New exact solutions for the Zhiber-Shabat equation using the extended F-expansion method

Ali H. Bhrawy[1,2] and Mustafa Obaid [1]

[1.] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia
[2.] Department of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt
(Bhrawy) alibhrawy@yahoo.co.uk, (Obaid) drmobaid@yahoo.com

Abstract: Extended F-expansion method is proposed to seek exact solutions of the Zhiber-Shabat (ZS) equation. As a result, many new and more general exact solutions are obtained. Interesting Jacobi doubly periodic wave solutions is obtained from the F-expansion (EFE) method with symbolic computation. It is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. In addition, as an illustrative sample, the properties for the Jacobi doubly periodic wave solutions of theses equations are shown with some figures.

[Bhrawy AH, Obaid M. **New exact solutions for the Zhiber-Shabat equation using the extended F-expansion method**. *Life Sci J* 2012;9(4):1154-1162] (ISSN:1097-8135). http://www.lifesciencesite.com. 172

Keywords: Extended F-expansion method; Exact solutions; Liouville equation; Dodd-Bullough-Mikhailov equation; Tzitzeica-Dodd-Bullough equation; sinh-Gordon equation; Zhiber-Shabat equation.

1. Introduction

Conte and Musette [1], studied Zhiber-Shabat (ZS) equation and obtained two kinds of solutions. Wazwaz [2] obtained six exact solutions of ZS equation by using tanh and extended tanh methods. Tang et al. [3] considered the existence of bounded travelling wave solutions of ZS equation and obtained several travelling wave solutions. Recently, Bin et al. [4] studied ZS equation using the bifurcation theory and the method of phase portraits analysis and obtained many solitary wave solutions, compacton solutions and smooth periodic wave solutions. Moreover, the $\frac{G'}{G}$-expansion method is introduced to solve ZS equation by Borhanifar and Moghanlu [5] .

In this paper, we will apply the extended F-expansion method to study ZS equation

$$u_{xt} + \beta e^{u} + \gamma e^{-u} + \lambda e^{-2u} = 0, \qquad (1)$$

where β, γ and λ are arbitrary constants. If we take $\gamma = \lambda = 0$, Eq. (1) reduced to the Liouville equation [6]. If we take $\lambda = 0$, Eq. (1) reduced to the sinh-Gordon equation [7]. And for $\gamma = 0$,(1) reduced to the well-known Dodd-Bullough-Mikhailov equation [8]. Moreover, for $\beta = 0$, $\gamma = -1$, $\lambda = 1$, (1) reduced to the Tzitzeica-Dodd-Bullough equation [8]. The previous equations plays an important role in many scientific applications such as solid state physics, nonlinear optics, dusty plasma, plasma physics, fluid dynamics, mathematical biology, nonlinear optics, dislocations in crystals, kink dynamics, and chemical kinetics, and quantum field theory. Moreover many authors have studied these equation (see for instance, [9]-[10]).

It is known that many physical phenomena are often described by nonlinear evolution equations (NLEEs). Integrable systems and NLEEs have recently attracted much attention of mathematicians as well as physicists. Many methods for obtaining explicit travelling solitary wave solutions to NLEEs have been proposed. Among these are the tanh methods [11]-[12], $\frac{G'}{G}$-expansion method [13]-[14], the exp-function method [15]-[16], Jacobi and extended Jacobi elliptic function expansion methods [17]-[18], the inverse scattering transform [19]- [20] and so on. Recently F-expansion method [21]-[25] was proposed to obtain periodic wave solutions of NLEEs, which can be thought of as a concentration of JEF expansion since F here stands for everyone of JEFs.

In this paper, we apply the extended F-expansion (EFE) method with symbolic computation to Eq. (1) for constructing their interesting Jacobi doubly periodic wave solutions. It is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. In addition the algorithm that we use here also a computerized method, in which generating an algebraic system.

2. Material and Methods

In this section, we introduce a simple description of the EFE method, for a given partial differential equation .

$$G(u, u_x, u_t, u_{xt}, \ldots) = 0. \qquad (2)$$

We like to know whether travelling waves (or stationary waves) are solutions of Eq. (2). The first step is to unite the independent variables x and t into one particular variable through the new variable

$$\zeta = x - vt, \quad u(x,t) = U(\zeta),$$

where v is wave speed, and reduce Eq. (2) to an ordinary differential equation(ODE)

$$G(U, U', U'', U''', \ldots) = 0. \tag{3}$$

Our main goal is to derive exact or at least approximate solutions, if possible, for this ODE. For this purpose, let us simply U as the expansion in the form,

$$u(x,t) = U(\zeta) = \sum_{i=0}^{N} a_i F^i + \sum_{i=1}^{N} a_{-i} F^{-i}, \tag{4}$$

where

$$F' = \sqrt{A + BF^2 + CF^4}, \tag{5}$$

the highest degree of $\dfrac{d^pU}{d\zeta^p}$ is taken as

$$O\left(\frac{d^pU}{d\zeta^p}\right) = N + p, \qquad p = 1,2,3,\cdots, \tag{6}$$

$$O\left(U^q \frac{d^pU}{d\zeta^p}\right) = (q+1)N + p, \quad q = 0,1,2,\cdots, p = 1,2,3,\cdots. \tag{7}$$

