An integrated FAHP-FMOMILP model for multi-product Multi-period lot sizing with Supplier selection in quantity discount environments

Ebrahim Kenarroudi

Department of Industrial Engineering, Islamic Azad University, Arak Branch, Arak, Iran
e_kenarroudi@yahoo.com

Abstract: Supplier selection is a multi-criteria decision making problem which includes both qualitative and quantitative factors. Supplier selection includes three main decisions: ordering one or several product? Which suppliers and how much of each supplier? Which time? In this research an integrated approach of Fuzzy analytic hierarchy process and Fuzzy multi-objective mixed integer linear programming for multi-product Multi-period lot sizing with supplier selection in quantity discount environments is proposed. In the first step, the suppliers were evaluated by qualitative and quantitative criteria and using fuzzy analytic hierarchy process technique. In the nest stage, the suppliers were selected and order quantity from each one was determined; fuzzy multi-objective mixed integer linear programming was applied for this purpose.

[Ebrahim Kenarroudi. **An integrated FAHP-FMOMILP model for multi-product Multi-period lot sizing with supplier selection in quantity discount environments**. *Life Sci J* 2012;9(3):1484-1494]. (ISSN: 1097-8135). http://www.lifesciencesite.com. 217

Keywords: Supplier selection, Fuzzy analytic hierarchy process (FAHP), fuzzy preference programming (FPP), fuzzy multi-objective mixed integer linear programming (FMOMILP), Multi-product Multi-period lot-sizing, quantity discount

1- Introduction

In production industries, cost of raw materials exceeds 70% of product manufacturing expenses, and at least 50% of qualitative pitfalls of the products originate from the purchased raw materials (Burton, 1988). Organizations must pursue strategies to achieve higher quality, reduced costs and shorter lead times to maintain a competitive position in the global market. Based on new strategies for purchasing and manufacturing, suppliers play a key role in achieving corporate competition. Hence, selecting the right suppliers is a vital component of these strategies (Amid et al., 2009).

There are essentially two types of supplier selection problems: single-source and multi-source (Ustun & Demirtas, 2008a, 2008b). In single-source case, all suppliers are able to supply all demands of customers. This ability of suppliers in commodity provision is not solely limited to their capability in supplying the amount of material for the customer but incorporates all criteria which convince the customer to purchase from a certain supplier. In this state, the only important decision is identification of the best supplier. Suppliers' evaluation models can be deployed for this purpose. Multi-attribute decision-making models account for majority of options proposed in such cases. Several suppliers shall be used for providing the materials in the case that none of suppliers is capable of providing all general demands of the customer due to limitations in capacity, quality, price, and other significant reasons or when the strategy of company's provision sector is to avoid reliance on a single source to prevent from material deficiency and maintain competition among the suppliers. Two problems are encountered in this case: selection of suppliers and determination of order quantity from each supplier (Ghodsypour & O'Brien, 1998).

The models introduced for selection of suppliers are divided into two general categories: mathematical programming models and ranking models. Ranking models can be used in situations when the decision to select the supplier is not complicated. Yet with further complexity of decision-making conditions, ranking models will not yield favorable responses due to addition of problem constraints and especially for the case of multi-source selection in which the decision needs to be made on simultaneous selection of suppliers and determination of their optimal order quantity. Mathematical programming models must be used to solve such problems. For the same reason, most of papers presented during the recent years in the field of supplier selection have used a mathematical programming model. (Chen, 2009; Hammami et al., 2009; Choi & Chang, 2006; Cakravastia et al., 2002; Amid et al., 2006, 2009; Wu et al., 2010; Liao & Rittscher, 2007; Basnet & Leung, 2005).

Failure to incorporate the qualitative criteria is among the major problems of mathematical models which impair their efficiency while through reviewing the models proposed for selection of suppliers this fact is revealed that most criteria used in selection are qualitative. To overcome this drawback, Ghodsypour & O'Brien (1998) proposed a two-stage integrated model. Initially, suppliers were evaluated and ranked by means of a multi-criteria model; for this purpose, they applied Analytic Hierarchy Process (AHP) technique to evaluate the tangible and intangible criteria. In the second stage, the obtained ranks were input in a mathematical programming model as coefficient of objective function where total value of purchasing (TVP) function is maximized. They utilized a linear programming model in the second stage for choosing suppliers and determining their order quantity. As such, AHP alleviates the defect mentioned in mathematical programming which pertains to its inability to consider the qualitative criteria affecting the selection. Accordingly, the results of mathematical modeling will be more reliable. Since then, use of integrated models increased due to enhancement of decision-making quality and their high level of reliability. The following research works can be implied as examples: Kokangul & Susuz, 2009; Lee et al., 2009; Wu et al., 2009; Ustun & Demirtas, 2008a, 2008b; Lin, 2009

