

Time-dependent two-dimensional Zakharov-Kuznetsov equation in the electron-positron-ion plasmasAli H. Bhrawy^{1,2} A.S. Alofi¹ and M.A. Abdelkawy²¹ Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia² Department of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt
(Bhrawy) alibhrawy@yahoo.co.uk, (Alofi) asmo1427@yahoo.com, (Abdelkawy) melkawy@yahoo.com

Abstract: The electrons and positrons are assumed to be dynamic, whereas positively charged ions are considered stationary. Using a computerized symbolic computation technique, we obtained several solutions of the Zakharov-Kuznetsov equation which describes the propagation of the electrostatic excitations in the electron-positron-ion plasmas. These solutions contain hyperbolic, triangular solutions. These solutions extended to ion-acoustic waves in quantum dusty plasmas consisting of electrons, ions, and negatively/positively charged dust particles. In addition, as an illustrative sample, the properties of the solutions of this equation are shown with some figures.

[Bhrawy AH, Alofi AS, Abdelkawy AM. **Time-dependent two-dimensional Zakharov-Kuznetsov equation in the electron-positron-ion plasmas.** *Life Sci J* 2012;9(4):1804-1813] (ISSN:1097-8135).
<http://www.lifesciencesite.com>. 274

Keywords: Extended F-expansion (EFE) method; Nonlinear partial differential equations; Nonlinear physical phenomena; Zakharov-Kuznetsov equation; Electron-positron (EP) plasma.

1. Introduction

Electron-positron (EP) plasma is a new state of matter with unique thermodynamic properties which different from those of ordinary electron-ion plasmas [1]. The study of electron-positron-ion (e-p-i) plasmas is important to understand the behavior of both astrophysical and laboratory plasmas. Electron-positron plasmas have been observed in the polar regions of neutron stars [2], active galactic nuclei [3], in magnetospheres of pulsars [4], polar regions of neutron stars [2], intense laser fields [5] and tokamak plasmas [6]-[7]. During the last few years, there have great interest in the field of pair (electron-positron) plasma, which is composed of electrons and positrons, having equal mass but having opposite charge [8]-[11].

The nonlinear wave phenomena can be observed in various scientific fields, such as plasma physics, optical fibers, fluid dynamics, chemical physics, etc. The nonlinear wave phenomena can be obtained in solutions of nonlinear evolution equations (NEEs). The exact solutions of these NEEs plays an important role in the understanding of nonlinear phenomena. In the past decades, many methods were developed for finding exact solutions of NEEs [12]-[20].

Although Porubov et al. [21]-[22] have obtained some exact periodic solutions to some nonlinear wave equations, they use the Weierstrass elliptic function and involve complicated deducing. A Jacobi elliptic function (JEF) expansion method, which is straightforward and effective, was proposed for constructing periodic wave solutions for some nonlinear evolution equations. The essential idea of this method is similar to the tanh method by replacing

the tanh function with some JEFs such as sn , cn and dn . For example, the Jacobi periodic solution in terms of sn may be obtained by applying the sn -function expansion. Many similarly repetitious calculations have to be done to search for the Jacobi doubly periodic wave solutions in terms of cn and dn [23].

Recently F-expansion method [24]-[27] was proposed to obtain periodic wave solutions of NLEEs, which can be thought of as a concentration of JEF expansion since F here stands for everyone of JEFs. In this paper, we apply the extended F-expansion (EFE) method with symbolic computation to system (4) for constructing their interesting Jacobi doubly periodic wave solutions. It is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. In addition the algorithm that we use here also a computerized method, in which generating an algebraic system.

2. Material and Methods**Governing equations**

The dynamics of the electrostatic (ES) solitons in the electron-positron-ion plasmas is governed by [28]

$$\frac{\partial n_{\pm}}{\partial T} + \nabla \cdot (n_{\pm} u_{\pm}) = 0, \quad \nabla \equiv \hat{X} \frac{\partial}{\partial X} + \hat{Y} \frac{\partial}{\partial Y} \quad (1)$$

$$m \left(\frac{\partial u_{\pm}}{\partial t} + u_{\pm} \cdot \nabla u_{\pm} \right) = \mp e \nabla \phi - \frac{1}{n_{\pm}} \nabla p_{\pm} \pm \frac{e}{c} (u_{\pm} \times B_0 \hat{X}) + 2m(u_{\pm} \times \Omega_0 \hat{X}), \quad (2)$$

and Poisson's equation

$$\nabla^2 \phi = 4\pi(n_- - n_+ - s_3 z_3 n_3), \quad (3)$$

where (X, Y) , $n_{\pm}$, $u_{\pm}$, ϕ , n_3 , e , c , B_0 , Ω_0 , $p_{\pm}$, m , z_3 and $s_3 = \pm 1$ are the coordinate of the propagation plane; the number densities of the positron, electron; the fluid velocity variables of the positron and electron; the ES potential; the number densities of ion; the magnitude of the electron charge; the speed of light; the magnitude of the ambient magnetic field; the rotation frequency; the pressures of the positron and electron; the electron mass; the ion charge number and the ion charge sign respectively.

The time-dependent two-dimensional ZK equation can be obtained from Eqs. (1)-(3), using multi-dimensional reductive technique [29] and the independent variables [30]

$$x = \varepsilon^{1/2}(X - \lambda T), \quad y = \varepsilon^{1/2}Y, \quad t = \varepsilon^{3/2}T,$$

where ε is a small parameter and $\lambda(\lambda > 0)$ is the phase velocity as

$$v_t + \delta v_{xx} + \mu v_{xxx} + \nu_{xyy} = 0, \quad (4)$$

where v is the 1st-order ES potential perturbation of the ES potential ϕ . Eq. (4) describe many physical phenomena, such as the weakly nonlinear ion-acoustic waves in a strongly magnetized lossless plasma comprised of the cold ions and hot isothermal electrons, the nonlinear dust-acoustic waves in a magnetized three-component dusty plasma consisting of negatively charged dust-particles and nonlinear ion-acoustic waves in a quantum magneto plasma.

