Comparison of analytical and experimental results of ductility factor in reinforced concrete structures

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Abstract: In seismic areas, ductility is an important factor in design of concrete members under flexure; it is due to the increase in capacity of plastic displacement. As a result, the inertial forces imposed on the structures can be decreased. The effective factors on ductility are; concrete compression strength f'_c, the percentage of tension and compression steel, ρ and ρ', the amount of stirrups confinement for concrete ρ_c , the stirrups spacing, brittle effect of concrete strength, yield stress of longitudinal bars f_y and the effect of width to the depth of the section b/h. Perhaps the most simple and general definition for section ductility of members is defined, as the ratio of curvatures at ultimate load to curvatures at yield load ($\mu=\varphi_u/\varphi_y$).In this paper, a proposed method was considered to calculate the flexural curvature ductility ratio of reinforced concrete (RC) sections. Based on the proposed method, computer software was produced to calculate the curvature ductility in confined RC beams. The method is based on actual characteristics of a concrete flexural section by considering almost all effective ductility parameters such as available experimental concrete compression diagrams. By the developed software, the ductility factor of 250 beams under efficient circumstances were investigated completely. The nonlinear multiple regression analyses were also performed for these 250 beams and a direct equation is introduced to determine the ductility factor. Based on the obtained experimental results a comparison was made between the proposed direct method and experimental results, and it was shown that a good agreement is available.

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1. INTRODUCTION

In seismic areas, ductility is an important factor in design of concrete members under flexure; it is due to the increase in capacity of plastic displacement. As a result, the inertial forces imposed on the structures can be decreased [1-2]. The effective factors on ductility are; concrete compression strength *f c*, the percentage of tension and compression steel, $\rho$and ρ', the amount of stirrups confinement for concrete ρc , the stirrups spacing, brittle effect of concrete strength, yield stress of longitudinal bars *fy* and the effect of width to the depth of the section *b/h* [3-10].

Beams ductility can be presented based on behaviour of members section or the entire members' behaviour. Prevalent criterion of beams ductility calculation according to entire members' behaviour are the ratio of ultimate displacement to yield displacement ($\mu= \Delta u/\Delta y$), ratio of ultimate rotation to yield rotation ($\mu=\theta u/\theta y$) and the value of structure absorbed energy. Perhaps the most simple and general definition for section ductility of members is defined, ratio of curvatures at ultimate load to curvatures at yield load ($\mu= \varphi u/\varphi y$). The entire members' behaviour reveals the actual behaviour of the structure but calculation of member section behaviour is simpler. However, the experimental results show that the difference between curvature and displacement value of ductility are quite small [6, 8] and hence, the curvature ductility is used generally to investigate the member behaviour.

The effect of concrete confinement with ties on flexure ductility was studied by many researchers [4, 9, 11-14]. But in this research, ductility is calculated based on the actual characteristics of a RC flexural section (Experimental strain-stress curves for confined and unconfined concrete) and act as a separator proposed curves, which divide the zone into; effective confined concrete core, unconfined concrete core and unconfined concrete cover. The method is also based on actual characteristics of a RC flexural section by considering almost all effective ductility parameters such as available experimental concrete compression diagrams.

The calculations of the accurate values of curvature ductility of members are usually complicated particularly in confined concrete beams and therefore, by the use of simplified formula can be much easier [15]. Lee and Pan presented an algorithm and simplified formulas for estimating the relationship between only the tension reinforcement and ductility of reinforced concrete beams. They considered the effects of concrete confinement and spilling of the concrete cover. Calculating of ductility based on Lee and Pan's method is time consuming and difficult.

Based on a proposed method in this research, computer software was produced to calculate the curvature ductility in confined beams. By the developed software, the ductility factor of 250 RC beams under efficient circumstances which are mentioned above were investigated completely. The nonlinear multiple regression analysing was also performed for these 250 RC beams data and a direct equation is introduced to determine the ductility factor.

To investigate the performance and accuracy of the proposed equation, fifteen HSC beams were cast and tested under bending and also the available results of nine HSC beams were selected from research by Tsong et al. [16].

2. CURVATURE DUCTILITY CALCULATION BASED ON PROPOSED METHOD

Perhaps the most simple and general definition for section ductility of members is defined as ratio of curvature at ultimate load to curvature at yield load ($\mu=\varphi u/\varphi y$) shown in Figure 1 where *My*, corresponds to the moment at the beginning of the yielding flat plateau in the moment-curvature curve and *Mu*, is the moment when the ultimate load was reached during testing.