Where A, B and C are constants, and N in Eq. (3) is a positive integer that can be determined by balancing the nonlinear term(s) and the highest order derivatives. Normally N is a positive integer, so that an analytic solution in closed form may be obtained. Substituting Eqs. (2)- (5) into Eq. (3) and comparing the coefficients of each power of $F(\zeta)$ in both sides, to get an over-determined system of nonlinear algebraic equations with respect to v, a_0, a_1, $\cdots$. Solving the over-determined system of nonlinear algebraic equations by use of Mathematica. The relations between values of A, B, C and corresponding JEF solution $F(\zeta)$ of Eq. (4) are given in Table 1. Substitute the values of A, B, C and the corresponding JEF solution $F(\zeta)$ chosen from table 1 into the general form of solution, then an ideal periodic wave solution expressed by JEF can be obtained.

Table 1: Relation between values of (A, B, C) and corresponding F

A	B	C	$F(\zeta)$
1	$-1-m^2$	m^2	sn(ζ), cd(ζ)= $\dfrac{cn(\zeta)}{dn(\zeta)}$
$1-m^2$	$2m^2-1$	$-m^2$	cn(ζ)
m^2-1	$2-m^2$	-1	dn(ζ)
m^2	$-1-m^2$	1	ns(ζ) $\dfrac{1}{sn(\zeta)}$, dc(ζ)= $\dfrac{dn(\zeta)}{cn(\zeta)}$
$-m^2$	$2m^2-1$	$1-m^2$	nc(ζ)= $\dfrac{1}{cn(\zeta)}$
-1	$2-m^2$	m^2-1	nd(ζ)= $\dfrac{1}{dn(\zeta)}$
1	$2-m^2$	$1-m^2$	sc(ζ)= $\dfrac{sn(\zeta)}{cn(\zeta)}$
1	$2m^2-1$	$-m^2(-1-m^2)$	sd(ζ)= $\dfrac{sn(\zeta)}{dn(\zeta)}$
$1-m^2$	$2-m^2$	1	cs(ζ)= $\dfrac{cn(\zeta)}{sn(\zeta)}$
$-m^2(1-m^2)$	$2m^2-1$	1	ds(ζ)= $\dfrac{dn(\zeta)}{sn(\zeta)}$
$\dfrac{1}{4}$	$\dfrac{1-2m^2}{2}$	$\dfrac{1}{4}$	ns(ζ)+cs(ζ)
$\dfrac{1-m^2}{4}$	$\dfrac{1+m^2}{2}$	$\dfrac{1-m^2}{2}$	nc(ζ)+sc(ζ)
$\dfrac{1}{4}$	$\dfrac{m^2-2}{2}$	$\dfrac{m^2}{4}$	ns(ζ)+ds(ζ)
$\dfrac{m^2}{4}$	$\dfrac{m^2-2}{2}$	$\dfrac{m^2}{4}$	sn(ζ)+ $ics(\zeta)$

where sn(ζ), cn(ζ) and dn(ζ) are the JE sine function, JE cosine function and the JEF of the third kind, respectively. And

$$cn^2(\zeta)=1-sn^2(\zeta), dn^2(\zeta)=1-m^2 sn^2(\zeta), \quad (8)$$

with the modulus m $(0<m<1)$.

When m $\rightarrow$ 1, the Jacobi functions degenerate to the hyperbolic functions, i.e.,

$sn\zeta \rightarrow tanh\zeta, \quad cn\zeta \rightarrow sech\zeta, \quad dn\zeta \rightarrow sech\zeta,$

when m$\rightarrow$0, the Jacobi functions degenerate to the triangular functions, i.e.,

$sn\zeta \rightarrow sin\zeta, \quad cn\zeta \rightarrow cos\zeta$ and $dn \rightarrow 1.$

3. Results

In this section, we will apply the extended method to study ZS equation (1)

$$u_{xt}+\beta e^u+\gamma e^{-u}+\lambda e^{-2u}=0, \quad (9)$$

if we use $\zeta = x - \nu t, \quad u(x,t)=U(\zeta)$ carries Eq. (9) into an ODE

$$-\nu U''+\beta e^U+\gamma e^{-U}+\lambda e^{-2U}=0, \quad (10)$$

if we use $V=e^U$ carries Eq. (10) into:-

$$-\nu(VV''-(V')^2+\beta V^3+\gamma V+\lambda=0. \quad (11)$$

Balancing the term VV'' with the term V^3 we obtain $N=2$ then

$$V(\zeta)=a_0+a_1F+a_{-1}F^{-1}+a_2F^2+a_{-2}F^{-2},$$
$$F'=\sqrt{A+BF^2+CF^4}. \quad (12)$$

