Sensitivity analysis of Super-efficiency DEA Models for Iranian banks

Hadi Mohammadi Zarandini, Mohammad Reza Sheikhnabi

Enghelabe Islami Technical College, Technical and Vocational University, Yaftabad, Tehran, Iran
Hadi_mohammadi_zarandini@yahoo.com

Abstract: Banks as the economy monetary sectors and also due to the speed of the reflection of the policies of these sectors have an effective role for making economic steady growth in the whole society. Due to the variety of services, the assessment of bank units is complicated. The current assessment methods and the evaluation of bank units are empirical methods and since they are not standardized, their results in different banks are not comparable with each other. In addition, these methods do not consider the efficiency of the units and only consider the output of the units. The methodology of the data envelopment analysis (DEA) is a scientific and nonparametric approach for evaluating efficiency or none efficiency of decision making units (DMU) which has many scientific applications in banks, hospitals, power stations, insurance, and also universities. In this paper, variation of input and output of an efficient bank with considering its efficient is verified. In this regard, strong bank efficiency is defined and the sensitive variations in input and output of the bank are studied in contrast with efficient banks.
[Hadi Mohammadi Zarandini, Mohammad Reza Sheikhnabi. **Sensitivity analysis of Superefficiency DEA Models for Iranian banks.** *Life Sci J* 2012;9(4):1519-1526] (ISSN:1097-8135). http://www.lifesciencesite.com. 230

KEYWORD: Data Envelopment Analysis, Super efficiency, Sensitive Analysis, Bank System.

1- Introduction:

One of the main issues of the economics which is in relation to the economic growth, price stability, and unemployment rate adjustment and obtain a main part of economic scientists attempt is to achieve efficiency and productivity. Achieving efficiency needs to consider optimum resources and factors of production. To consider the development of the world, particularly, and the rapid growth of the production, the revision and improvement of the methods for the optimum scarce resources such as raw materials, skilled labor, and time are the factors for units of enterprises to the whole economy of a country.

Banks as the administrators of economy monetary sectors with providing possible investment and making the capital for trade activities play an important role in the development of the economy of the country. Regarding the rapid reflection of the policies of this unit in the whole economy of the country, the control of its fulfillment can be significant. Without the assessment and improving the methods and avoid destroying the resources in banks, these institutes cannot be successful in fulfillment of their dynamic role in the economic development process. The available banks assessment and evaluation methods are frequently empirical and do not have any scientific background.

In this paper, in section 2, we study a method for the assessment of the banks fulfillment on the basis of DEA that has about two decade's antecedent. Then, in section 3 the sensitivity analysis of efficient units with considering simultaneous variations in inputs and outputs with efficiency are discussed. At the end, in section 4 the mentioned methodology is used with real data of 113 branches of one bank in Tehran-Iran.

2- Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a mathematical planning method for evaluating decision making units (DMU). This method has been found by Charnes, Cooper and Rhodes (CCR) in 1978[3]. They developed Farrell nonparametric method that has been designed for evaluating (DMU) with two inputs and one output. They added mathematical planning and removed some limitations which Farrell's method caused. Then, in 1984, Banker, Charnes, and Cooper (BCC)[2] considered very important concept of return to scale and developed the applied domain of DEA. In this way, DEA subject started and during the two last decades developed both in applying and theoretical point of view. Today, the managers use DEA as an effective tool for assessment of DMU revenue. DEA different models with input view and the output oriented model of DMU assess them. This systematic view occurred with the aid of DEA model that they, in turn, are basically the planning problem. Suppose that there are n DMU that j^{th} unit of s dimensional outputs vector y_{ij} produce component x_{ij} from m dimensional input. The CCR and BCC models with input oriented are as follow, respectively:

The Input Oriented CCR

$$Min\,\theta - \varepsilon(1s^- + 1s^+)$$
$$\sum_{j=1}^{n} x_j\lambda_j + s^- = x_r\theta$$
$$\sum_{j=1}^{n} y_j\lambda_j - s^+ = y_r$$
$$\lambda_j \geq 0,\ j = 1,2,\ldots,n$$
$$s^+ \geq 0\ ,\ s^- \geq 0$$

The Input Oriented BCC

$$Min\,\theta - \varepsilon(1s^- + 1s^+)$$
$$\sum_{j=1}^{n} x_j\lambda_j + s^- = x_r\theta$$
$$\sum_{j=1}^{n} y_j\lambda_j - s^+ = y_r$$
$$\lambda_j \geq 0,\ j = 1,2,\ldots,n$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$s^+ \geq 0\ ,\ s^- \geq 0$$

In the above models, ε is a very small none Archimedean . s^+ and s^- are s and m dimensional slack variables corresponding to output and input constraints, respectively. θ and λ_i are the other variables.