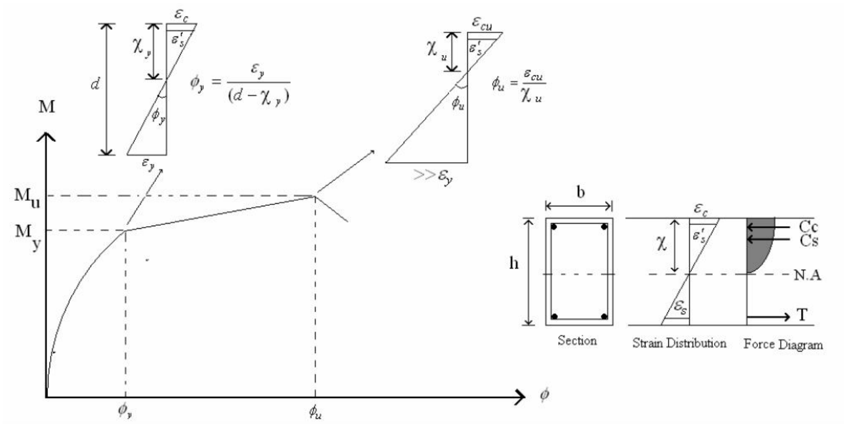


Figure 1. Moment-curvature curve and strain distribution

3. EFFECTIVE CONFINED CONCRETE CORE

3.1. Separator Curves at Transverse Level

There have been many attempts to describe the separator curves of confined concrete. Sheikh and Uzumeri, Sheikh and Yeh and Hwang and Yun made analytical and experimental studies on the mechanism of concrete confinement by considering the various parameters [9, 13-14]. They introduced the concept of the effectively confined concrete area. Fafities and Shah and Woods et al. experimentally investigated the confinement effects of HSC columns [4, 17]. Effective confined concrete area that causes an increase in both member strength and ductility is less than core nominal area which is placed centre to centre of two adjacent transverse bars [13]. Effective confined area of concrete is calculated according to parameters, such as shape of transverse and distance between longitudinal bars. At the section of transverse level with regard to the distance between the longitudinal bars, some of compressive concrete core area is unconfined when it is under bending. The unconfined concrete in core area is hatched and is shown in Figure 2. Longitudinal bars confined concrete effectively in its vicinity. It is assumed that unconfined concrete stress for hatched area out of inside concrete core is uniform.

In this research, a confinement model is proposed to divide concrete inside tie into effective confined concrete core and unconfined concrete core. Therefore, the compression concrete zone of rectangular section under bending is divided by the separator proposed curves.

The relation for separator curves of confined and unconfined concrete areas is as:

$$Y = aX^n \quad (1)$$

where, terms *a* and *n* are the experimental constants.

It is possible to obtain the values of *a* and *n* by considering the coordinate of the stirrup corner (point M) in Figure 2, and the separator curves which is lying between a triangular and elliptical shape, in which the values of A (the hatched area) and θ are respectively equal to *C2/5.5* and 45° as Sheikh and Uzumeri suggested [13].

Hence;

$$a = \frac{1}{1.75}\left(\frac{2}{c}\right)^{0.75} \quad and \ n = 1.75 \quad (2)$$

Having the values of *a* and *n*, and substituting in Eq. 1, the following equation is obtained:

$$Y = \frac{1}{1.75}\left(\frac{2}{c}\right)^{0.75} X^{1.75} \quad or \quad Y = \frac{1}{1.75}\left(\frac{2}{c}\right)^{0.75} |X|^{1.75} \quad (3)$$

where $c > 0, -\frac{c}{2} \le X \le \frac{c}{2}$

Now, the Eq. 4 can be written by considering the known values of *a* and *n* and the coordinate of point M.

$$h = \frac{1}{1.75} \times \left(\frac{2}{c}\right)^{0.75} \times \left(\frac{c}{2}\right)^{1.75} \Rightarrow h = \frac{c}{3.5} \quad (4)$$

3.2. Separator Proposed Curves at Midway Between Ties (i.e., critical section)

It is clear that, confinement of concrete is improved if transverse (stirrup) reinforced layers are placed relatively close together along the longitudinal axis of the beam. There will be some critical spacing of transverse reinforcement layers above which the section midway between the transverse sets will be ineffectively confined, and therefore, it seems the available equation will be inappropriate.

