 0$.

6- (Closeness). The set T is closed.

7- (Minimum extrapolation). T is the smallest set that satisfies axiom 1-6. (Where T is, T = {(X, Y)| output vector Y≥ 0 can produced from input vector X ≥0}).

Now, the PPS can be defined on the basis of the following.

The minimal PPS (PPS_{TO}) that satisfies axioms (1) − (7) is:

$PPS_{TO} = \{(X, Y)| Y = \bar{Y}\lambda + \sum_{t=1}^{k} \pi_t\, Q_t - e, X = \bar{X}\lambda + \sum_{t=1}^{k} \pi_t\, P_t + d, \lambda \in R_+^n, \pi \in R_+^k, d \in R_+^m \text{ and } e \in R_+^s\}$, (see Podinovski (2004)).

Based on PPSTO, for assessing the relative efficiency of DMUP (p = 1, 2, ..., n) that is defined from this PPS, we have the following model:

DEA model with trade-offs technology and input orientation

$$\begin{aligned} & Min\ \theta_p \\ \text{S.t}\quad & \bar{X}\lambda + \sum_{t=1}^{k} \pi_t\, P_t \leq \theta_p X_p \\ & \bar{Y}\lambda + \sum_{t=1}^{k} \pi_t\, Q_t \geq Y_p \qquad (3) \\ & \lambda, \pi \geq 0,\ \theta_p\ sign\ free \end{aligned}$$

DEA model with trade-offs technology and output orientation

$$\begin{aligned} & Max\ \theta_p \\ \text{S.t}\quad & \bar{X}\lambda + \sum_{t=1}^{k} \pi_t\, P_t \leq X_p \\ & \bar{Y}\lambda + \sum_{t=1}^{k} \pi_t\, Q_t \geq \theta_p Y_p \qquad (4) \\ & \lambda, \pi \geq 0,\ \theta_p\ sign\ free \end{aligned}$$

4. Common Weight in Data Envelopment Analysis

Definition 1**:** The virtual positive anti ideal DMU is a DMU with maximize inputs of all of DMUs as its input and minimize outputs of all DMUs as its That is if we show positive ideal DMU with $\overline{DMU} = (\bar{X}, \bar{Y})$ then $\bar{x}_i = \max\{x_{ij} | j = 1,2, \dots, n\}, (i = 1,2, \dots, m)$ and $\bar{y}_r = \min\{y_{rj} | j = 1,2, \dots, n\}, (r = 1,2, \dots, s)$.

Definition 2: An ideal level is one straight line that passes through the origin and positive ideal DMU with slope 1.0. In Fig.1 the vertical and horizontal axes are set to be the virtual output (weighted sum of s outputs) and the virtual input (weighted sum of m inputs), respectively and ox is an ideal line and $\overline{DMU} = (\sum_{i=1}^{m} \bar{x}_i v_i', \sum_{r=1}^{s} \bar{y}_r u_r')$ is an ideal DMU. The notation of a decision variable with superscript symbols”,” represents an arbitrary assigned value. For any DMU_N, DMU_M, if given one set of weights u_r' $(r = 1,2,\dots,s)$ and v_i' $(i = 1,2,\dots,m)$ then the coordinate of points M', N' and N0 in Fig. 1 are $(\sum_{i=1}^{m} x_{iM} v_i', \sum_{r=1}^{s} y_{rM} u_r')$ and $(\sum_{i=1}^{m} x_{iN} v_i', \sum_{r=1}^{s} y_{rN} u_r')$. The virtual gaps, between points M' and M'^p on the horizontal axes and vertical axes, are denoted as $\Delta_{M'}^{I}$ and $\Delta_{M'}^{O}$, respectively. Similarly, for points N' and N'^p , the gaps are $\Delta_{N'}^{I}$ and $\Delta_{N'}^{O}$. We observe that there exists a total virtual gap $\Delta_{M'}^{I} + \Delta_{M'}^{O} + \Delta_{N'}^{O} + \Delta_{N'}^{O}$ to the ideal line. Let the notation of a decision variable with superscript " * " represents the optimal value of the variable. We want to determine an optimal set of weights u_r^* $(r = 1,2,\dots,s)$ and v_i^* $(i = 1,2,\dots,m)$ such that both points M^* and N^* below the ideal line could be as close to their projection points, M^{*p} and N^{*p} on the ideal line, as possible. In other words, by adopting the optimal weights, the total virtual gaps $\Delta_{M^*}^{I} + \Delta_{M^*}^{O} + \Delta_{N^*}^{O} + \Delta_{N^*}^{O}$ to the ideal line is the shortest to both DMUs. As for the constraint, the numerator is the weighted sum of outputs plus the vertical gap Δ_j^O and the denominator is the weighted sum of inputs minus the horizontal virtual gap Δ_j^I. The constraint implies that the direction closest to the ideal line is upwards and leftwards at the same time. The ratio of the numerator to the denominator equals 1.0, which means that the projection point on the ideal line is reached. Therefore we have following model:

$$\Delta^* = \min \sum\nolimits_{j=1}^{n} \Delta_j^I + \Delta_j^O$$

$$\begin{aligned}
\text{S.t}\quad & \frac{\sum_{r=1}^{s} u_r \bar{y}_r}{\sum_{i=1}^{m} v_i \bar{x}_i} = 1 \\
& \frac{\sum_{r=1}^{s} u_r y_{rj} + \Delta_j^O}{\sum_{i=1}^{m} v_i x_{ij} - \Delta_j^I} = 1, \qquad j = 1,2,\dots,n \qquad (5) \\
& \Delta_j^I, \Delta_j^O \geq 0, \qquad j = 1,2,\dots,n \\
& u_r \geq \epsilon > 0, \qquad r = 1,2,\dots,s \\
& v_i \geq \epsilon > 0, \qquad i = 1,2,\dots,m
\end{aligned}$$

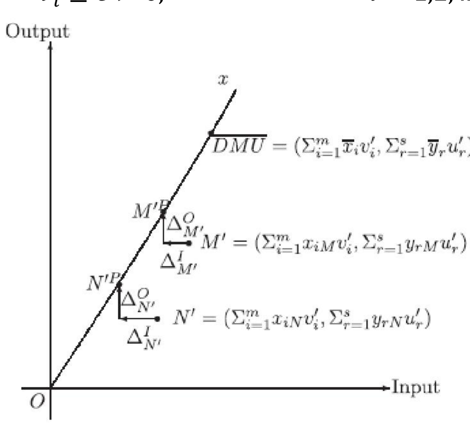

Fig. 1. Gap analysis showing *DMU* below the virtual ideal line.

ϵ is positive Archimedean infinitesimal constant. The ratio form of constrains (5) can be rewritten in a linear form, so we have the following model:

$$\Delta^* = \min \sum\nolimits_{j=1}^{n} \Delta_j^I + \Delta_j^O$$

$$\begin{aligned}
\text{S.t}\quad & \sum_{r=1}^{s} u_r \bar{y}_r - \sum_{i=1}^{m} v_i \bar{x}_i = 0 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \Delta_j^I + \Delta_j^O = 0 \\
& j = 1,2,\dots,n \quad (6) \\
& \Delta_j^I, \Delta_j^O \geq 0, \qquad j = 1,2,\dots,n \\
& u_r \geq \epsilon > 0, \qquad r = 1,2,\dots,s \\
& v_i \geq \epsilon > 0, \qquad i = 1,2,\dots,m
\end{aligned}$$

Then, if we let $\Delta_j^I + \Delta_j^O$, be Δ_j (6) is then simplified to the following linear programming (7).

$$\Delta^* = \min \sum\nolimits_{j=1}^{n} \Delta_j$$

$$\begin{aligned}
\text{S.t}\quad & \sum_{r=1}^{s} u_r \bar{y}_r - \sum_{i=1}^{m} v_i \bar{x}_i = 0 \qquad (*) \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \Delta_j = 0 \\
& j = 1,2,\dots,n \quad (7) \\
& \Delta_j \geq 0, \qquad j = 1,2,\dots,n \\
& u_r \geq \epsilon > 0, \qquad r = 1,2,\dots,s \\
& v_i \geq \epsilon > 0, \qquad i = 1,2,\dots,m
\end{aligned}$$

If a DMUj was on positive ideal then we use definition of the CWA efficiency score of DMUj that Liu and Peng (2006) was defined as following equation:

$$\theta_j^{*(CRS)} = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} \qquad j = 1,2,\dots,n \qquad (8)$$

Therefore the CWA efficiency score of it is 1.0. So that constrain (*) in (7) become redundant and this model become same the CWA model in paper of Liu and Peng (2006). On the other hand, the ideal line is the benchmark line. We result CWA model is special case of (7) in this paper. Therefore DMUj is CWA efficient if $\Delta_j^* = 0$ or $\theta_j^* = 1$ otherwise, DMUj is CWA inefficient.

Definition 3: The performance of DMUj is better than DMUi if $\Delta_j < \Delta_i$. (for more information about this subject see jahanshahloo and et. al 2010).