During the recent years, the issue of sensitivity and stability of data envelopment analysis (DEA) result has been extensively studied. Some studies, Ali and Seiford (1993) and Smith (1997), focus on the sensitivity of DEA results to the variable and model selection. Most of the DEA sensitivity analysis studies focus on the misspecification of efficiency classification of a test decision making unit. However, we shall note that DEA is an external method in the sense that all extreme points are characterized as efficient. If data entry errors occur for various DMUs, the resulting may vary substantially (see, e.g., Sexton et al., 1986). In the current proposal, as in many other DEA sensitivity studies, we say that the calculated frontiers of DEA models are stable if the frontier DMUs that determine the DEA frontier remain on the frontier after the particular data perturbations are made for all DMUs.

By updating the inverse of the basis matrix associated with a specific efficient DMU in a DEA linear programming problem, Charnes **et al. (**1995) study the sensitivity of DEA model to single output change. This is followed by a series of sensitivity analysis articles by Charnes and Neralic (1990)[5] in which sufficient conditions preserving efficiency are determined.

Another type of DEA sensitivity analysis is based on super–efficiency DEA approach in which a test DMU is not included in reference set (Andersen and Petersen, 1993; Seiford and Zhu, 1999[12]). Charnes **et al.** (1992), Rousseau and Semple (1995) and Charnes **et al. (**1996) develop a super-efficiency DEA sensitivity analysis technique for the situation where simultaneous proportional change is assumed in all inputs and outputs for a specific DMU under the consideration. This data variation condition reduced by Zhu (1996), Seiford and Zhu (1998)[10],[11] to a situation where small inputs or outputs variations can be changed. In addition, this necessary condition for preserving the efficiency of the considered DMU was proved

The DEA sensitivity analysis methods are all developed for the situation where data variations are only applied to the test efficient DMU and the data for the remaining DMUs are assumed fixed. Obviously, this assumption may not be realistic, since possible data errors may occur in each DMU. Seiford and Zhu (1998) generalize the technique in Zhu (1996) and Seiford and Zhu (1998) to the worst-case scenario where the efficiency of the test DMU is deteriorating while the efficiencies of the other DMUs are improving. In their method, same maximum percentage data change of a test DMU and the remaining DMUs is assumed and sufficient conditions for preserving an extreme-efficient DMUs efficiency are determined. Note that Thompson **et al.** (1994) use the SCSC (strong complementary slackness condition) multipliers to analyze the stability of CCR model when the data for all efficient and all inefficient DMU are simultaneously changed in opposite directions and in same percentages. Although the data variation condition is more restrictive in Seiford and Zhu (1998) than that in Thompson et al. (1994), the super-efficiency based approach may generate a larger stability than the SCSC method does. Also the SCSC method is dependent upon a particular SCSC solution, among other and, therefore, the resulting analysis may vary.

For the DEA sensitivity analysis based upon the inverse of basis matrix, is referred to Neralic (1994). It is well-known that certain super-efficiency DEA models may be infeasible for some extreme-efficient DMUs. Seiford and Zhu (1999) develop the necessary and sufficient conditions for infeasibility of various super-efficiency DEA models. Although the super-efficiency DEA models employed in Charnes et al.

(1992) and Charnes et al. (1996) and did not encounter the infeasibility problem, the models used in Seiford and Zhu (1998a). Seiford and Zhu (1998a)[10] discovered the relationship between infeasibility and stability of efficiency classification. That is, infeasibility means that the CCR efficiency of the test DMU remains stable to data changes in the test DMU. Furthermore, Seiford and Zhu (1998b) [11]showed that this relationship is also true and the simultaneous data change case and other DEA models, such as BCC model of Banker et al. (1984) and additive model of Charnes et al. (1985b). This finding is critical since super-efficiency DEA models in Seiford and Zhu (1998b) are frequently infeasible for real-world data sets, indicating efficiency stability with respective to data variations in inputs/outputs associated with infeasibility. As a result, DEA sensitivity analysis can be easily applied if we employ various super-efficiency DEA models. By using super-efficiency DEA models, the sensitivity analysis of DEA efficiency classification can be easily achieved. Since the approach uses optimal values to various super-efficiency DEA models, our approach provides "what-if" tool to the standard DEA analysis kit. We are able to know what may happen to DMUs efficiency, if data variation occurs in all DMUs as a result of new strategic planning. The new sensitivity analysis technique can well be applied to inefficient DMUs if we are interested in preserving the inefficiency of inefficient DMUs. (see Liang, L., Zha, Y., Cook, W.D. and Zhu, Joe, A. in press).