The concrete confinement between the stirrups sets (ie, the spacing between two adjacent stirrups) is affecting on buckled place between stirrups sets. The minimum effective confinement lies between two stirrups. This is clearly illustrated in Figure 3. The maximum value of Y is located at a section midway between stirrups sets. Here, such a section is called a critical section. A suggested value of *0.25S tan*θ is reported by Sheikh and Uzumeri [13], where θ is 45°. Hence, for analyzing purpose, the critical section can be calculated by considering of actual effective concrete confinement area at transverse level which is equal to area that 0.25S at sides of width and height of the section.

Now, Figure 4, illustrates a proposed critical section along the beam's length and it is possible to define the equations for the proposed separator curves in such a section, for both confined concrete and unconfined effective concrete core. To obtain, the distance *O*' from axis *X* and *Y*, the following operations are treated;

$$MG = d' + d''' - d'', \qquad d''' = 0.25S, \qquad MG = d' + 0.25S - d''$$

$$C_1 = b - 2MG, \quad H_1 = \frac{C_1}{3.5}$$
$$C_2 = h - 2MG, \quad H_2 = \frac{C_2}{3.5}, \quad C_3 = C_2, \quad H_3 = H_2 \qquad (5)$$

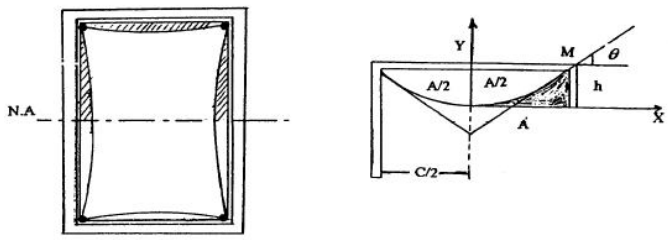


Figure 2. Unconfined concrete in core and separator curve

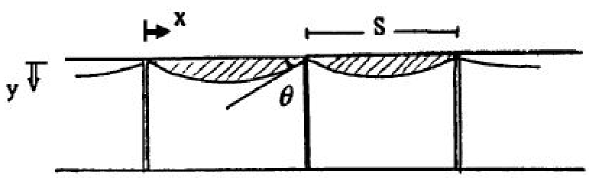


Figure 3. Unconfined concrete midway between the transverse sets

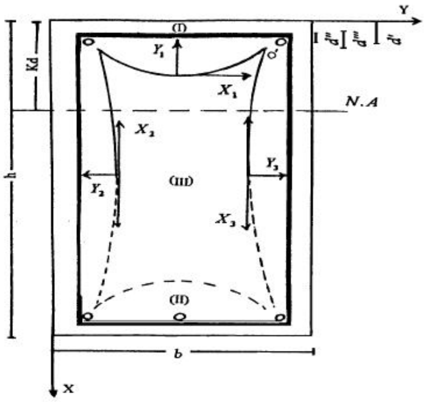


Figure 4. Proposed critical section midway between the transverse sets

C1, *H1*, *C2*, *H2*, *C3* and *H3* are the base and height of the separator curves 1, 2 and 3 respectively. Therefore, based on Eq. 3, it is possible to derive the separator curves 1, 2 and 3 for each local coordinate curve (Figure 4).

The equation for separator curve 1:

$$Y_1 = \frac{1}{1.75}\left(\frac{2}{C_1}\right)^{0.75}|X_1|^{1.75}, \qquad Y = MG + H_1 - Y_1, \qquad X = \frac{b}{2} + X_1 \tag{6}$$

The equation for separator curve 2:

$$Y_2 = \frac{1}{1.75}\left(\frac{2}{C_2}\right)^{0.75}|X_2|^{1.75}, \qquad Y = MG + H_2 - Y_2, \qquad X = \frac{b}{2} + X_2 \tag{7}$$

The equation for separator curve 3:

$$Y_3 = \frac{1}{1.75}\left(\frac{2}{C_3}\right)^{0.75} |X_3|^{1.75}, \qquad Y = MG + H_3 - Y_3, \qquad X = \frac{b}{2} + X_3 \tag{8}$$

3.3. Flowchart to Determine the Moment-Curvature

By using separator curves; experimental stress-strain data of confined, unconfined concrete; and bars stress-strain relationship; a computer program was developed to calculate the moment-curvature curves for confined reinforced concrete beams (i.e., for both high strength and normal strength concrete (HSC and NSC)), and therefore, the curvature ductility is obtained based on moment-curvature curve. The proposed algorithm is demonstrated by the flowchart shown in Figure 5.