Extended F-expansion method

In this section, we introduce a simple description of the EFE method, for a given partial differential equation

$$G(u, u_x, u_y, u_z, u_t, u_{xy}, \dots) = 0. \quad (5)$$

We like to know whether travelling waves (or stationary waves) are solutions of Eq. (5). The first step is to unite the independent variables x , y , z and t into one particular variable through the new variable

$$\zeta = \alpha x + \beta y + \gamma z + \nu t, \quad u(x, y, z, t) = U(\zeta),$$

where ν is wave speed, and reduce Eq. (5) to an ordinary differential equation(ODE)

$$G(U, U', U'', U''', \dots) = 0. \quad (6)$$

Our main goal is to derive exact or at least approximate solutions, if possible, for this ODE. For

(2) this purpose, let us simply U as the expansion in the form,

$$u(x, y, z, t) = U(\zeta) = \sum_{i=0}^N a_i F^i + \sum_{i=1}^N a_{-i} F^{-i}, \quad (7)$$

where

$$F' = \sqrt{A + BF^2 + CF^4}, \quad (8)$$

the highest degree of $\frac{d^p U}{d\zeta^p}$ is taken as

$$O\left(\frac{d^p U}{d\zeta^p}\right) = N + p, \quad p = 1, 2, 3, \dots, \quad (9)$$

$$O\left(U^q \frac{d^p U}{d\zeta^p}\right) = (q+1)N + p, \quad q = 0, 1, 2, \dots, p = 1, 2, 3, \dots, \quad (10)$$

where A , B and C are constants, and N in Eq. (6) is a positive integer that can be determined by balancing the nonlinear term(s) and the highest order derivatives. Normally N is a positive integer, so that an analytic solution in closed form may be obtained. Substituting Eqs. (5)-(8) into Eq. (6) and comparing the coefficients of each power of $F(\zeta)$ in both sides, to get an over-determined system of nonlinear algebraic equations with respect to ν , a_0 , a_1 , $\dots$. Solving the over-determined system of nonlinear algebraic equations by use of Mathematica. The relations between values of A , B , C and corresponding JEF solution $F(\zeta)$ of Eq. (7) are given in table 1. Substitute the values of A , B , C and the corresponding JEF solution $F(\zeta)$ chosen from table 1 into the general form of solution, then an ideal periodic wave solution expressed by JEF can be obtained.

Table 1: Relation between values of (A, B, C) and corresponding F

A	B	C	$F(\zeta)$
1	$-1 - m^2$	m^2	$\text{sn}(\zeta),$ $\text{cd}(\zeta) = \frac{\text{cn}(\zeta)}{\text{dn}(\zeta)}$
$1 - m^2$	$2m^2 - 1$	$-m^2$	$\text{cn}(\zeta)$
$m^2 - 1$	$2 - m^2$	-1	$\text{dn}(\zeta)$
m^2	$-1 - m^2$	1	$\text{ns}(\zeta) = \frac{1}{\text{sn}(\zeta)},$ $\text{dc}(\zeta) = \frac{\text{dn}(\zeta)}{\text{cn}(\zeta)}$
$-m^2$	$2m^2 - 1$	$1 - m^2$	$\text{nc}(\zeta) = \frac{1}{\text{cn}(\zeta)}$
-1	$2 - m^2$	$m^2 - 1$	$\text{nd}(\zeta) = \frac{1}{\text{dn}(\zeta)}$

1	$2-m^2$	$1-m^2$	$\text{sc}(\zeta) = \frac{\text{sn}(\zeta)}{\text{cn}(\zeta)}$
1	$2m^2-1$	$\frac{m^2}{(1+m^2)}$	$\text{sd}(\zeta) = \frac{\text{sn}(\zeta)}{\text{dn}(\zeta)}$
$1-m^2$	$2-m^2$	1	$\text{cs}(\zeta) = \frac{\text{cn}(\zeta)}{\text{sn}(\zeta)}$
$\frac{-m^2}{(1-m^2)}$	$2m^2-1$	1	$\text{ds}(\zeta) = \frac{\text{dn}(\zeta)}{\text{sn}(\zeta)}$
$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$\text{ns}(\zeta) + \text{cs}(\zeta)$
$\frac{1-m^2}{4}$	$\frac{1+m^2}{2}$	$\frac{1-m^2}{2}$	$\text{nc}(\zeta) + \text{sc}(\zeta)$
$\frac{1}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\text{ns}(\zeta) + \text{ds}(\zeta)$
$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\text{sn}(\zeta) + i\text{cs}(\zeta)$

where $\text{sn}(\zeta)$, $\text{cn}(\zeta)$ and $\text{dn}(\zeta)$ are the JE sine function, JE cosine function and the JEF of the third kind, respectively. And

$$\text{cn}^2(\zeta) = 1 - \text{sn}^2(\zeta), \quad \text{dn}^2(\zeta) = 1 - m^2 \text{sn}^2(\zeta), \quad (11)$$

with the modulus m ($0 < m < 1$).