Suppose we have l Trade Offs (P_{ih}, Q_{rh}) $h = 1,2,\dots,l$ $i = 1,2,\dots,m$ $r = 1,2,\dots,s$ the linear program of DEA model for evaluating common weight:

$$\Delta^* = \min \sum_{j=1}^{n} \Delta_j + \sum_{h=1}^{l} \Delta_h$$

$$\begin{aligned}
\text{S.t}\quad & \sum_{r=1}^{s} u_r \bar{y}_r - \sum_{i=1}^{m} v_i \bar{x}_i = 0 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \Delta_j = 0 \qquad j = 1,2,\dots,n \qquad (9) \\
& \sum_{r=1}^{s} u_r q_{rh} - \sum_{i=1}^{m} v_i p_{ih} + \Delta_h = 0 \qquad h = 1,2,\dots,l \\
& \Delta_j \geq 0, \qquad j = 1,2,\dots,n \\
& \Delta_h \geq 0, \qquad h = 1,2,\dots,l \\
& u_r \geq \epsilon > 0, \qquad r = 1,2,\dots,s \\
& v_i \geq \epsilon > 0, \qquad i = 1,2,\dots,m
\end{aligned}$$

Let u_r^*, v_i^* weights obtaining from solving model (9), therefore efficiency of DMUj in Trade Off models of DEA by using common weight is:

$$\theta_j^{*(TO)} = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} \quad j = 1,2,\dots,n \quad (10)$$

5. Computing of Efficiency by using common weights in different period and different models of DEA

We can compute $\theta_{k(t)}^{*t(TO)}$(ideal DMU and DMUs in period t, frontier period = t), Likewise Previous Section, where x_{ij}^t, $y_{rj}^t$are substituted x_{ij}, y_{rj}. ($\theta_{k(t+1)}^{*t+1(TO)}$ (ideal DMU and DMUs in period t+1, frontier period = t+1))

DEA model of Trade Off technology in input orientation, ideal DMU and DMUs in period t, frontier period = t+1.

Phase 1:

$$\Delta^{*(t)} = \min \sum_{j=1}^{n} {\Delta_j}^t + \sum_{h=1}^{l} {\Delta_h}^t$$

S.t

$$\sum_{r=1}^{s} u_r^{t+1} \bar{y}_r^t - \sum_{i=1}^{m} v_i^{t+1} \bar{x}_i^t = 0$$
$$\sum_{r=1}^{s} u_r^{t+1} y_{rj}^{t+1} - \sum_{i=1}^{m} v_i^{t+1} x_{ij}^{t+1} + {\Delta_j}^t = 0 \quad j = 1,2,\dots,n \quad (11)$$
$$\sum_{r=1}^{s} u_r^{t+1} q_{rh}^{t+1} - \sum_{i=1}^{m} v_i^{t+1} x_{ih}^{t+1} + {\Delta_h}^t = 0 \quad h = 1,2,\dots,l$$
$${\Delta_j}^t \geq 0, \quad j = 1,2,\dots,n$$
$${\Delta_h}^t \geq 0, \quad h = 1,2,\dots,l$$
$$u_r^{t+1} \geq \epsilon > 0, \quad r = 1,2,\dots,s$$
$$v_i^{t+1} \geq \epsilon > 0, \quad i = 1,2,\dots,m$$

Phase 2: Therefore by solving model (11) we obtain $v_i^{*(t+1)}$, $u_r^{*(t+1)}$. So efficiency by using common weight is:

$$\theta_{j(t)}^{*t+1(TO)} = \frac{\sum_{r=1}^{s} u_r^{*(t+1)} y_{rj}^t}{\sum_{i=1}^{m} v_i^{*(t+1)} x_{ij}^t} \quad j = 1,2,\dots,n \quad (12)$$

DEA model of Trade Off technology in input orientation, ideal DMU and DMUs in period t+1, frontier period = t.

Phase 1:

$$\Delta^{*(t+1)} = \min \sum_{j=1}^{n} {\Delta_j}^{t+1} + \sum_{h=1}^{l} {\Delta_h}^{t+1}$$

S.t

$$\sum_{r=1}^{s} u_r^t \bar{y}_r^{t+1} - \sum_{i=1}^{m} v_i^t \bar{x}_i^{t+1} = 0$$
$$\sum_{r=1}^{s} u_r^t y_{rj}^t - \sum_{i=1}^{m} v_i^t x_{ij}^t + {\Delta_j}^{t+1} = 0 \quad j = 1,2,\dots,n \quad (13)$$
$$\sum_{r=1}^{s} u_r^t q_{rh}^t - \sum_{i=1}^{m} v_i^t x_{ih}^t + {\Delta_h}^{t+1} = 0 \quad h = 1,2,\dots,l$$
$${\Delta_j}^{t+1} \geq 0, \quad j = 1,2,\dots,n$$
$$u_r^t \geq \epsilon > 0, \quad r = 1,2,\dots,s$$
$$v_i^t \geq \epsilon > 0, \quad i = 1,2,\dots,m$$

Phase 2: Therefore by solving model (13) we obtain $v_i^{*(t)}$, $u_r^{*(t)}$. So efficiency by using common weight is:

$$\theta_{j(t+1)}^{*t(to)} = \frac{\sum_{r=1}^{s} u_r^{*(t)} y_{rj}^{t+1}}{\sum_{i=1}^{m} v_i^{*(t)} x_{ij}^{t+1}} \quad j = 1,2,\dots,n \quad (14)$$

Likewise we can compute $\theta_{j(t+1)}^{*t(VRS)}$ and $\theta_{j(t)}^{*t+1(VRS)}$.