Substituting Eq. (12) into Eq. (11) and comparing the coefficients of each power of F in both sides, to get an over-determined system of nonlinear algebraic equations with respect to ν, a_i, $i=1$, -1, $i=2$, -2. Solving the over-determined system of nonlinear algebraic equations by use of Mathematica, we obtain three groups of constants:

1.

$$a_1=a_{-1}=0, \quad a_0=\frac{2B\nu-\sqrt{4(B^2+12AC)\nu^2-3\beta\gamma}}{3\beta},$$
$$a_2=\frac{2C\nu}{\beta}, \quad a_{-2}=\frac{2A\nu}{\beta},$$
$$\lambda=\frac{1}{27\beta^2}(2(3\beta\gamma+8B^2\nu^2+96AC\nu^2)\sqrt{4(B^2+12AC)\nu^2-3\beta\gamma}$$
$$-16B^3\nu^3+576ABC\nu^3), \quad (13)$$

2.

$$a_1=a_{-1}=a^2=0,$$
$$a_0=\frac{2B\nu-\sqrt{4(B^2-3AC)\nu^2-3\beta\gamma}}{3\beta}, \quad a_{-2}=\frac{2A\nu}{\beta},$$
$$\lambda=\frac{1}{27\beta^2}(2(3\beta\gamma+8B^2\nu^2+96AC\nu^2)\sqrt{4(B^2-3AC)\nu^2-3\beta\gamma}$$
$$-16B^3\nu^3+72ABC\nu^3), \quad (14)$$

3.

$$a_1=a_{-1}=a^{-2}=0,$$
$$a_0=\frac{2B\nu-\sqrt{4(B^2-3AC)\nu^2-3\beta\gamma}}{3\beta}, \quad a_2=\frac{2C\nu}{\beta},$$
$$\lambda=\frac{1}{27\beta^2}(2(3\beta\gamma+8B^2\nu^2+96AC\nu^2)\sqrt{4(B^2-3AC)\nu^2-3\beta\gamma}$$
$$-16B^3\nu^3+72ABC\nu^3). \quad (15)$$

Now, the solutions of ZS equation (9) can be written as follows:

$$u_1= \ln[-\frac{2(1+m^2)\nu+\sqrt{4(12m^2+(1+m^2)^2)\nu^2-3\beta\gamma}}{3\beta}$$
$$+\frac{2m^2\nu}{\beta}sn^2(x-\nu t)+\frac{2\nu}{\beta}ns^2(x-\nu t)], \quad (16)$$

$$u_2= \ln[-\frac{2(1+m^2)\nu+\sqrt{4(12m^2+(1+m^2)^2)\nu^2-3\beta\gamma}}{3\beta}$$
$$+\frac{2m^2\nu}{\beta}cd^2(x-\nu t)+\frac{2\nu}{\beta}dc^2(x-\nu t)], \quad (17)$$

$$u_3= \ln[-\frac{2(1-2m^2)\nu+\sqrt{4(12m^2(m^2-1)+(2m^2-1)^2\nu^2)-3\beta\gamma}}{3\beta}$$
$$-\frac{2m^2\nu}{\beta}cn^2(x-\nu t)$$
$$+\frac{2(1-m^2)\nu}{\beta}nc^2(x-\nu t)], \quad (18)$$

$$u_4= \ln[-\frac{2(m^2-2)\nu+\sqrt{4((2-m^2)^2+12(1-m^2))\nu^2-3\beta\gamma}}{3\beta}$$
$$-\frac{2\nu}{\beta}dn^2(x-\nu t)$$
$$+\frac{2(m^2-1)\nu}{\beta}nd^2(x-\nu t)], \quad (19)$$

$$u_5 = \ln[-\frac{2(m^2-2)\nu+\sqrt{4(12(1-m^2)+(2-m^2)^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{2\nu}{\beta}cs^2(x-\nu t) + \frac{2\nu(1-m^2)}{\beta}sc^2(x-\nu t)], \quad (20)$$

$$u_6 = \ln[-\frac{2(1-2m^2)\nu+\sqrt{4(12m^2(1+m^2)+(1-2m^2)^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{2\nu}{\beta}ds^2(x-\nu t)+\frac{2\nu m^2(1+m^2)}{\beta}sd^2(x-\nu t)], \quad (21)$$

$$u_7 = \ln[-\frac{2(m^2-0.5)\nu+\sqrt{4(0.75+(0.5-m^2)^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{\nu}{2\beta}(ns(x-\nu t)+cs(x-\nu t))^2 + \frac{\nu}{2\beta}(ns(x-\nu t)+cs(x-\nu t))^{-2}], \quad (22)$$