When $m \rightarrow 1$, the Jacobi functions degenerate to the hyperbolic functions, i.e.,

$$\text{sn}\zeta \rightarrow \tanh\zeta, \quad \text{cn}\zeta \rightarrow \text{sech}\zeta, \quad \text{dn}\zeta \rightarrow \text{sech}\zeta,$$

when $m \rightarrow 0$, the Jacobi functions degenerate to the triangular functions, i.e.,

$$\text{sn}\zeta \rightarrow \sin\zeta, \quad \text{cn}\zeta \rightarrow \cos\zeta \quad \text{and} \quad \text{dn} \rightarrow 1.$$

3. Results

In this section, we will apply the extended method to study the time-dependent two-dimensional ZK equation (4)

$$v_t + \delta v v_x + \mu v_{xxx} + \gamma v_{xyy} = 0, \quad (12)$$

if we use $\zeta = \alpha x + \beta y + \nu t$, $\phi(x, y, t) = V(\zeta)$ carries PDE (12) into the ODE

$$\nu V' + \alpha \delta V V' + \alpha(\beta^2 \gamma + \alpha^2 \mu) V''' = 0, \quad (13)$$

where by integrating once we obtain, upon setting the constant of integration to zero,

$$2\nu V + \alpha \delta V^2 + 2\alpha(\beta^2 \gamma + \alpha^2 \mu) V'' = 0, \quad (14)$$

Balancing the term V'' with the term V^2 we obtain $N=2$ then

$$U(\zeta) = a_0 + a_1 F + a_{-1} F^{-1} + a_2 F^2 + a_{-2} F^{-2}, \quad (15)$$

$$F' = \sqrt{A + BF^2 + CF^4}.$$

Substituting Eq. (15) into Eq. (14) and comparing the coefficients of each power of F in both sides, to get an over-determined system of nonlinear algebraic equations with respect to ν , a_i , $i = 1, -1, -2, 2$. Solving the over-determined system of nonlinear algebraic equations by use of Mathematica, we obtain three groups of constants:

1.

$$a_{-1} = a_{-2} = 0, \quad a_0 = -\frac{4(\alpha^2 \mu + \beta^2 \gamma)(B \pm \sqrt{B^2 + 12AC})}{\delta},$$

$$a_2 = -\frac{12C(\alpha^2 \mu + \beta^2 \gamma)}{\delta}, \quad a_{-2} = -\frac{12A(\alpha^2 \mu + \beta^2 \gamma)}{\delta},$$

$$\nu = \pm 4\alpha(\alpha^2 \mu + \beta^2 \gamma)\sqrt{B^2 + 12AC}, \quad (16)$$

2.

$$a_{-1} = a_{-2} = a_2 = 0, \quad a_0 = -\frac{4(\alpha^2 \mu + \beta^2 \gamma)(B \pm \sqrt{B^2 - 3AC})}{\delta},$$

$$a_{-2} = -\frac{12A(\alpha^2 \mu + \beta^2 \gamma)}{\delta}, \quad \nu = \pm 4\alpha(\alpha^2 \mu + \beta^2 \gamma)\sqrt{B^2 - 3AC}, \quad (17)$$

3.

$$a_{-1} = a_{-2} = a_2 = 0, \quad a_0 = -\frac{4(\alpha^2 \mu + \beta^2 \gamma)(B \pm \sqrt{B^2 - 3AC})}{\delta}, \quad (18)$$

$$a_2 = -\frac{12C(\alpha^2 \mu + \beta^2 \gamma)}{\delta}, \quad \nu = \pm 4\alpha(\alpha^2 \mu + \beta^2 \gamma)\sqrt{B^2 - 3AC},$$

if we use Eqs. (16)-(18) we obtained the 1st-order ES potential perturbation of the ES potential of Eq. (12) as:-

$$v_1 = \frac{4(\alpha^2 \mu + \beta^2 \gamma)(1+m^2 \pm \sqrt{12m^2 + (1+m^2)^2})}{\delta} - \frac{12(\alpha^2 \mu + \beta^2 \gamma)}{\delta}$$

$$\times [m^2 \text{sn}^2(\alpha x + \beta y \pm 4(\alpha^2 \mu + \beta^2 \gamma)\sqrt{(12m^2 + (1+m^2)^2)t})$$

$$+ \text{ns}^2(\alpha x + \beta y \pm 4(\alpha^2 \mu + \beta^2 \gamma)\sqrt{(12m^2 + (1+m^2)^2)t})], \quad (19)$$

$$v_2 = \frac{4(\alpha^2 \mu + \beta^2 \gamma)(1+m^2 \pm \sqrt{12m^2 + (1+m^2)^2})}{\delta} - \frac{12(\alpha^2 \mu + \beta^2 \gamma)}{\delta}$$

$$\times [m^2 \text{cd}^2(\alpha x + \beta y \pm 4(\alpha^2 \mu + \beta^2 \gamma)\sqrt{(12m^2 + (1+m^2)^2)t})$$

$$+ d^2(\alpha x + \beta y \pm 4(\alpha^2 \mu + \beta^2 \gamma)\sqrt{(12m^2 + (1+m^2)^2)t})], \quad (20)$$

$$v_3 = -\frac{4(\alpha^2 \mu + \beta^2 \gamma)((2m^2 - 1) \pm \sqrt{12m^2(m^2 - 1) + (1 - 2m^2)^2})}{\delta}$$

$$+ \frac{12(\alpha^2 \mu + \beta^2 \gamma)}{\delta} [m^2 \text{cn}^2(\alpha x + \beta y \pm 4(\alpha^2 \mu + \beta^2 \gamma)$$

$$\times \sqrt{(12m^2(m^2 - 1) + (1 - 2m^2)^2)t})$$

$$- (1 - m^2) \text{nc}^2(\alpha x + \beta y \pm 4(\alpha^2 \mu + \beta^2 \gamma)\sqrt{(12m^2(m^2 - 1) + (1 - 2m^2)^2)t})], \quad (21)$$

$$v_4 = -\frac{4(\alpha^2\mu + \beta^2\gamma)((2-m^2) \pm \sqrt{(2-m^2)^2 - 12(m^2-1)})}{\delta} + \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [dn^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{(2-m^2)^2 - 12(m^2-1)}) + (1-m^2)nd^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{(2-m^2)^2 - 12(m^2-1)})], \quad (22)$$