Contained in the various evaluation methods proposed in the available literature, price, delivery performance, and quality are the most common criteria (Wang & Yang, 2009). Hence, most of programming models were based on these criteria; these criteria are generally taken as objectives of model otherwise are regarded as constraints. Purchase costs are among the most important criteria considered as objective function in most researches. For this reason, many researchers take into account the discount conditions in their studies. Chaudhry et al. (1993) were the pioneers of considering discount conditions in their mathematical programming model. Many researchers were conducted afterwards in this scope; some instances include: Sadrian & Yoon,1994; Rosenthal et al., 1995; Xu et al.,2000; Tempelmeier, 2002; Xia & wu, 2007; Crama et al., 2004; Choi & Chang, 2006; Ganeshan et al., 1999; Chang, 2006; Amid et al., 2009; Wang & Yang, 2009.

In the current research, a two-stage integrated model is proposed for selecting the suppliers and determining their optimal order quantity. This model applies to general state of decision-making about selection of suppliers i.e. multi-products, multi-suppliers, and multi-periods. In the first stage, the suppliers are evaluated by means of some qualitative and quantitative criteria. In this stage, suppliers are evaluated for different products separately and the calculated weights of these evaluations enter as the overall priorities for suppliers into the second stage where order quantity are selected and allocated. FAHP technique is used in the present study to evaluate the suppliers. In the second stage, the superior suppliers are selected by a mathematical programming model and optimal order quantity of each one is determined. A FMOMILP model is used in this stage for decision optimization; this model consists of six objective functions including: cost reduction, increase of total value of purchasing, reduction of products' defect, improvement of on-time delivery, reduction of lead-time, and enhancement of guarantee level. These objective functions are the most important quantitative criteria affecting selection of suppliers, which can be converted into objective functions because of having quantitative values.

2. Fuzzy Analytic Hierarchy Process

2.1. Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) is a theory of measurement through pairwise comparisons and relies on the judgments of experts to derive priority scales (Saaty, 2008). AHP is a multi-attribute decision-making technique which is based on human brain's analysis for intricate and fuzzy problems. This method was primarily introduced by Thomas. L. Saaty in 1971(saaty, 1980).

The procedures of AHP to solve a complex problem involve six essential steps (lee 2009):

1. Define the unstructured problem and state clearly the objectives and outcomes;
2. Decompose the problem into a hierarchical structure with decision elements;
3. Employ pairwise comparisons among decision elements and form comparison matrices;
4. Use the value method to estimate the relative weights of the decision elements;
5. Check the consistency property of matrices to ensure that the judgments of decision makers are consistent;
6. Aggregate the relative weights of decision elements to obtain an overall rating for the alternatives.

2.2. Fuzzy set theory and triangular fuzzy number

Fuzzy set theory (FST) was introduced by Zadeh (1965) and is a class of objects with a continuum of grades of membership (Zadeh, 1965). A fuzzy set A in X is characterized by membership function $f_A(x)$ which associates with each point in X a real number in the interval [0,1], with the value of $f_A(x)$ at x representing the "grade of membership" of x in A (Zadeh, 1965).

A triangular fuzzy number (TFN) $\tilde{A}$ is defined by three real numbers $l \le m \le u$, and characterized by a linear piecewise continuous membership function $\mu_{\tilde{A}}(x)$ of the type (Mikhailov & Tsvetinov, 2004):

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l} & l \le x \le m \\ \frac{u-x}{u-m} & m \le x \le u \\ 0 & Otherwise \end{cases} \quad (1)$$

The fuzzy number $\tilde{A}$ can be expressed as a triple (l, m, u), where m is the most possible value of the fuzzy number; and l and u are the lower and the upper bounds, respectively (Mikhailov, 2003).