$$u_8 = \ln[\frac{1}{3\beta}(2(0.5+0.5m^2)\nu - \sqrt{4(12(0.5-0.5m^2)(0.25-0.25m^2)+(0.5+0.5m^2)^2)\nu^2-3\beta\gamma}) + \frac{2\nu(0.5-0.5m^2)}{\beta}(nc(x-\nu t)+sc(x-\nu t))^2 + \frac{2\nu(0.5-0.5m^2)}{\beta}(nc(x-\nu t)+sc(x-\nu t))^{-2}], \quad (23)$$

$$u_9 = \ln[-\frac{2(0.5m^2-1)\nu+\sqrt{4(0.75m^2+(1-0.5m^2)^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{m^2\nu}{2\beta}(ns(x-\nu t)+ds(x-\nu t))^2 + \frac{\nu}{2\beta}(ns(x-\nu t)+ds(x-\nu t))^{-2}], \quad (24)$$

$$u_{10} = \ln[-\frac{2(1-0.5m^2)\nu+\sqrt{4(0.75m^4+(1-0.5m^2)^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{m^2\nu}{2\beta}(sn(x-\nu t)+ics(x-\nu t))^2 + \frac{m^2\nu}{2\beta}(sn(x-\nu t)+ics(x-\nu t))^{-2}], \quad (25)$$

$$u_{11} = \ln[-\frac{2(1+m^2)\nu+\sqrt{4((1+m^2)^2-3m^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{2\nu}{\beta}ns^2(x-\nu t)], \quad (26)$$

$$u_{12} = \ln[-\frac{2(1+m^2)\nu+\sqrt{4((1+m^2)^2-3m^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{2\nu}{\beta}dc^2(x-\nu t)], \quad (27)$$

$$u_{13} = \ln[-\frac{2(1-2m^2)\nu+\sqrt{4(3m^2(1-m^2)+(2m^2-1)^2\nu^2)-3\beta\gamma}}{3\beta} + \frac{2(1-m^2)\nu}{\beta}nc^2(x-\nu t)], \quad (28)$$

$$u_{14} = \ln[-\frac{2(m^2-2)\nu+\sqrt{4((2-m^2)^2-3(1-m^2))\nu^2-3\beta\gamma}}{3\beta} + \frac{2(m^2-1)\nu}{\beta}nd^2(x-\nu t)], \quad (29)$$

$$u_{15} = \ln[-\frac{2(m^2-2)\nu+\sqrt{4(3(m^2-1)+(2-m^2)^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{2\nu(1-m^2)}{\beta}sc^2(x-\nu t)], \quad (30)$$

$$u_{16} = \ln[-\frac{2(1-2m^2)\nu+\sqrt{4((1-2m^2)^2-3m^2(1+m^2))\nu^2-3\beta\gamma}}{3\beta} + \frac{2\nu m^2(1+m^2)}{\beta}sd^2(x-\nu t)], \quad (31)$$

$$u_{17} = \ln[-\frac{2(m^2-0.5)\nu+\sqrt{4((0.5-m^2)^2-\frac{3}{16})\nu^2-3\beta\gamma}}{3\beta} + \frac{\nu}{2\beta}(ns(x-\nu t)+cs(x-\nu t))^{-2}], \quad (32)$$

$$u_{18} = \ln[\frac{1}{3\beta}(2(0.5+0.5m^2)\nu - \sqrt{4(3(0.5m^2-0.5)(0.25-0.25m^2)+(0.5+0.5m^2)^2)\nu^2-3\beta\gamma}) + \frac{2\nu(0.5-0.5m^2)}{\beta}(nc(x-\nu t)+sc(x-\nu t))^{-2}], \quad (33)$$

$$u_{19} = \ln[-\frac{2(0.5m^2-1)\nu+\sqrt{4((1-0.5m^2)^2-\frac{3m^2}{16})\nu^2-3\beta\gamma}}{3\beta} + \frac{\nu}{2\beta}(ns(x-\nu t)+ds(x-\nu t))^{-2}], \quad (34)$$

$$u_{20} = \ln[-\frac{2(1-0.5m^2)\nu+\sqrt{4((1-0.5m^2)^2-\frac{3m^4}{16})\nu^2-3\beta\gamma}}{3\beta} + \frac{m^2\nu}{2\beta}(sn(x-\nu t)+ics(x-\nu t))^{-2}], \quad (35)$$

$$u_{21} = \ln[-\frac{2(1+m^2)\nu+\sqrt{4((1+m^2)^2-3m^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{2m^2\nu}{\beta}sn^2(x-\nu t)], \quad (36)$$