$$v_5 = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2-m^2 \pm \sqrt{12(1-m^2) + (2-m^2)^2})}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [(1-m^2)sc^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \times \sqrt{12(1-m^2) + (2-m^2)^2} t) + cs^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{12(1-m^2) + (2-m^2)^2} t)], \quad (23)$$

$$v_6 = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2m^2 - 1 \pm \sqrt{12m^2(1+m^2) + (1-2m^2)^2})}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [m^2(1+m^2)sd^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \times \sqrt{12m^2(1+m^2) + (1-2m^2)^2} t) + ds^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{12m^2(1+m^2) + (1-2m^2)^2} t)], \quad (24)$$

$$v_7 = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5 - m^2 \pm \sqrt{0.75 + (0.5 - m^2)^2})}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} [(ns(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75 + (0.5 - m^2)^2} t) + cs(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75 + (0.5 - m^2)^2} t))^2 + (ns(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75 + (0.5 - m^2)^2} t) + cs(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75 + (0.5 - m^2)^2} t))^{-2}], \quad (25)$$

$$v_8 = -\frac{4(\alpha^2\mu + \beta^2\gamma)}{\delta} \times \frac{(0.5 + 0.5m^2 \pm \sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2})}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [(0.5 - 0.5m^2) \times (nc(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2} t) + sc(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2} t))^2 + (0.25 - 0.25m^2)(nc(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2} t) + sc(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2} t))^{-2}], \quad (26)$$

$$v_9 = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5m^2 - 1 \pm \sqrt{0.75m^2 + (-1 + 0.5m^2)^2})}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} [m^2(ns(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75m^2 + (-1 + 0.5m^2)^2} t) + ds(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75m^2 + (-1 + 0.5m^2)^2} t))^2 + (ns(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75m^2 + (-1 + 0.5m^2)^2} t) + ds(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75m^2 + (-1 + 0.5m^2)^2} t))^{-2}], \quad (27)$$

$$v_{10} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5m^2 - 1 \pm \sqrt{0.75m^4 + (-1 + 0.5m^2)^2})}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} [m^2(sn(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75m^4 + (-1 + 0.5m^2)^2} t) + ics(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75m^4 + (-1 + 0.5m^2)^2} t))^2 + (sn(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75m^4 + (-1 + 0.5m^2)^2} t) + ics(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{0.75m^4 + (-1 + 0.5m^2)^2} t))^{-2}], \quad (28)$$

$$v_{11} = \frac{4(\alpha^2\mu + \beta^2\gamma)(1 + m^2 \pm \sqrt{(1+m^2)^2 - 3m^2})}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [m^2sn^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{(1+m^2)^2 - 3m^2} t)], \quad (29)$$

$$v_{12} = \frac{4(\alpha^2\mu + \beta^2\gamma)(1 + m^2 \pm \sqrt{(1+m^2)^2 - 3m^2})}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [m^2cd^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{(1+m^2)^2 - 3m^2} t)], \quad (30)$$

$$v_{13} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2m^2 - 1 \pm \sqrt{3m^2(1-m^2) + (1-2m^2)^2})}{\delta} + \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [m^2cn^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \times \sqrt{3m^2(1-m^2) + (1-2m^2)^2} t)], \quad (31)$$

$$v_{14} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2-m^2 \pm \sqrt{(2-m^2)^2 - 3(1-m^2)})}{\delta} + \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [dn^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \times \sqrt{(2-m^2)^2 - 3(1-m^2)} t)], \quad (32)$$

$$v_{15} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2-m^2 \pm \sqrt{3(m^2-1) + (2-m^2)^2})}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [(1-m^2)sc^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \times \sqrt{3(m^2-1) + (2-m^2)^2} t)], \quad (33)$$

$$v_{16} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2m^2 - 1(\alpha^2\mu + \beta^2\gamma) \pm \sqrt{(1-2m^2)^2 - 3m^2(1+m^2)})}{\delta} \\ - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [m^2(1+m^2)sd^2(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma) \\ \times \sqrt{(1-2m^2)^2 - 3m^2(1+m^2)t})], \quad (34)$$

$$v_{17} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5m^2 \pm \sqrt{(0.5m^2)^2 - \frac{3}{16}})}{\delta} \\ - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} [(ns(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{(0.5m^2)^2 - \frac{3}{16}}t) \\ + cs(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{((0.5m^2)^2 - \frac{3}{16})^2})], \quad (35)$$

$$v_{18} = -\frac{4(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ \times \frac{(0.5 + 0.5m^2 \pm \sqrt{3(0.5m^2 - 0.5)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2})}{\delta} \\ - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [(0.5 - 0.5m^2)(nc(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma) \\ \times \sqrt{3(0.5m^2 - 0.5)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2}t) \\ + sc(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma) \\ \times \sqrt{3(0.5m^2 - 0.5)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2}t))^2], \quad (36)$$

$$v_{19} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5m^2 - 1 \pm \sqrt{((1-0.5m^2)^2 - \frac{3m^2}{16}})}{\delta} \\ - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} [m^2(ns(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{(1-0.5m^2)^2 - \frac{3m^2}{16}}t) \\ + ds(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{(1-0.5m^2)^2 - \frac{3m^2}{16}}t))^2], \quad (37)$$

$$v_{20} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5m^2 - 1 \pm \sqrt{(1-0.5m^2)^2 - \frac{3m^4}{16}})}{\delta} \\ - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} [m^2(sn(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma) \\ \sqrt{(1-0.5m^2)^2 - \frac{3m^4}{16}}t) \\ + ics(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{(1-0.5m^2)^2 - \frac{3m^4}{16}}t))^2], \quad (38)$$