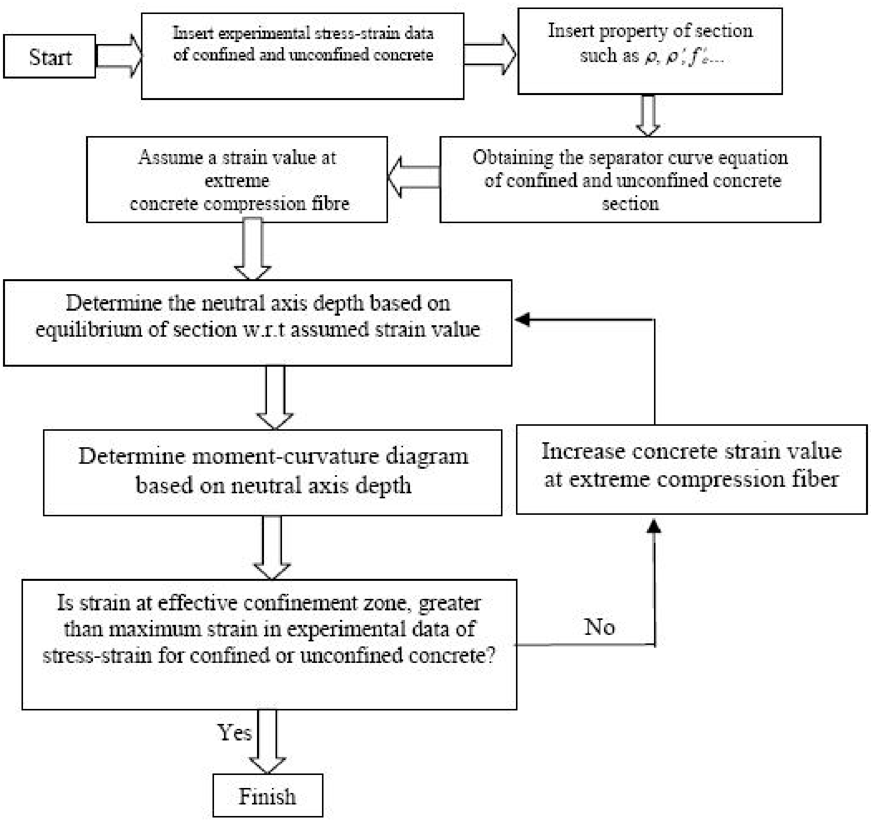


Figure 5. Flowchart to obtain moment-curvature diagram

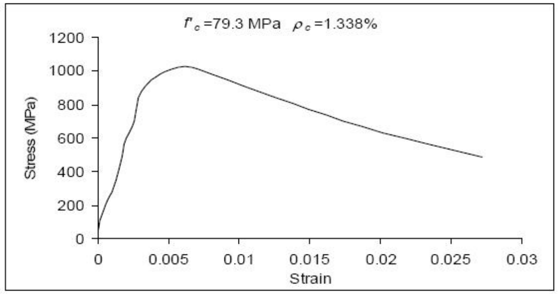
By use of computer software which is based on proposed method, 250 beams with various assumed properties (shown in Table 1) were studied to determine the curvature ductility ($\mu\varphi$).

The electrical strain gauges were fixed on the surfaces of concrete cylindrical specimens and tested under compression and the stress-strain diagrams under the load were plotted. Two diagrams of total are shown in Figure 6. Behaviour of Steel was applied perfectly, where equation for steel was given in reference [18].

Table 1: Selected beams results studied by computer software

f_c'	ρ	ρ'	ρ_c	s/d	μ_ϕ	f_c'	ρ	ρ'	ρ_c	s/d	μ_ϕ
79.3	0.0440	0.0293	0.01338	0.25	15.2	41.3	0.0261	0.0087	0.0163	0.50	12.7
79.3	0.0440	0.0293	0.01338	0.50	16.8	41.3	0.0261	--	0.0163	0.25	8.0
79.3	0.0440	0.0293	0.00735	0.50	11.0	41.3	0.0261	--	--	--	2.6
79.3	0.0440	0.0147	0.00735	0.25	7.0	41.3	0.0174	0.0087	--	--	5.7
79.3	0.0440	0.0147	0.00735	0.50	8.0	26.6	0.0061	0.0087	0.0163	0.50	15.9
79.3	0.0440	--	0.00735	0.50	5.4	26.6	0.0061	--	0.0163	0.25	12.7
79.3	0.0147	--	--	--	7.0	26.6	0.0122	--	--	--	3.8