$$u_{22} = \ln[-\frac{2(1+m^2)\nu+\sqrt{4((1+m^2)^2-3m^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{2m^2\nu}{\beta}cd^2(x-\nu t)], \quad (37)$$

$$u_{23} = \ln[-\frac{2(1-2m^2)\nu+\sqrt{4(3m^2(1-m^2)+(2m^2-1)^2\nu^2)-3\beta\gamma}}{3\beta} - \frac{2m^2\nu}{\beta}cn^2(x-\nu t)], \quad (38)$$

$$u_{24} = \ln[-\frac{2(m^2-2)\nu+\sqrt{4((2-m^2)^2-3(1-m^2))\nu^2-3\beta\gamma}}{3\beta} - \frac{2\nu}{\beta}dn^2(x-\nu t)], \quad (39)$$

$$u_{25} = \ln[-\frac{2(m^2-2)\nu+\sqrt{4(3(m^2-1)+(2-m^2)^2)\nu^2-3\beta\gamma}}{3\beta} + \frac{2\nu}{\beta}cs^2(x-\nu t)], \quad (40)$$

$$u_{26} = \ln[-\frac{2(1-2m^2)\nu+\sqrt{4((1-2m^2)^2-3m^2(1+m^2))\nu^2-3\beta\gamma}}{3\beta} + \frac{2\nu}{\beta}ds^2(x-\nu t)], \quad (41)$$

$$u_{27} = \ln[-\frac{2(m^2-0.5)\nu+\sqrt{4((0.5-m^2)^2-\frac{3}{16})\nu^2-3\beta\gamma}}{3\beta} + \frac{\nu}{2\beta}(ns(x-\nu t)+cs(x-\nu t))^2], \quad (42)$$

$$u_{28} = \ln[\frac{1}{3\beta}(2(0.5+0.5m^2)\nu - \sqrt{4(3(0.5m^2-0.5)(0.25-0.25m^2)+(0.5+0.5m^2)^2)\nu^2-3\beta\gamma}) + \frac{2\nu(0.5-0.5m^2)}{\beta}(nc(x-\nu t)+sc(x-\nu t))^2], \quad (43)$$

$$u_{29} = \ln[-\frac{2(0.5m^2-1)\nu+\sqrt{4((1-0.5m^2)^2-\frac{3m^2}{16})\nu^2-3\beta\gamma}}{3\beta} + \frac{m^2\nu}{2\beta}(ns(x-\nu t)+ds(x-\nu t))^2], \quad (44)$$

$$u_{30} = \ln[-\frac{2(1-0.5m^2)\nu+\sqrt{4((1-0.5m^2)^2-\frac{3m^4}{16})\nu^2-3\beta\gamma}}{3\beta} + \frac{m^2\nu}{2\beta}(sn(x-\nu t)+ics(x-\nu t))^2]. \quad (45)$$

The modulus of solitary wave solutions u_1, u_2, u_{16} and u_{22} are displayed in figures 1, 2, 3 and 4 respectively, with values of parameters listed in their captions.

Fig. 1 (a)

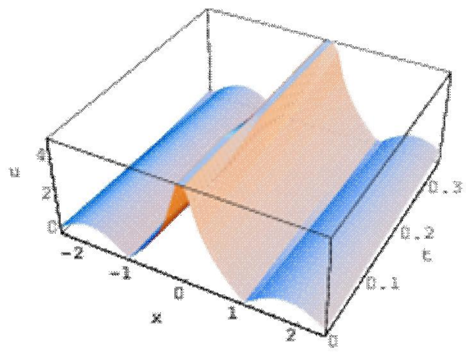

Fig. 1 (b)

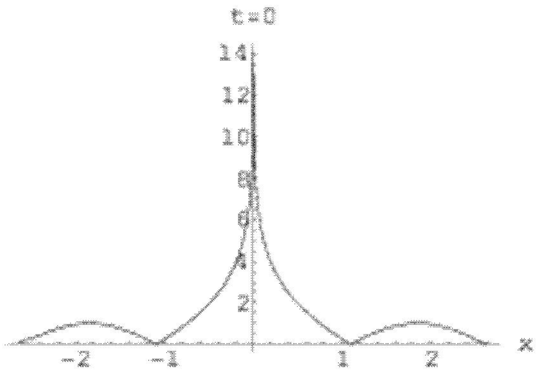

Fig. 1 The modulus of solitary wave solution u_1 (Eq. 16) where $m=\beta=\gamma=0.5$.