$$v_{21} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1+m^2 \pm \sqrt{(1+m^2)^2 - 3m^2})}{\delta} \\ - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [ns^2(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{(1+m^2)^2 - 3m^2}t)], \quad (39)$$

$$v_{22} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1+m^2 \pm \sqrt{(1+m^2)^2 - 3m^2})}{\delta} \\ - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [dc^2(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{((1+m^2)^2 - 3m^2)t})], \quad (40)$$

$$v_{23} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2m^2 - 1 \pm \sqrt{3m^2(1-m^2) + (1-2m^2)^2})}{\delta} \\ - (1-m^2)nc^2(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{3m^2(1-m^2) + (1-2m^2)^2}t)], \quad (41)$$

$$v_{24} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2-m^2 \pm \sqrt{(2-m^2)^2 - 3(1-m^2)})}{\delta} \\ + \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [(1-m^2)nd^2(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma) \\ \times \sqrt{(2-m^2)^2 - 3(1-m^2)t})], \quad (42)$$

$$v_{25} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2-m^2 \pm \sqrt{3(m^2-1) + (2-m^2)^2})}{\delta} \\ - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [cs^2(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{3(m^2-1) + (2-m^2)^2}t)], \quad (43)$$

$$v_{26} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2m^2 - 1 \pm \sqrt{(1-2m^2)^2 - 3m^2(1+m^2)})}{\delta} \\ - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [ds^2(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma) \\ \times \sqrt{(1-2m^2)^2 - 3m^2(1+m^2)t})], \quad (44)$$

$$v_{27} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5-m^2 \pm \sqrt{(0.5-m^2)^2 - \frac{3}{16}})}{\delta} \\ - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} [(ns(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{(0.5-m^2)^2 - \frac{3}{16}}t) \\ + cs(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma)\sqrt{(0.5-m^2)^2 - \frac{3}{16}}t))^2], \quad (45)$$

$$v_{28} = -\frac{4(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ \times \frac{(0.5 + 0.5m^2 \pm \sqrt{3(0.5m^2 - 0.5)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2})}{\delta} \\ - \frac{12(0.25 - 0.25m^2)(\alpha^2\mu + \beta^2\gamma)}{\delta} (nc(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma) \\ \times \sqrt{3(0.5m^2 - 0.5)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2}t) \\ + sc(\alpha\gamma + \beta\gamma \pm 4(\alpha^2\mu + \beta^2\gamma) \\ \times \sqrt{3(0.5m^2 - 0.5)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2}t))^{-2}, \quad (46)$$

$$v_{29} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5m^2 - 1 \pm \sqrt{(1-0.5m^2)^2 - \frac{3m^2}{16}})}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} [(ns(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{(1-0.5m^2)^2 - \frac{3m^2}{16}}t) + ds(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{(1-0.5m^2)^2 - \frac{3m^2}{16}}t))^{-2}], \quad (47)$$

$$v_{30} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5m^2 - 1 \pm \sqrt{(1-0.5m^2)^2 - \frac{3m^4}{16}})}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} [(sn(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{(1-0.5m^2)^2 - \frac{3m^4}{16}}t) + ics(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma) \sqrt{(1-0.5m^2)^2 - \frac{3m^4}{16}}t))^{-2}]. \quad (48)$$

The modulus of electrostatic potentials ϕ_1 , ϕ_2 , ϕ_3 and ϕ_4 are displayed in figures 1, 2, 3 and 4 respectively, with values of parameters listed in their captions.

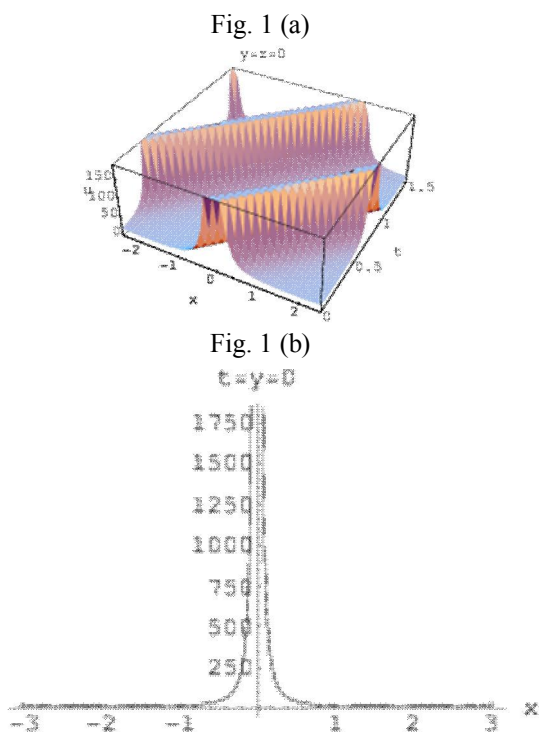


Fig. 1 The modulus of solitary wave solution ϕ_1 (Eq. 19) where $\alpha = \beta = \varepsilon = \gamma = \lambda = m = \delta = 0.5$.

Fig. 2 (a)

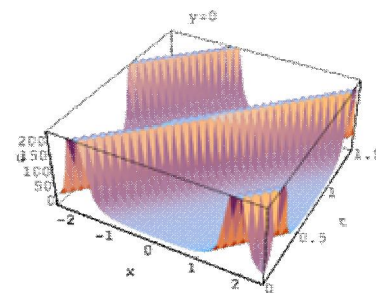


Fig. 2 (b)

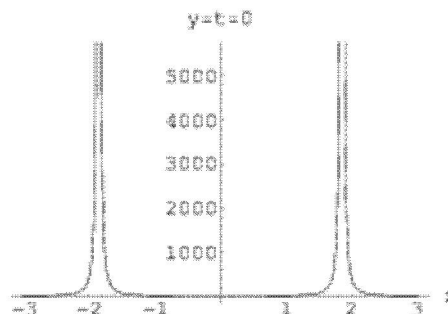


Fig. 2 The modulus of solitary wave solution ϕ_2 (Eq. 20) where $\alpha = \beta = \varepsilon = \gamma = \lambda = m = \delta = 0.5$.