Suppose that there are n DMU such that j^{th} unit of s dimensional outputs vector y_{ij} produce component x_{ij} from m dimensional input. The CCR and BCC models with input oriented as follows, respectively:

The Input Oriented CCR

$$Min\ \theta - \varepsilon(1s^- + 1s^+)$$
$$\sum_{j=1}^{n} x_j\lambda_j + s^- = x_r\theta$$
$$\sum_{j=1}^{n} y_j\lambda_j - s^+ = y_r$$
$$\lambda_j \ge 0,\ j = 1,2,\dots,n$$
$$s^+ \ge 0\ ,\ s^- \ge 0$$

The Input Oriented BCC

$$Min\,\theta - \varepsilon(1s^- + 1s^+)$$
$$\sum_{j=1}^{n} x_j\lambda_j + s^- = x_r\theta$$
$$\sum_{j=1}^{n} y_j\lambda_j - s^+ = y_r$$
$$\lambda_j \ge 0,\ j = 1,2,\dots,n$$
$$\sum_{j=1}^{n}\lambda_j = 1$$
$$s^+ \ge 0\ ,\ s^- \ge 0$$

Where ε very small non-Archimedean is number, s^+ and s^- are s dimensional slack variables corresponding to output and input constraints, respectively, θ and λ are the other variables. Suppose I is a set of inputs data that are variable and O is a set of outputs that are also variable. In this case, we have:

$$DMU_O$$

$1-$ *Input Oriented Case*

$$\begin{cases} y^{\wedge}_{ro} = \tau\, y_{ro}, 0 \le \tau \le 1, & r \in O \\ y^{\wedge}_{ro} = y_{ro} & r \notin O \end{cases}$$

$2-$ *Output Oriented Case*

$$\begin{cases} x^{\wedge}_{xo} = \delta\, x_{io}\,,\ \delta \ge 1, & i \in I \\ x^{\wedge}_{xo} = x_{io} & i \notin I \end{cases}$$

$$DMU_J,\ J \ne O$$

$$\begin{cases} y^{\wedge}_{ro} = y_{ro} - (1-\tau)y_{ro}, & r \in O \\ y^{\wedge}_{ro} = y_{ro} & r \notin O \end{cases}$$

$$\begin{cases} x^{\wedge}_{io} = x_{io} + (\delta-1)x_{io}, & i \in I \\ x^{\wedge}_{io} = x_{io} & i \notin I \end{cases}$$

$1-Input\ Variable\ Case$

$$\begin{cases} y_{rj}^{\wedge} = y_{rj}/\tau, 0 \prec \tau \le 1, & r \in O \\ y_{rj}^{\wedge} = y_{ro} & r \notin O \end{cases}$$

$$\begin{cases} y_{rj}^{\wedge} = y_{rj} + \dfrac{1-\tau}{\tau} y_{rj}, & r \in O \\ y_{rj}^{\wedge} = y_{rj} & r \notin O \end{cases}$$

$2-Output\ Variable\ Case$

$$\begin{cases} x_{xj}^{\wedge} = x_{ij}/\delta, \delta \ge 1, & i \in I \\ x_{xj}^{\wedge} = x_{ij} & i \notin I \end{cases}$$

$$\begin{cases} x_{ij}^{\wedge} = x_{ij} - \dfrac{(1-\delta)}{\delta} x_{ij}, & i \in I \\ x_{ij}^{\wedge} = x_{ij} & i \notin I \end{cases}$$

Now for these two cases we have:

Theorem 1: If $1 \le \delta \le \sqrt{\beta^*}$ then the DMU remain efficient, where $\sqrt{\beta^*}$ is an upper bound of variations of inputs and by the above modified the follow efficiency model is obtained:

$$\begin{aligned} &\beta^* = Min\ \beta \\ &\sum_{\substack{j=1 \\ j \ne o}}^{n} x_{ij}\lambda_j \le \beta x_{io}, \quad i \in I \\ &\sum_{\substack{j=1 \\ j \ne o}}^{n} x_{ij}\lambda_j \le x_{io}, \quad i \notin I \\ &\sum_{\substack{j=1 \\ j \ne o}}^{n} y_{rj}\lambda_j \ge y_{ro}, \quad r = 1,2,\dots,s \\ &\beta \ge 0, \quad \lambda_j \ge 0 \end{aligned}$$

Theorem 2: If $\sqrt{\alpha^*} \le \tau \le 1$, then the DMU remain efficient, where $\sqrt{\alpha^*}$ is a lower bound of variations of outputs, and by the above modified model follow efficiency models obtained as:

$$\begin{aligned} &\alpha^* = Max\ \alpha \\ &\sum_{\substack{j=1 \\ j \ne o}}^{n} x_{ij}\lambda_j \le x_{io}, \quad i = 1,2,\dots,m \\ &\sum_{\substack{j=1 \\ j \ne o}}^{n} y_{rj}\lambda_j \ge \alpha x_{ro}, \quad r \in O \\ &\sum_{\substack{j=1 \\ j \ne o}}^{n} y_{rj}\lambda_j \ge y_{ro}, \quad r \notin O \\ &\alpha \ge 0, \quad \lambda_j \ge 0 \end{aligned}$$

Assessment of efficiency and the analysis of sensitive bank system

In this study 113 branches of an Iranian trading bank are considered. In the interior classification of the bank, the branches are consisted in a region in Tehran and, therefore, all the branches are selected from that region and they are compared in trading and economic positions. In the following, the inputs and outputs which have been employed with the help of bank specialists are introduced.