Fig. 2 (a)

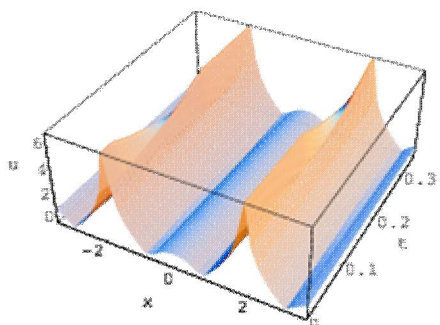

Fig. 2 (b)

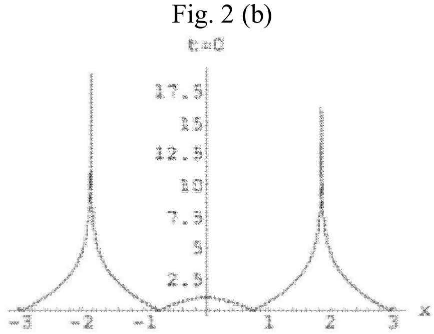

Fig. 2 The modulus of solitary wave solution u_2 (Eq. 17) where $m=\beta=\gamma=0.5$.

Fig. 3 (a)

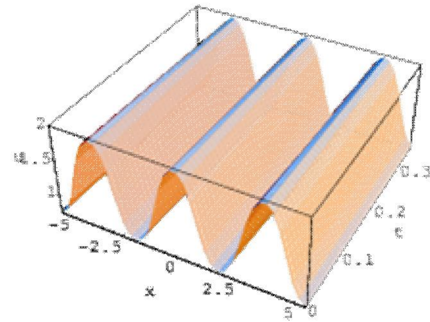

Fig. 3 (b)

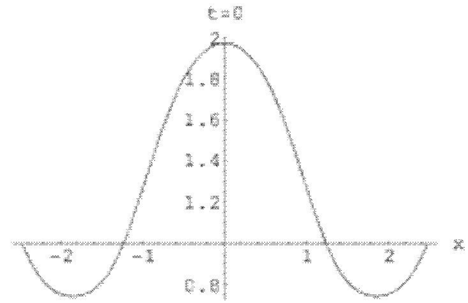

Fig. 3 The modulus of solitary wave solution u_{16} (Eq. 31) where $m=\beta=\gamma=0.5$.

Fig. 4 (a)

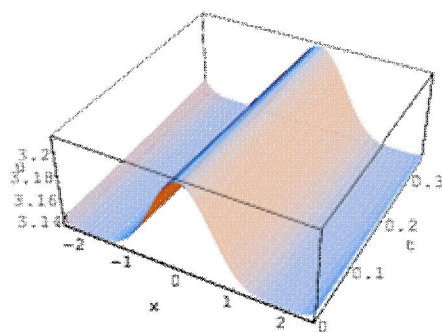

Fig. 4 (b)

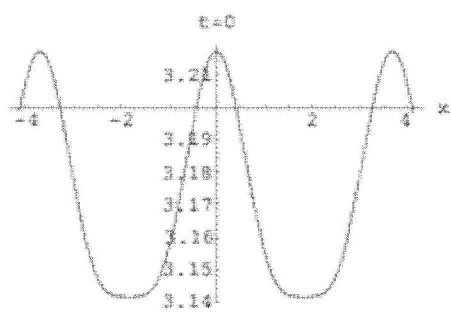

Fig. 4 The modulus of solitary wave solution u_{22} (Eq. 37) where $m=\beta=\gamma=0.5$.

3.1 Soliton solutions

Some solitary wave solutions can be obtained, if the modulus m approaches to 1 in Eqs. (16)-(43)

$$u_{31} = \ln[\frac{4\nu+\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}\tanh^2(x-\nu t)], \quad (46)$$

$$u_{32} = \ln[\frac{4\nu+\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}\coth^2(x-\nu t)], \quad (47)$$

$$u_{33} = \ln[\frac{2\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}-\frac{2\nu}{\beta}sech^2(x-\nu t)], \quad (48)$$

$$u_{34} = \ln[\frac{2\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}csch^2(x-\nu t)], \quad (49)$$

$$u_{35} = \ln[\frac{2\nu-i\sqrt{5\nu^2+3\beta\gamma}}{3\beta}+\frac{4\nu}{\beta}\sinh^2(x-\nu t)], \quad (50)$$

$$u_{36} = \ln[\frac{4\nu-\sqrt{64\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}\tanh^2(x-\nu t)+\frac{2\nu}{\beta}\coth^2(x-\nu t)], \quad (51)$$

$$u_{37} = \ln[\frac{-\nu-\sqrt{\frac{\nu^2}{4}-3\beta\gamma}}{3\beta}+\frac{\nu}{2\beta}(\tanh(x-\nu t)+icsch(x-\nu t))^2], \quad (52)$$

$$u_{38} = \ln[\frac{\nu-\sqrt{\frac{\nu^2}{4}-3\beta\gamma}}{3\beta}+\frac{\nu}{2\beta}(\coth(x-\nu t)+csch(x-\nu t))^2], \quad (53)$$

$$u_{39} = \ln[\frac{-\nu-\sqrt{\frac{\nu^2}{4}-3\beta\gamma}}{3\beta}+\frac{\nu}{2\beta}(\tanh(x-\nu t)+icsch(x-\nu t))^{-2}], \quad (54)$$