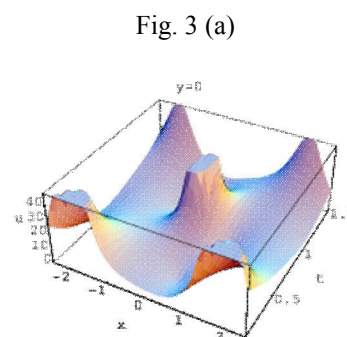


Fig. 3 (a)

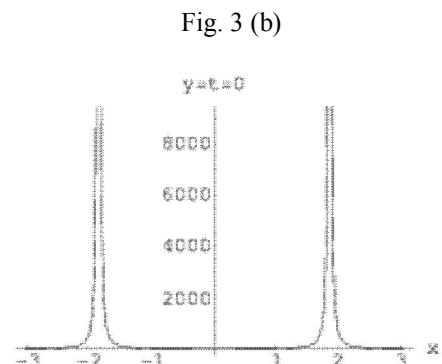


Fig. 3 The modulus of solitary wave solution ϕ_3 (Eq. 21) where $\alpha = \beta = \varepsilon = \gamma = \lambda = m = \delta = 0.5$.

Fig. 4 (a)

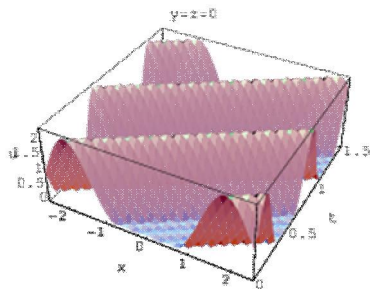


Fig. 4 (b)

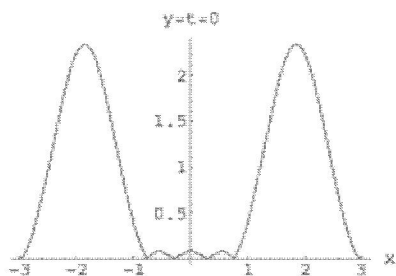


Fig. 4 The modulus of solitary wave solution ϕ_4 (Eq. 22) where $\alpha = \beta = \varepsilon = \gamma = \lambda = m = \delta = 0.5$.

3.1 Soliton solutions

Some solitary wave solutions can be obtained, if the modulus m approaches to 1 in Eqs. (19)-(48)

$$v_{31} = \frac{4(\alpha^2\mu + \beta^2\gamma)(2 \pm 4)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [\tanh^2(\alpha x + \beta y \pm 16(\alpha^2\mu + \beta^2\gamma)t) + \coth^2(\alpha x + \beta y \pm 16(\alpha^2\mu + \beta^2\gamma)t)], \quad (49)$$

$$v_{32} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm 1)}{\delta} + \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [\operatorname{sech}^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (50)$$

$$v_{33} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm 1)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [\operatorname{csch}^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (51)$$

$$v_{34} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm 5)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [2\sinh^2(\alpha x + \beta y \pm 20(\alpha^2\mu + \beta^2\gamma)t) + \operatorname{csch}^2(\alpha x + \beta y \pm 20(\alpha^2\mu + \beta^2\gamma)t)], \quad (52)$$

$$v_{35} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(-0.5 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \times [(\coth(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t) + \operatorname{csch}(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t))^2 + (\coth(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t) + \operatorname{csch}(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t))^{-2}], \quad (53)$$

$$v_{36} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(-0.5 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \times [(\tanh(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t) + \operatorname{icsch}(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t))^2 + (\tanh(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t) + \operatorname{icsch}(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t))^{-2}], \quad (54)$$

$$v_{37} = \frac{4(\alpha^2\mu + \beta^2\gamma)(2 \pm 1)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [\tanh^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (55)$$

$$v_{38} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm 1)}{\delta} + \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [\operatorname{sech}^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (56)$$

$$v_{39} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm i\sqrt{5})}{\delta} - \frac{24(\alpha^2\mu + \beta^2\gamma)}{\delta} \times [\sinh^2(\alpha x + \beta y \pm 4i(\alpha^2\mu + \beta^2\gamma)\sqrt{5}t)], \quad (57)$$

$$v_{40} = -\frac{(\alpha^2\mu + \beta^2\gamma)(-2 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \times [(\coth(\alpha x + \beta y \pm (\alpha^2\mu + \beta^2\gamma)t) + \operatorname{csch}(\alpha x + \beta y \pm (\alpha^2\mu + \beta^2\gamma)t))^2], \quad (58)$$

$$v_{41} = -\frac{(\alpha^2\mu + \beta^2\gamma)(-2 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \times [(\tanh(\alpha x + \beta y \pm (\alpha^2\mu + \beta^2\gamma)t) + \operatorname{icsch}(\alpha x + \beta y \pm (\alpha^2\mu + \beta^2\gamma)t))^2], \quad (59)$$

$$v_{42} = \frac{4(\alpha^2\mu + \beta^2\gamma)(2 \pm 1)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [\coth^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (60)$$

$$v_{43} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm i\sqrt{5})}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ [csch^2(\alpha x + \beta y \pm 4i(\alpha^2\mu + \beta^2\gamma)\sqrt{5}t)], \quad (61)$$

$$v_{44} = -\frac{(\alpha^2\mu + \beta^2\gamma)(-2 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ \times [(\coth(\alpha x + \beta y \pm (\alpha^2\mu + \beta^2\gamma)t) \\ + csch(\alpha x + \beta y \pm (\alpha^2\mu + \beta^2\gamma)t))^{-2}], \quad (62)$$

$$v_{45} = -\frac{(\alpha^2\mu + \beta^2\gamma)(-2 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ \times [(\tanh(\alpha x + \beta y \pm (\alpha^2\mu + \beta^2\gamma)t) \\ + icsch(\alpha x + \beta y \pm (\alpha^2\mu + \beta^2\gamma)t))^{-2}], \quad (63)$$