Introducing Inputs

The following inputs have been achieved on the basis of research and bank specialists (table 3):

1- The number of personnel of each unit with considering a particular weight for each of them:
  1-1 Personnel with high education
  1-2 Personnel with high experience
2- Regional position of the considered unit (branch) from the trade and economical view
3- Congestion of the branches of the other banks close to the under study unit
4- Infrastructure of the considered unit
5- Cost of the considered unit:
  5-1 Personnel costs
  5-2 Administrative costs
  5-3 Operational costs

6- The number of computer terminals used in the considered unit

In this study, the selected inputs with considering limited information are as follows:

1- Personnel costs
2- Number of terminals
3- Rate of renting

The Personnel costs are the function of the number of staffs and a combination of branch staffs. For this reason, this input has been used in place of the number of staffs. The number of branch computer terminals is the second input that is used in the same way as received. The cost of renting is another input which is a function of regional trade position and infrastructure of the branch. Particularly, regional trade position has an effect on the rate of renting cost of branch. This input has been used as an indicator for comparing economical position of regions that the branches have been established there. Rent costs are considered up to date. Concerned information to input has been achieved from the related centers in bank and then without any changes has been used.

Introducing Outputs

Considering the main duties of the trade banks, branch outputs are studied in the following 3 sections:

1- Outputs concerned to the branch activities in the equipment of sources section
2- Outputs concerned to the branch activities in the allocation of sources section Outputs concerned to the branch activities in the services section.

Finally, regarding informational limitations and the bank specialists' point of view, the following outputs are considered (table 6);

1- Sources
2- Consumptions
3- Services
4- Account numbers

Considering Computing results and Conclusion

In this section the results are discussed. In order to solve linear programming, the GAMS software is used. According to the results, the following branches of banks in the sample obtained efficient (Table4).The branches are numbers 7, 8, 21, 22, 25, 26, 29, 32, 34, 37, 41, 49, 50, 57, 62, 71, 78, 80, 83, 84, 85, 92, 111, 112, and 113. These make assessment standard of efficient frontier for the other branches. Different combinations of efficiency branches provide possibility for the presentation revenue improvement approaches of inefficiency branches. Beside efficiency result of the models with input oriented, the effectiveness references presentation and the concerned computational provide revenue improvement possibility approaches in such a way that it can be reduced of the inefficiency branches so as to approach to frontier. As an example, the branch 1 with 95 percent of efficiency is inefficient and its assessment references are the branches 26, 29, 62 and 85 with corresponding weight .586, .235, .160 and .018, respectively.

Revenue improvement approaches with input oriented

For considering branches operating situation with the rate of efficiency and their references, the revenue improvement approaches with input oriented are computed and also presented. Table 1 shows the operating situation of the branch of 64. Particularly with allocating the corresponding weights to reference branches, the assessment reference branches show that they correspond with a point on efficient frontier.

Table (1): the revenue of branch number 64

Branch 64 Efficiency rate %86		inputs			Outputs			
Branch current situation		25000000	12	17675020	2836	6737	253183	19190151
References	weights							
37	.361	1000000	6	9580191	1077	5822	1359328	13855106
84	.639	7000000	11	18276182	3798	11862	6024026	29902424
Branch desirable situation		8083000	9.1	15136929	2816	9681	4340070	24109342
Reducing inputs so as to Transform the current situation To desirable one		16917000	2.8	2538091				

In fact, the desired situation suggests the best one for the branch. It produces the current outputs with unknown rate. In order to revenue improvement, the branch number 64 reduction rate of inputs is recommended.

We embark to improve the revenue with input oriented and the branches are leads to frontiers reality that has been described by them. The following are the results:

Table (2): summery result

Inputs Situation	current situation	desired
Branches personality cost	804698493	711433241
Branches computer terminal	360	291
Branches rental costs	276936516	40030000

With this improvement we should obtain the current available outputs with reduction %11.59 in branches personnel costs and %10.16 in branches computer terminal and %30.81 in branches rental structural costs. In table 4 the efficiency is presented with using BCC model. In this table, the efficiency of DMUs and their indicator units are specified. As an example, the DMU number 14 has 70% efficiency and its indicator units are the DMU numbers 32, 34, 62, 83, and 85. Obviously, this says the linear combination of the indicator units where all of them are effective. There are at most 70% inputs of the DMU and number 14 produce at least the same outputs. The coefficient of this linear combination is the same value λ that obtained from solving the model. The coefficient λ for the DMU numbers 32,34,62,83, and 85 are .148, .117, .156, .455, and .123, respectively. As an example, the unit 14 has personnel cost 8864374 Rials (Iranian money unit), whereas the linear combination of the personnel cost of the indicator units of number 14 with cited coefficient is 735200 Rials. It shows that this unit is effective and the money saving in the personnel cost will be 1512373 Rials. In summary, if all inefficient units with the reduction of their inputs get efficiency and the corresponding outputs have no any changes, then the monthly money saving in personnel cost will be 93265252 Rials. That is, there is a possibility that the reduction of about %19 is due to the reduction in personnel cost by reducing the number of personnel. In addition, there are 69 computer terminals out of total number of 360. Also, there is possible reduction in rate rental units, as we said before, the upper bounds $g_o(\delta-1)$ and $g(=\frac{\delta-1}{\delta})$ for the input variations and $h_o(=1-\tau)$, $h(=\frac{1-\tau}{\tau})$ for the outputs are obtained, where the upper bounds g_o and g_1 with considering theorem and the upper bounds h_o and h_1 by using theorem will be computed. There are three inputs and four outputs for the 113 bank units. Therefore, there are 20 cases for the only variations and the only output variations and the simultaneous inputs and outputs variation. For example, we consider variations in three inputs and four outputs, simultaneously. Table 5 is an example. It shows:

$(g_o, g) = (\%31.114, \%32.75)$, $(h_o h) = (\%47.08, \%88.98)$.

That is, three inputs of the DMU 34 can simultaneously be increased as %23.75. Also four outputs of DMU 34 can be decreased as %47.08 and four outputs of the other units can be increased %88.98; while, unit 34 remained efficient.

Table (3): trade banks input

DMU	P.C	T.N	R.R	DMU	P.C	T.N	R.R	DMU	P.C	T.N	R.R	DMU	P.C	T.N	R.R
1	1048247	4	3500000	29	5059262	3	1000000	57	4833471	3	3000000	85	16488749	5	1500000
2	11091919	7	1500000	30	6538932	4	4000000	58	5675406	5	6000000	86	543265	5	5000000
3	6006368	7	9000000	31	7743787	4	4500000	59	6845551	4	4000000	87	5677375	4	1000000
4	4983049	6	10000000	32	4728702	6	6500000	60	4648552	5	9000000	88	6041356	5	1000000
5	595421	4	3000000	33	5710969	4	3600000	61	8067707	5	3000000	89	13551643	7	1000000
6	12361292	8	12000000	34	5376670	6	8000000	62	6897099	4	2500000	90	7201054	8	5000000
7	5667654	3	7000000	35	5853293	4	5000000	63	5104826	4	6000000	91	5920688	4	500000
8	4576087	4	5000000	36	5413185	4	1000000	64	17675020	12	25000000	92	19213993	11	3000000
9	8758295	6	1200000	37	9580191	4	8000000	65	4643428	4	8000000	93	5324270	4	3000000
10	6556536	4	7000000	38	15788314	7	4000000	66	7069601	4	8000000	94	10724288	6	5000000
11	4247944	4	2000000	39	4319690	5	2500000	67	7798662	5	5000000	95	8682941	6	8000000
12	9186560	5	5000000	40	9169764	7	6000000	68	24765351	11	5000000	96	5754220	4	3500000
13	14483093	6	3500000	41	4271776	3	5000000	69	7513519	6	8000000	97	7458563	5	5000000
14	8864374	6	7000000	42	8248442	4	7000000	70	8352050	6	6000000	98	6368766	4	1000000
15	4509167	5	3500000	43	6898955	5	6000000	71	6631905	3	8000000	99	6313385	5	7000000
16	5843595	4	45000000	44	6389470	4	2500000	72	5864453	5	5000000	100	4142405	4	1500000
17	6811667	4	6000000	45	7733840	4	5600000	73	4075674	4	3000000	101	5567881	4	3000000
18	5166614	5	1800000	46	5178158	5	6000000	74	5587501	5	1500000	102	4635193	4	3000000
19	5696379	4	2500000	47	8411440	4	5000000	75	8538411	4	6000000	103	4521143	4	6000000
20	8663787	5	1000000	48	6951224	4	4000000	76	10649418	8	10000000	104	7483051	5	10000000
21	6187368	4	1500000	49	3350041	3	2000000	77	8548086	4	4000000	105	8144612	6	7000000
22	4795112	3	5000000	50	13232901	8	4500000	78	5483076	3	600000	106	4410235	4	5000000
23	5856757	5	2000000	51	4456491	4	3000000	79	6968171	5	5000000	107	4422029	5	4000000
24	4950695	6	3500000	52	6291246	4	1500000	80	2846195	4	1700000	108	8577974	4	1200000
25	5223122	3	1000000	53	5424185	5	1500000	81	6662196	5	10000000	109	5468282	4	1000000
26	6491851	4	80000	54	6094183	5	9000000	82	6830834	6	1000000	110	5764381	5	2000000
27	4611285	4	800000	55	5175868	5	3000000	83	3875326	3	1500000	111	2692566	3	1500000
28	4736197	4	5000000	56	8487390	5	4000000	84	18276182	1	70000	112	4892159	3	3000000
												113	1868551	6	8000000