5. New Method for computing Expanded Malmquist Index based on Common Weights in different models of DEA:

According computing of $\theta_{j(t)}^{*t(VRS)}$, $\theta_{j(t)}^{*t(CRS)}$, $\theta_{j(t)}^{*t(TO)}$ .... in previous section, we know:

$$EC_{\theta^*} = \frac{\theta_{(t+1)}^{*t+1(CRS)}}{\theta_{(t)}^{*t(CRS)}} \quad (15)$$

$$PEC_{\theta^*} = \frac{\theta_{(t+1)}^{t+1(VRS)}}{\theta_{(t)}^{t(VRS)}} \quad (16)$$

$$TC_{\theta^*} = \left[\frac{\theta_{(t)}^{*t(CRS)}}{\theta_{(t)}^{*t+1(CRS)}} \times \frac{\theta_{(t+1)}^{*t(CRS)}}{\theta_{(t+1)}^{*t+1(CRS)}}\right]^{\frac{1}{2}} \quad (17)$$

$$SEC_{\theta^*} = \left[\frac{\theta_{(t)}^{*t(VRS)}}{\theta_{(t)}^{*t(CRS)}} \times \frac{\theta_{(t+1)}^{*t+1(CRS)}}{\theta_{(t+1)}^{*t+1(VRS)}}\right] \quad (18)$$

Where EC_{θ^*} is Efficiency Change based on θ^*, PEC_{θ^*} is Pure Efficiency Change based on θ^*, $TC_{\theta^*}$is Technology Change based on $\theta^*$and SEC_{θ^*} is Scale Efficiency Change based on θ^*. The Malmquist Index and its FGLR and FGNZ decompositions are as follows (for more details, see Fare and et al., 1992, 1994). By similar way we can compute Malmquist Index.

Malmquist Index based on θ^* (MI_{θ^*}) = $EC_{\theta^*} \times TC_{\theta^*}$ (19)

Malmquist Index based on θ^* (MI_{θ^*})= $PEC_{\theta^*} \times SEC_{\theta^*} \times TC_{\theta^*}$ (20)

We define:

$$EEC_{\theta^*} = \frac{\theta_{(t+1)}^{*t+1(TO)}}{\theta_{(t)}^{*t(TO)}} \quad (21)$$

$$ETC_{\theta^*} = \left[\frac{\theta_{(t)}^{*t(TO)}}{\theta_{(t)}^{t+1(TO)}} \times \frac{\theta_{(t+1)}^{t(TO)}}{\theta_{(t+1)}^{t+1(TO)}}\right]^{\frac{1}{2}} \quad (22)$$

$$REC_{\theta^*} = \left[\frac{\theta_{(t)}^{t(CRS)}}{\theta_{(t)}^{t(TO)}} \times \frac{\theta_{(t+1)}^{t+1(TO)}}{\theta_{(t+1)}^{t+1(CRS)}}\right] \quad (23)$$

Where EEC_{θ^*} is Expanded Efficiency Change based on θ^*, ETC_{θ^*} is Expanded Technology Change based on $\theta^*$and EEC_{θ^*} is Regulation Efficiency Change based on θ^*. So

$$\text{Expanded Malmquist Index based on } \theta^*(EMI_{\theta^*}) = EEC_{\theta^*} \times ETC_{\theta^*} \quad (24)$$

Or

$$\text{Expanded Malmquist Index based on } \theta^*(EMI_{\theta^*}) = EC_{\theta^*} \times REC_{\theta^*} \times ETC_{\theta^*} \quad (25)$$

If EMI_{θ^*} >1, it shows DMU had progress.
If EMI_{θ^*} <1, it shows DMU had regress.
If EMI_{θ^*} =1, it shows DMU had not changing.

We define Malmquist Index Disparity and Expanded Malmquist Index Disparity:

$$EMID = \frac{EMI_\theta - EMI_{\theta^*}}{EMI_\theta} \times 100 \quad (26)$$

6. Conclusion

For obtaining relative Efficiency of DMUs, we use means of weights and by using this method we compute Malmquist Index. The result seems be quite satisfactoriness. By using new method (common weights in Trade Off models in DEA) we can rank DMUs by logical criteria, that you can see the result from performance this method in numerical example.

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