$f_y = 420\text{MPa}$ and $b/h = 0.65$



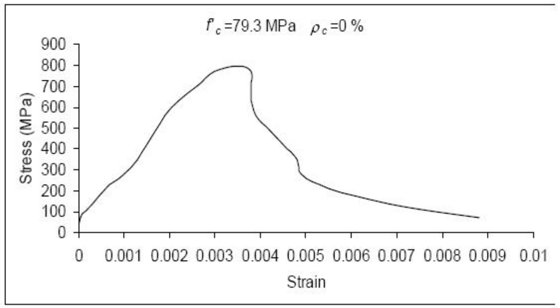


Figure 6. Experimental concrete stress-strain diagram

4. ANALYZING THE DATA BY NONLINEAR REGRESSION METHOD

Any regression analysis should be preceded by a great deal of thought devoted to what variables should be included in the analysis, how these variables might influence the dependent variable, the correlation among the independent variables and ease of using a predictive model based on the selected independent variables. Therefore the first step in the regression analysis should be the development of the form of the predictive model based on a rational analysis of the problem. Regression analysis can then be used to develop the parameters of the model, test the importance of the variables included and develop confidence intervals for the predictions. A nonlinear model is defined as an equation that is nonlinear in the coefficients or a combination of linear and nonlinear in the coefficients. For example Gaussians, ratios of polynomials and power functions are all nonlinear. In matrix form, nonlinear models are given by the formula:

$$y = f(x, \beta) + \varepsilon \qquad (9)$$

Where y is an n-by-1 vector of observations, f is a function of β and X, β is a m-by-1 vector of unknown parameters, x is the n-by-m matrix made up of n observations on each of m independent variables and ε is an n-by-1 vector of errors.

Nonlinear models are more difficult to fit than linear models because the coefficients cannot be estimated using simple matrix techniques. Instead, an iterative approach is required that follows these steps: a) Start with an initial estimate for each coefficient. For some nonlinear models, a heuristic approach is provided that produces reasonable starting values. For other models, random values on the interval [0,1] are provided. b) Produce the fitted curve for the current set of coefficients. The fitted response value *y*ˆ is given by:

$$\hat{y} = f(x, b) \qquad (10)$$

and involves the calculation of the Jacobian of f(x,b), which is defined as a matrix of partial derivatives taken with respect to the coefficients. c) Adjust the coefficients and determine whether the fit improves. The direction and magnitude of the adjustment depend on the fitting method [19]. Many methods can be used to solve these problems such as Gauss-Newton. This method is potentially faster than the other methods, but it assumes that the residuals are close to zero.

The magnitude of ductility is dependent upon the different variables such as; concrete compression strength *f c*, the percentage of tension and compression steel, ρ and ρ′, the amount of stirrups confinement for concrete ρ*c*, the stirrups spacing, brittle effect of concrete strength, yield stress of longitudinal bars *fy*, and the effect of ratio of width to the height of the section, *b/h*. Here, a constant longitudinal yield stress value of 420 MPa is assumed.

Table 2: Details of testing program of tested beams [5-6] and Comparison of experimental and direct proposed equation results

Beam No	f'_c (MPa)	d (mm)	d' (mm)	A_s	ρ (%)	ρ/ρ_0	A'_s	ρ' (%)	μ (exp)	μ $(eq.11)$	$Error$ (%)
BC1	56.31	254	42	2Φ14	0.61	0.13	2Φ14	0.61	11.84	11.15	6.19
B1	69.50	254	--	2Φ14	0.61	0.13	--	--	10.25	10.46	2.00
BC2	63.48	250	47	2Φ20	1.25	0.24	2Φ14	0.61	6.84	8.96	23.66
B2	70.50	250	--	2Φ20	1.25	0.24	--	--	5.38	7.88	31.72
BC3	63.21	251	42	4Φ18	2.03	0.36	3Φ18	1.01	5.75	7.56	23.94
B3	70.80	251	--	4Φ18	2.03	0.36	--	--	4.52	5.50	17.82
BC4	71.45	250	47	4Φ20	2.51	0.43	3Φ20	1.24	5.60	7.40	24.32
B4	72.80	250	--	2Φ20	2.51	0.43	--	--	2.82	4.37	35.47
BC5	65.54	259	42	4Φ22	3.05	0.52	3Φ25	1.60	5.31	6.71	20.86
B5	71.50	259	--	4Φ22	3.05	0.52	--	--	--	3.04	--
BC6	73.77	256	40	4Φ28	4.81	0.79	2Φ14	0.61	3.20	1.21	62.50
B6	71.00	256	--	4Φ28	4.81	0.79	--	--	1.03	1	3.00
BC10	73.42	256	40	4Φ28	4.81	0.79	2Φ20	1.23	3.29	2.76	19.20
BC11	72.98	256	40	4Φ28	4.81	0.79	2Φ28	2.41	4.33	5.78	25.09
BC12	74.35	256	40	4Φ28	4.81	0.79	3Φ28	3.61	6.50	8.99	27.69