3.2 Triangular periodic solutions

Some trigonometric function solutions can be obtained, if the modulus m approaches to zero in Eqs. (19)-(48)

$$v_{46} = \frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm 1)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ [csc^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (64)$$

$$v_{47} = \frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm 1)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ [sec^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (65)$$

$$v_{48} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2 \pm 4)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ [\tan^2(\alpha x + \beta y \pm 16(\alpha^2\mu + \beta^2\gamma)t) \\ + \cot^2(\alpha x + \beta y \pm 16(\alpha^2\mu + \beta^2\gamma)t)], \quad (66)$$

$$v_{49} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(0.5 \pm)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ \times [(\csc(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t) \\ + \cot(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t))^2 \\ + (\csc(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t) \\ + \cot(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t))^{-2}], \quad (67)$$

$$v_{50} = -\frac{2(\alpha^2\mu + \beta^2\gamma)(1 \pm \sqrt{7})}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ \times [0.5(\sec(\alpha x + \beta y \pm 2(\alpha^2\mu + \beta^2\gamma)\sqrt{7}t) \\ + \tan(\alpha x + \beta y \pm 2(\alpha^2\mu + \beta^2\gamma)\sqrt{7}t))^2 \\ + 0.25(\sec(\alpha x + \beta y \pm 2(\alpha^2\mu + \beta^2\gamma)\sqrt{7}t) \\ + \tan(\alpha x + \beta y \pm 2(\alpha^2\mu + \beta^2\gamma)\sqrt{7}t))^{-2}], \quad (68)$$

$$v_{51} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ [\sin^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (69)$$

$$v_{52} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(1 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ \times [(\sin(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t) \\ + i \cot(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t))^{-2}], \quad (70)$$

$$v_{53} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2 \pm 1)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ [\tan^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (71)$$

$$v_{54} = -\frac{2(\alpha^2\mu + \beta^2\gamma)(1 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ \times [(\csc(\alpha x + \beta y \pm 2(\alpha^2\mu + \beta^2\gamma)t) \\ + \cot(\alpha x + \beta y \pm 2(\alpha^2\mu + \beta^2\gamma)t))^2], \quad (72)$$

$$v_{55} = -\frac{(\alpha^2\mu + \beta^2\gamma)(2 \pm i\sqrt{2})}{\delta} - \frac{6(\alpha^2\mu + \beta^2\gamma)}{\delta} \\ \times (\sec(\alpha x + \beta y \pm i(\alpha^2\mu + \beta^2\gamma)\sqrt{2}t) \\ + \tan(\alpha x + \beta y \pm i(\alpha^2\mu + \beta^2\gamma)\sqrt{2}t))^2, \quad (73)$$

$$v_{56} = -\frac{4(\alpha^2\mu + \beta^2\gamma)(2 \pm 1)}{\delta} - \frac{12(\alpha^2\mu + \beta^2\gamma)}{\delta} [\cot^2(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t)], \quad (74)$$

$$v_{57} = -\frac{2(\alpha^2\mu + \beta^2\gamma)(1 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \times [(\csc(\alpha x + \beta y \pm 2(\alpha^2\mu + \beta^2\gamma)t) + \cot(\alpha x + \beta y \pm 2(\alpha^2\mu + \beta^2\gamma)t))^{-2}], \quad (75)$$

$$v_{58} = -\frac{(\alpha^2\mu + \beta^2\gamma)(2 \pm i\sqrt{2})}{\delta} - \frac{12(0.25 - 0.25m^2)(\alpha^2\mu + \beta^2\gamma)}{\delta} \times (\sec(\alpha x + \beta y \pm i(\alpha^2\mu + \beta^2\gamma)\sqrt{2}t) + \tan(\alpha x + \beta y \pm i(\alpha^2\mu + \beta^2\gamma)\sqrt{2}t))^{-2}, \quad (76)$$

$$v_{59} = \frac{4(0.25 - 0.25m^2)(1 \pm 1)}{\delta} - \frac{3(\alpha^2\mu + \beta^2\gamma)}{\delta} \times (\sin(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t) + i \cot(\alpha x + \beta y \pm 4(\alpha^2\mu + \beta^2\gamma)t))^{-2}. \quad (77)$$

4. Discussions

By introducing appropriate transformations and using extended F-expansion method, we have been able to obtain in a unified way with the aid of symbolic computation system-mathematica, a series of solutions including single and the combined Jacobi elliptic function. Also, extended F-expansion method is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. When $m \rightarrow 1$, the Jacobi functions degenerate to the hyperbolic functions and given the solutions by the extended hyperbolic functions methods. When $m \rightarrow 0$, the Jacobi functions degenerate to the triangular functions and given the solutions by extended triangular functions methods.

Acknowledgements:

This article was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah. The authors, therefore, acknowledge with thanks DSR technical and financial support.