$$u_{40} = \ln[\frac{\nu-\sqrt{\frac{\nu^2}{4}-3\beta\gamma}}{3\beta}+\frac{\nu}{2\beta}(\coth(x-\nu t)+csch(x-\nu t))^{-2}], \quad (55)$$

$$u_{41} = \ln[\frac{2\nu-\sqrt{100\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}csch^2(x-\nu t)+\frac{4\nu}{\beta}\sinh^2(x-\nu t)], \quad (56)$$

$$u_{42} = \ln[-\frac{\nu+\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{\nu}{2\beta}(\coth(x-\nu t)+csch(x-\nu t))^2 + \frac{\nu}{2\beta}(\coth^2(x-\nu t)+csch^2(x-\nu t))^{-2}], \quad (57)$$

$$u_{43} = \ln[\frac{-\nu-\sqrt{\frac{\nu^2}{4}-3\beta\gamma}}{3\beta}+\frac{\nu}{2\beta}(\tanh(x-\nu t)+icsch(x-\nu t))^2 + \frac{\nu}{2\beta}(\tanh(x-\nu t)+icsch(x-\nu t))^{-2}]. \quad (58)$$

3.2 Triangular periodic solutions

Some trigonometric function solutions can be obtained, if the modulus m approaches to zero in Eqs. (16)-(43)

$$u_{44} = \ln[\frac{2\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}\cot^2(x-\nu t)], \quad (59)$$

$$u_{45} = \ln[\frac{-4\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}\csc^2(x-\nu t)], \quad (60)$$

$$u_{46} = \ln[\frac{2\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{\nu}{\beta}\sin^2(x-\nu t)], \quad (61)$$

$$u_{47} = \ln[\frac{-2\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}\csc^2(x-\nu t)], \quad (62)$$

$$u_{48} = \ln[\frac{-2\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}\sec^2(x-\nu t)], \quad (63)$$

$$u_{49} = \ln[\frac{4\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}\tan^2(x-\nu t)], \quad (64)$$

$$u_{50} = \ln[\frac{2\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}(\csc(x-\nu t)+\cot(x-\nu t))^{-2}], \quad (65)$$

$$u_{51} = \ln[\frac{\nu-i\sqrt{2\nu^2+3\beta\gamma}}{3\beta}+\frac{\nu}{\beta}(\sec(x-\nu t)+\tan(x-\nu t))^{-2}], \quad (66)$$

$$u_{52} = \ln[\frac{2\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}(\csc(x-\nu t)+\cot(x-\nu t))^{2}], \quad (67)$$

$$u_{53} = \ln[\frac{\nu-i\sqrt{2\nu^2+3\beta\gamma}}{3\beta}+\frac{\nu}{\beta}(\sec(x-\nu t)+\tan(x-\nu t))^{2}], \quad (68)$$

$$u_{54} = \ln[\frac{2\nu-\sqrt{4\nu^2-3\beta\gamma}}{3\beta}+\frac{2\nu}{\beta}(\csc(x-\nu t)+\cot(x-\nu t))^{2}+\frac{2\nu}{\beta}(\csc(x-\nu t)+\cot(x-\nu t))^{-2}], \quad (69)$$

$$u_{55} = \ln[\frac{\nu-i\sqrt{2\nu^2+3\beta\gamma}}{3\beta}+\frac{\nu}{\beta}(\sec(x-\nu t)+\tan(x-\nu t))^{2}+\frac{\nu}{\beta}(\sec(x-\nu t)+\tan(x-\nu t))^{-2}]. \quad (70)$$

Note: The solutions u_{31}, u_{32}, u_{36}, u_{44} and u_{49} are the same as in Wazwaz [2]. And the solutions of the special equations in [2] can be obtained where the parameters are taken as special values as pointed in introduction.

4. Discussions

By introducing appropriate transformations and using extended F-expansion method, we have been able to obtain in a unified way with the aid of symbolic computation system-mathematica, a series of solutions including single and the combined Jacobi elliptic function. Also, extended F-expansion method is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. When $m \to 1$, the Jacobi functions degenerate to the hyperbolic functions and given the solutions by the extended hyperbolic functions methods. When $m \to 0$, the Jacobi functions degenerate to the triangular functions and given the solutions by extended triangular functions methods.

Acknowledgements:

This article was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah. The authors, therefore, acknowledge with thanks DSR technical and financial support.