Table (4): Branches efficiency assessment with using BBC model

DM	eff	ref	wei	ref	wei	ref	wei	ref	wei	ref	wei	ref	wei
1	95%	26	.586	29	.235	62	.160	85	.018				
2	66%	8	.253	21	.185	26	.128	78	.303	92	.131		
3	67%	32	.463	34	.093	83	.102	111	.343				
4	78%	32	.429	34	.124	85	.000	111	.438	113	.009		
5	75%	7	.003	25	.285	71	.073	83	.639				
6	59%	8	.253	21	.185	26	.128	78	.303	92	.131		
7*	100%	7	1.000										
8*	100%	8	1.000										
9	58%	26	.264	29	.222	34	.072	83	.427	85	.015		
10	80%	25	.427	29	.442	34	.055	83	.063	85	.014		
11	77%	26	.036	32	.011	62	.006	83	.310	85	.002	111	.635
12	61%	25	.778	29	.201	34	.008	85	.013				
13	97%	21	.291	78	.298	85	.128	92	.282				
14	70%	32	.148	34	.117	62	.156	83	.455	85	.123		
15	82%	26	.078	32	.130	34	.098	80	.322	85	.010	111	.362
16	85%	32	.011	34	.044	62	.145	83	.754	85	.046		
17	75%	26	.264	29	.222	34	.072	83	.427	85	.015		
18	84%	26	.370	32	.102	85	.002	111	.526				
19	75%	57	.342	78	.163	83	.326	85	.001	111	.169		
20	71%	25	.094	26	.191	29	.582	34	.112	85	.121		
21*	100%	21	1.000										
22*	100%	22	1.000										
23	75%	21	.049	26	.347	34	.072	80	.142	111	.390		
24	73%	26	.105	32	.204	34	.011	80	.659	111	.021		
25*	100%	25	1.000										
26*	100%	26	1.000										
27	94%	1	.445	26	.213	111	.324						
28	90%	32	.171	62	.077	83	.736	85	.001	111	.015		
29*	100%	29	1.000										
30	76%	25	.550	29	.196	34	.005	83	.241	85	.008		
31	88%	25	.641	29	.167	34	.163	85	.024				
32*	100%	32	1.000										
33	83%	26	.091	34	.008	62	.182	83	.715	85	.003		
34*	100%	34	1.000										
35	82%	26	.212	29	.146	62	.049	83	.591	85	.002		
36	88%	8	.112	21	.122	26	.275	78	.134	111	.357		
37*	100%	37	1.000										
38	99%	26	.507	84	.355	85	.086	92	.053				
39	85%	26	.152	32	.131	34	.007	80	.667	111	.043		
40	66%	26	.342	34	.244	80	.313	85	.098	92	.003		
41*	100%	41	1.000										
42	85%	25	.459	29	.396	34	.095	85	.049				
43	90%	32	.231	34	.101	62	.394	83	.210	85	.065		
44	91%	25	.686	26	.136	34	.156	85	.210	85	.065		
45	76%	7	.257	25	.565	29	.154	85	.024				
46	97%	32	.180	34	.204	80	.197	85	.106	111	.218	113	.096
47	82%	32	.058	34	.040	83	.883	111	.018				
48	77%	25	.764	29	.205	34	.027	85	.004				
49*	100%	49	1.000										
50*	100%	50	1.000										
51	85%	32	.080	34	.054	83	.667	111	.200				
52	78%	21	.004	26	.085	78	.377	83	.524	85	.009		
53	82%	21	.132	26	.315	34	.011	80	.542				
54	69%	32	.102	62	.127	83	.617	85	.002	111	.152		
55	72%	26	.131	32	.092	34	.058	83	.147	111	.573		
56	65%	25	.636	29	.050	34	.047	83	.217	85	.051		
57*	100%	57	1.000										