Table 3: Details of testing program of tested beams [16] and Comparison of experimental and direct proposed equation results

Beam No	f'_c (MPa)	A_s	ρ (%)	A'_s	ρ' (%)	Spacing (cm)	ρ_c (%)	μ (exp)	μ $(eq. 11)$	$Error$ (%)
C-1	61.80	$2\Phi6$	3.39	$2\Phi4$	1.51	4	5.47	25.63	35.05	26.87
C-2	60.80	$2\Phi6$	3.39	$2\Phi4$	1.51	8	2.73	15.00	19.54	23.23
C-3	61.80	$2\Phi6$	3.39	$2\Phi4$	1.51	16	1.37	11.13	14.03	20.67
C-4	63.80	$2\Phi6$	3.39	$2\Phi5$	2.36	4	5.47	41.82	37.37	11.91
C-5	61.80	$2\Phi6$	3.39	$2\Phi5$	2.36	8	2.73	17.84	21.35	16.44
C-6	63.80	$2\Phi6$	3.39	$2\Phi5$	2.36	16	1.37	11.17	16.35	31.68
C-7	62.80	$2\Phi6$	3.39	$2\Phi6$	3.39	4	5.47	46.60	39.99	16.5
C-8	61.80	$2\Phi6$	3.39	$2\Phi6$	3.39	8	2.73	25.00	24.49	2.08
C-9	63.80	$2\Phi6$	3.39	$2\Phi6$	3.39	16	1.37	24.8	19.03	30.32
C-10	61.80	$2\Phi6$	3.39	$2\Phi4$	1.51	4	5.47	25.63	35.05	26.87

The computer software, based on proposed method was testified for 250 RC beams of mentioned variables. Nonlinear regression method is used to analyze these 250 beams data. The analysis results provide the following direct equation to determine the ductility factor:

$$\mu = 1027.48\rho_c^{1.24} + 2.33\left(\frac{b}{h}\right)^{2.13} + 3.04\left(\frac{s}{d}\right)^{0.68} - 76.92\rho^{0.49} + 276.00\rho'^{1.02} + 2447.55\left(\frac{f_c'}{E_c}\right)^{0.93} + 8.39$$

R-square and average errors of the proposed equation are 0.91 and 13 percent respectively.

5. COMPARISON OF EXPERIMENTAL AND DIRECT PROPOSED EQUATION

To evaluate the accuracy of proposed direct Eq. 11, the experimental results of tests reported by Akbarzadeh and Maghsoudi, and Tsong et al. [5-6, 16] are investigated. Table 2 and 3 present the detailed testing programs. Curvature ductility is defined as the ratio of curvatures at ultimate load to curvatures at yield load ($\mu = \varphi u / \varphi y$). The experimental yielding curvature, φy, corresponds to the curvature at the beginning of the yielding flat plateau in the moment-curvature curve. The experimental ultimate curvature, φu, is the curvature when the ultimate load was reached during testing. For the tested beams, experimental curvature ductilities and the obtained ductility amount of these beams based on proposed equation are compared and shown in Table 2 and 3. The average error for experimental and proposed direct Eq. (11) is 20 percent, which indicates that a good agreement is available.

6. CONCLUSIONS

In seismic areas, flexural ductility is an important factor in design of concrete members. The calculation of the accurate values of ductility of a member is usually complicated and therefore a direct and accurate approach to obtain such value is necessarily required particularly in seismic regions. The proposed direct Eq. (11) for calculating the curvature ductility satisfies this requirement with an average error as low as 20 %.

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