Corresponding Author:
Ali H. Bhrawy Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia, alibhrawy@yahoo.coo.uk

References

1. Conte R, Musette M. Link between solitary waves and projective Riccati equations, J. Phys. A, 1992, 25, 5609-5623.
2. Wazwaz AM. The tanh method for travelling wave solutions to the Zhiber-Shabat equation and other related equations, Commun. Nonlinear Sci. Numer. Simul., 2008, 13, 584-592.
3. Tang YN, Xu W., Shen JW, Gao L. Bifurcations of traveling wave solutions for Zhiber-Shabat equation, Nonlinear Anal., 2007, 67, 648-656.
4. Bin H, Yao L, Weiguo R. New exact bounded travelling wave solutions for the Zhiber-Shabat equation, Nonlinear Analysis, 2009, 71, 1636-1648.
5. Borhanifar A, Moghanlu AZ. Application of the -expansion method for the Zhiber-Shabat equation and other related equations,

Mathematical and Computer Modelling, 2011, 54 2109-2116.

6. Chen H., Shizgal BD, A spectral solution of the Sturm-Liouville equation: comparison of classical and nonclassical basis sets, Journal of Computational and Applied Mathematics, 2001, 136(1–2), 17-35.
7. Wazwaz AM. The tanh method: exact solutions of the sine-Gordon and the sinh-Gordon equations, Applied Mathematics and Computation, 2005, 167(2), 1196-1210.
8. Wazwaz AM. The tanh method: solitons and periodic solutions for the Dodd-Bullough–Mikhailov and the Tzitzeica-Dodd-Bullough equations, Chaos, Solitons and Fractals, 2005, 25(1), 55-63.
9. Salort D, Dispersion and Strichartz estimates for the Liouville equation, Journal of Differential Equations, 2007, 233(2), 543-584.
10. Assis PG, Ferreira LA. The Bullough-Dodd model coupled to matter fields, Nuclear Physics B, 2008, 800(3), 409-449.
11. Khater AH, Callebaut DK, Abdelkawy MA. Two-dimensional force-free magnetic fields described by some nonlinear equations, Physics Plasmas, 2010, 17, 122902.
12. Bekir A. The tanh-coth method combined with the Riccati equation for solving non-linear equation, Chaos, Solitons and Fractals, 2009, 40(3), 1467-1474.
13. Ebadi G, Biswas A. The method and -soliton solution of the Davey-Stewartson equation, Mathematical and Computer Modelling, 2011, 53 (5–6), 694-698.
14. Ebadi G, Biswas A. Application of the - expansion method for nonlinear diffusion equations with nonlinear source, Journal of the Franklin Institute, 2010, 347(7), 1391-1398.
15. Bhrawy AH, Biswas A, Javidi M, Ma W-X, Pinar Z, Yildirim A. New Solutions for - Dimensional and -Dimensional Kaup-Kupershmidt Equations, Resul. Math., 2012, DOI 10.1007/s00025-011-0225-7.
16. A. Ebaid, Exact solitary wave solutions for some nonlinear evolution equations via Exp-function method, Phys. Lett. A, 2007, 365, 213-219.
17. Bhrawy AH, Abdelkawy MA, Biswas A. Cnoidal and Snoidal Wave Solutions to Coupled Nonlinear Wave Equations by the Extended Jacobi's Elliptic Function Method, Communications in Nonlinear Science and Numerical Simulation, In press (2012) http://dx.doi.org/10.1016/j.cnsns.2012.08.034
18. Bhrawy AH, Tharwat MM, Yildrim A, Abdelkawy MA. A Jacobi elliptic function method for nonlinear arrays of vortices, Indian J Phys, In press (2012) DOI 10.1007/s12648-012-0173-4
19. Sohail A, Rees JM, Zimmerman WB. Analysis of capillary-gravity waves using the discrete periodic inverse scattering transform, Colloids and Surfaces A: Physicochemical and Engineering Aspects, 2011, 391(1-3), 42-50.
20. Osborne AR. Soliton physics and the periodic inverse scattering transform, Physica D: Nonlinear Phenomena, 1995, 86(1-2), 81-89.
21. Rui W, He B, Long Y. The binary F-expansion method and its application for solving the (n + 1)-dimensional sine-Gordon equation, Comm. Nonl. Sci. Numer. Simul, 2009, 14(4), 1245-1258.
22. Zhang S, Xia T A generalized F-expansion method with symbolic computation exactly solving Broer-Kaup equations, Applied Mathematics and Computation, 2007, 189(1), 836-843.
23. Zhang J-F, Dai C-Q. Qin Yang, Jia-Min Zhu, Variable-coefficient F-expansion method and its application to nonlinear Schrödinger equation, Optics Communications, 2005, 252(4-6), 408-421.
24. Shen Y, Cao N. The double F-expansion approach and novel nonlinear wave solutions of soliton equation in (2 + 1)-dimension, Applied Mathematics and Computation, 2008, 198(2), 683-690.
25. Wang M, Li X. Extended F-expansion method and periodic wave solutions for the generalized Zakharov equations, Physics Letters A, 2005, 343(1-3), 48-54.

9/6/2012