DM	eff	ref	wei	ref	wei	ref	wei	ref	wei	ref	wei	ref	wei
58	82%	32	.272	34	.044	62	.151	83	.530	85	.002		
59	85%	25	.710	29	.148	34	.130	85	.012				
60	76%	32	.226	34	.046	83	.230	111	.498				
61	68%	25	.410	26	.160	34	.055	83	.329	85	.046		
62*	100%	62	1.000										
63	76%	34	.016	83	.973	85	.000	111	.011				
64	86%	37	.361	84	.639								
65	87%	32	.065	62	.286	85	.001	85	.012				
66	76%	25	.426	29	.543	62	.018	85	.012				
67	73%	25	.389	26	.241	29	.197	34	.117	83	.039	85	.017
68	77%	26	.326	84	.336	85	.039	92	.300				
69	71%	32	.195	34	.101	62	.381	83	.130				
70	73%	32	.120	34	.121	62	.629	83	.130				
71*	100%	71	1.000										
72	70%	32	.136	62	.088	83	.642	111	.134				
73	84%	34	.118	83	.319	85	.003	111	.560				
74	76%	21	.050	26	.344	34	.008	80	.391	111	.206		
75	79%	25	.570	29	.370	34	.058	85	.005				
76	54%	32	.105	34	.185	62	.427	83	.264	85	.019		
77	86%	25	.329	26	.098	29	.220	62	.351	85	.003		
78*	100%	78	1.000										
79	79%	32	.085	34	.076	62	.485	83	.355				
80*	100%	80	1.000										
81	75%	32	.000	34	.158	62	.252	83	.580	85	.009		
82	64%	32	.236	34	.023	62	.089	83	.650	85	.002		
83*	100%	83	1.000										
84*	100%	84	1.000										
85*	100%	85	1.000										
86	72%	32	.174	34	.019	62	.687	111	.120				
87	84%	21	.021	26	.337	78	.202	83	.131	111	.309		
88	83%	32	.142	34	.172	62	.141	83	.521	85	.025		
89	78%	26	.282	34	.032	37	.686						
90	70%	32	.649	34	.184	62	.116	83	.051				
91	79%	26	.056	29	.281	62	.106	83	.557	85	.001		
92*	100%	92	1.000										
93	97%	32	.154	34	.016	62	.364	83	.646	85	.005		
94	71%	26	.635	37	.116	62	.231	85	.018				
95	69%	26	.032	34	.190	62	.459	83	.293	85	.025		
96	78%	25	.056	26	.100	29	.158	62	.021	83	.662	85	.003
97	85%	32	.039	34	.135	62	.722	83	.101	85	.004		
98	88%	25	.273	26	.464	34	.008	83	.246	85	.009		
99	68%	29	.062	34	.121	83	.805	85	.011				
100	91%	26	.262	32	.048	111	.690						
101	79%	25	.234	29	.077	34	.046	83	.641	85	.002		
102	85%	32	.083	62	.159	83	.354	111	.403				
103	96%	32	.231	34	.022	62	.075	83	.672				
104	76%	26	.372	34	.070	62	.184	83	.361				
105	81%	32	.182	34	.134	37	.121	62	.563				
106	83%	32	.096	34	.012	83	.630	111	.262				
107	89%	26	.092	32	.438	111	.470						
108	95%	25	.311	26	.615	34	.048	85	.027				
109	85%	21	.034	25	.072	26	.376	78	.086	111	.432		
110	76%	21	.154	26	.254	34	.051	80	.226	111	.315		
111*	100%	111	1.000										
112*	100%	112	1.000										
113*	100%	113	1.000										

Table (5):sensitive analytical results regarding to three inputs and four outputs

Effi. DMU	In.inpout DMU_0	De.input DMU_J	De.output DMU_0	In.output DMU_J
26	infeasible			
32	%3.44	%3.32	%14.24	%16.60
34	%31.14	%23.75	%47.07	%88.98
37	infeasible			
62	infeasible			
84	%11.35	%10.19	%12.82	%14.70
85	Without any change			
92	infeasible			

Table (6): Outputs of 113 trade bank

DMU	SOU	COM	S.N	A.N	DMU	SOU	COM	S.N	A.N	DMU	SOU	COM	S.N	A.N	DMU	SOU	COM	S.N	A.N
1	8072802	1137288	3553	1027	29	3369191	73004	2572	745	57	1537353	547768	1386	1661	85	1207845	30613605	1816	1377
2	3796848	8802238	1771	1111	30	2810359	441499	1887	1267	58	4507971	551102	2238	762	86	2805270	108529	1718	986
3	4388834	328775	3650	3473	31	3832076	1120866	3576	1112	59	3886103	741356	3249	919	87	1997840	293928	2070	926
4	3275816	723422	2873	1237	32	6183678	1299979	2553	2163	60	2255696	134776	1569	1044	88	4856578	1160545	3306	1292
5	2144232	130902	1497	1043	33	3029892	340393	2068	1003	61	3792390	2779127	2342	1085	89	12465404	906861	4585	2160
6	558342	1669955	4685	2848	34	6742874	428067	12680	1458	62	10235516	178267	2249	983	90	5957086	597212	4102	1370
7	2379602	978280	1569	1666	35	3850715	227202	2173	1001	63	1327064	127459	1976	861	91	2924174	134378	2163	648
8	1847615	369477	1341	762	36	2814899	841857	1363	669	64	19190101	2531283	6737	2836	92	14981686	43515415	4755	1888
9	3866057	1132261	2907	1372	37	13855106	1359328	5822	1071	65	2901544	195778	844	512	93	3972444	448796	1964	1104
10	2629585	737234	2660	881	38	25500395	8219533	5822	1365	66	6149032	566435	1863	1917	94	12923013	1140798	2623	1683
11	1364925	158713	793	707	39	2481749	217056	1542	1055	67	5454577	1139455	3768	1027	95	7802798	1495416	3782	1064
12	3085390	899475	1828	1438	40	6353515	3229342	4745	1215	68	18779101	13973829	6074	2483	96	2756073	241804	1845	1046
13	5944605	20520010	2161	944	41	2404629	326794	1163	739	69	6470746	230553	3220	1296	97	5585589	498031	3278	1285
14	5509590	7677767	2718	452	42	3922389	2641086	2778	2003	70	6470746	230553	3220	1643	98	2279105	500717	3247	895
15	2761161	5721165	2239	1074	43	5363843	4651810	3206	1106	71	2131713	481673	2043	762	99	3460702	648227	3020	922
16	3482386	2739362	2056	933	44	4939518	1038649	3753	903	72	2772727	184187	1606	773	100	2114675	567999	1266	761
17	3137177	578727	1783	841	45	2856027	1947496	1445	894	73	1578610	287373	2211	3557	101	3413493	211528	1932	1083
18	2979151	277762	1681	995	46	3361165	7527834	3632	1013	74	3061077	291655	1981	885	102	2528746	41547	1167	675
19	1642125	257041	1211	666	47	2767980	128683	2138	701	75	3373373	185792	2663	792	103	3476094	205468	2098	813
20	4858125	764296	3475	1557	48	2282079	254094	2253	816	76	6827827	1402650	3147	2063	104	6239642	933676	3445	1271
21	2636044	4588414	1777	706	49	1087799	165857	723	483	77	4633627	293017	2174	1089	105	7836688	300411	3721	1641
22	2449765	318999	1601	918	50	16553107	15447880	8038	1181	78	1647497	241576	1561	659	106	2090219	78817	1424	733
23	2606134	456496	2475	1163	51	3097595	71839	1893	1027	79	5400530	135597	2471	1327	107	3302432	417763	1665	980
24	2832667	230500	1522	1245	52	3026974	485292	1837	1111	80	990122	49881	897	750	108	3244055	1640191	4021	881
25	4144835	53852	896	578	53	2904981	1123450	1949	3473	81	4963290	612598	3394	899	109	2732872	373473	1996	890
26	5707467	7861	4549	1240	54	2801642	356528	1152	1237	82	3643806	537143	2033	1280	110	3361719	1517184	2100	707
27	1931856	156395	1121	680	55	2372087	74417	1761	1043	83	1913860	100350	1550	832	111	582473	21207	270	348
28	3148362	347568	1707	1092	56	3494077	3153101	2076	2848	84	29902424	6024026	11862	3798	112	23672613	232131	792	334
															113	162874	232131	435	42