Corresponding Author:

Ali H. Bhrawy Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia, alibhrawy@yahoo.co.uk

References

1. Shah A, Saeed R. Ion acoustic shock waves in a relativistic electron-positron-ion plasmas, *Physics Letters A* 373 (2009) 4164-4168.
2. Michel FC. *Theory of Neutron Star Magnetosphere*, Chicago University Press, Chicago, 1991
3. Miller HR, Witta PJ. *Active Galactic Nuclei*, Springer-Verlag, Berlin, 1987, p. 202.
4. Michel FC. *Theory of Pulsar Magnetospheres*, *Rev. Mod. Phys.* 54 (1982) 1-66.
5. Berezhiani V, Tskhakaya DD, Shukla PK. Pair production in a strong wake field driven by an intense short laser pulse, *Phys. Rev. A* 46 (1992) 6608-6612.
6. Greaves RG, Tinkle MD, Surko CM. Creation and uses of positron plasmas, *Phys. Plasmas* 1 (1994) 1439 8pp.
7. Helander P, Ward DJ. Positron Creation and Annihilation in Tokamak Plasmas with Runaway Electrons, *Phys. Rev. Lett.* 90 (2003) 135004 4pp.
8. El-Shamy EF, El-Bedwehy NA. On the linear and nonlinear characteristics of electrostatic solitary waves propagating in magnetized electron-positron-ion plasmas, *Physics Letters A* 374 (2010) 4425-4429.
9. Kourakis I, Verheest F, Cramer N. Nonlinear perpendicular propagation of ordinary mode electromagnetic wave packets in pair plasmas and electron-positron-ion plasmas, *Phys. Plasmas* 14 (2007) 022306 10pp.
10. Sahu B, Roychoudhury R, Quantum ion acoustic shock waves in planar and nonplanar geometry, *Phys. Plasmas* 14 (2007) 072310.
11. Shkula PK, Mamun AA, Stenflo L. Vortices in a strongly magnetized electron-positron-ion plasma, *Phys. Scr.* 68 (2003) 295-298.
12. Baldock RA, Robson BA, Barrett RF. Application of an inverse-scattering method to determine the effective interaction between composite particles, *Nuclear Physics A*, 366 (2) (1981) 270-280.
13. Abdelkawy MA, Alghamdi MA, Bhrawy AH. Jacobi doubly periodic wave solutions for three versions of Benjamin-Bona-Mahony equation, *Scientific Research and Essays Vol. 7* (27), (2012) pp. 2417-2423
14. Bhrawy AH, Tharwat MM, Yildirim A, Abdelkawy MA. A Jacobi elliptic function method for nonlinear arrays of vortices, *Indian J*

- Phys, In press (2012) DOI 10.1007/s12648-012-0173-4
15. Bhrawy AH, Abdelkawy MA, Biswas A. Cnoidal and Snoidal Wave Solutions to Coupled Nonlinear Wave Equations by the Extended Jacobi's Elliptic Function Method, Communications in Nonlinear Science and Numerical Simulation, In press (2012) <http://dx.doi.org/10.1016/j.cnsns.2012.08.034>
 16. Johnpillai AG, Yildirim A, Biswas A. Chiral solitons with Bohm potential by Lie group analysis and traveling wave hypothesis, Romanian Journal of Physics, Volume 57, Numbers 3-4, 545-554. (2012).
 17. Kohl R, Biswas A, Milovic D, Zerrad E. Optical soliton perturbation in a non-Kerr law media, Optics and Laser Technology, Volume 40, Issue 4, 647-662. (2008).
 18. Crutcher S, Oseo A, Yildirim A, Biswas A. Oscillatory parabolic law spatial optical solitons. Journal of Optoelectronics and Advanced Materials, Volume 14, Numbers 1-2, 29-40. (2012).
 19. Ebadi G, Kara AH, Petkovic MD, Yildirim A, Biswas A. Solitons and conserved quantities of the Ito equation. Proceedings of the Romanian Academy, Series A. Volume 13, Number 3, 215-224. (2012).
 20. Khater AH, Callebaut DK, Bhrawy AH, Abdelkawy MA. Nonlinear periodic solutions for isothermal magnetostatic atmospheres, Journal of Computational and Applied Mathematics, (2012) 10.1016/j.cam.2012.10.012
 21. Porubov AV. Periodical solution to the nonlinear dissipative equation for surface waves in a convecting liquid layer, Phys. Lett. A. 221 (1996) 391-394.
 22. Porubov AV, Velarde MG. Exact periodic solutions of the complex Ginzburg- Landau equation, J. Math. Phys. 40 (1999) 884-896.
 23. E. Fan , Multiple travelling wave solutions of nonlinear evolution equations using a unified algebraic method, J. Phys. A: Math. Gen. 35 (2002) 6853-6872.
 24. Abdou MA. An improved generalized F-expansion method and its applications, Journal of Computational and Applied Mathematics, 214(1) (2008) 202-208
 25. Bhrawy AH, Alhuthali MSh, Abdelkawy MA. New solutions for (1+1)-dimensional and (2+1)-dimensional Ito equations, Mathematical problems in engineering, (2012) In press
 26. Bhrawy AH, Obaid M. New exact solutions for the Zhiber-Shabat equation using the extended F-expansion method. Life Sci J 2012;9(4):1154-1162
 27. Wang M, Li X. Extended F-expansion method and periodic wave solutions for the generalized Zakharov equations, Physics Letters A, 343(1-3) (2005) 48-54
 28. Kourakis I, Moslem WM, Abdelsalam UM, Sabry R, Shukla PK. Nonlinear dynamics of rotating multi-component pair plasmas and e-p-i plasmas, Plasma Fusion Res. 4 (2009) 1-11.
 29. Washimi H, Taniuti T, Propagation of ion-acoustic solitary waves of small amplitude, Phys. Rev. Lett. 17 (1966) 996-998.
 30. Awawdeh F, New exact solitary wave solutions of the Zakharov-Kuznetsov equation in the electron-positron-ion plasmas, Applied Mathematics and Computation 218 (2012) 7139-7143.

9/6/2012