Reference

1. Mohammadi, H. (1999). Sensitive analysis and power of efficient units in data envelopment analysis. MSc Desertation, *Amirkabir University of Technology*. Tehran-Iran.
2. Banker, R.D.,A,. Charns and W.W.Cooper, "some models for estimating technical and scale inefficiencies in Data envelopment analysis," Management Science 30,1078-1092, 1984
3. Charnes, A, W.W. Cooper and E. Rhodes, "measuring the efficiency of decision making units," European J. of Oper. (1978) 429-444.
4. Farrell,M., " The measuring of productivity efficiency," Journal of Statistical Society, Series A(General), Vol. 120(1957), pp.253-281
5. Charnes, A, and L.Neralic, "sensitivity analysis of the additive model in data envelopment analysis", European J. of Operational Research 48 (1990), 332-341.
6. Cooper, W.W., Li, S., Seiford, L.M., Thrall, R.M. and Zhu, Joe, Sensitivity and stability analysis in DEA: some recent developments, Journal of Productivity Analysis, Vol. 15, (2001), 217-246.
7. Cooper, W.W., Li, Shanling, Seiford, L.M. and Zhu, Joe, Sensitivity Analysis in DEA, in Handbook on Data Envelopment Analysis, eds W.W. Cooper, L.M. Seiford and J. Zhu, Chapter 3, 75-97, Kluwer Academic Publishers, Boston. 2004.
8. Cook, W.D. and Zhu, Joe, Rank order data in DEA: A general framework, European Journal of Operational Research, Vol. 174, Issue 2 (2006), 1021-1038.
9. Huang, Z.M., Li, S.X. and Zhu, Joe, A Special Issue on "Data Envelopment Analysis: Theories and Applications" in honor of William W. Cooper, International Journal of Information Technology and Decision Making, Vol. 4, No. 3 (2005), 311-316
10. Seiford , Lawrence M. and joe Zhu, " Stability regions for maintaining efficiency in data envelopment analysis", European J. of Operational Research Vol. 108, No. 1, (1998a), 127-139.
11. Seiford , Lawrence M. and joe Zhu, " Sensitivity analysis of DEA models for simultaneous change in all the data,", Journal of the Operational Society, Vol. 49, (1998b), 1060-1071.
12. Seiford , Lawrence M. and joe Zhu, " Infeasibility of super-efficiency data envelopment analysis models", INFOR, Vol. 37, No.2, (1999), 174-187.
13. Zhu.J. "Super-efficiency and DEA sensitivity analysis.", European J. of Operational Research, Vol.129 (2001), pp 443-455.
14. Liang, L., Zha, Y., Cook, W.D. and Zhu, Joe, A Modified Super-efficiency DEA Model for Infeasibility, Journal of Operational Research Society 60, 276-281(February 2009).

9/22/2012