2.3. Fuzzy preference programming method

In many cases the preference model of the human decision-maker is uncertain and fuzzy and it is relatively difficult crisp numerical values of the comparison ratios to be provided. The decision-maker may be uncertain about his level of preference due to incomplete information or knowledge, inherent complexity and uncertainty within the decision environment, lack of an appropriate measure or scale. A natural way to cope with uncertain judgments is to express the comparison ratios as fuzzy sets or fuzzy numbers, which incorporate the vagueness of the human thinking (Mikhailov, 2003).

Van Laarhoven and Pedrycz (1983) were the first persons who used fuzzy concept in pairwise comparisons in their research works. Evaluation of fuzzy priorities in pairwise comparison matrix is most important stage in solving FAHP models. They benefited from fuzzy logarithmic least squares method for this purpose. Many other FAHP methods were subsequently proposed by researchers, including: Geometric mean method (Buckley, 1985), interval arithmetic (Cheng & Mon, 1994), extend analysis method (Chang 1996), Fuzzy least squares method (Xu et al., 2000), Fuzzy preference programming (Mikhailov, 2003).

Fuzzy preference programming method is used in the current paper due to its numerous merits compared to alternative techniques. The most important of these advantages is the measurement of consistency indexes for the fuzzy pairwise comparison matrixes. It is not possible to determine the consistency ratios of fuzzy pairwise comparison matrixes in other AHP methods without conducting an additional study (Dagdeviren & Yüksel, 2010) and as opposed to other approaches it does not require constructing fuzzy comparison matrices and derives the priority vector from the incomplete judgment set (Cakir & Canbolat, 2008)

Suppose the pairwise comparisons matrix $F = \{\tilde{a}_{ij}\}$ for n elements which contained $m \le n(n-1)/2$ judgments about fuzzy pairwise comparison where $i = 1,2,..., n-1,\ j = 2,3,..., n,\ j > i$ and $a_{ij} = (l_{ij}, m_{ij}, u_{ij})$ are triangular fuzzy numbers. In this state, the non-linear program proposed by Mikhailov & Tsvetinov (2004) is written in the form of equation (2) for obtaining the relative weights of pairwise comparison matrix.

$$\begin{aligned} &\text{Maximize} \quad \lambda \\ &\text{Subject to} \\ &(m_{ij} - l_{ij})\lambda w_j - w_i + l_{ij} w_j \le 0, \\ &(u_{ij} - m_{ij})\lambda w_j + w_i - u_{ij} w_j \le 0, \\ &\sum_{k=1}^{n} w_k = 1, w_k > 0, k = 1,2,..., n. \\ &i = 1,2,..., n-1, j = 2,3,..., n, j > i. \end{aligned} \quad (2)$$

The optimal value λ^* can be used for measuring the consistency of the initial set of fuzzy judgments. The optimal value λ^*, if it is positive, indicates that all solution ratios completely satisfy the fuzzy judgments, i.e. $l_{ij} \le (w_i^*/w_j^*) \le u_{ij}$, which means that the initial set of fuzzy judgements is rather consistent. A negative value of λ^* shows that the solutions ratios approximately satisfy all double-side inequalities $l_{ij} \le w_i/w_j \le u_{ij}$, i.e. the fuzzy judgements are strongly inconsistent (Mikhailov & Tsvetinov, 2004).

3. Benefits, opportunities, costs and risks

In decision-making, there are often criteria that are opposite in direction to other criteria as in benefits (B)versus costs (C), and in opportunities (O) versus risks (R), and sometimes need to be distinguished by using negative numbers (Saaty 2003b). For the same reason, Saaty (2004) proposed a model with criteria B, O, C, and R criteria for determining the priorities of variables. An advantage of BOCR is the merging ability in different decision-making problems including AHP. B, O, C, and R are involved in a hierarchy as major priorities after objective and each of them consists of criteria and sub-criteria.

The important point in hierarchical structures with BOCR merits is the fact that if significance degrees of B, O, C and R are not equal, they cannot be mutually compared with the objective in order to achieve the weights. To obtain weight of each priority, a series of control criteria are defined with respect to the objective. So, the relative weights of these priorities can be derived through pairwise comparisons between control criteria compared to objective and BCOR merits compared to control criteria (Lee, 2009).

In order to calculate weights of alternatives, the weights obtained for each alternative with respect to priorities are merged with the weights obtained for each of criteria with respect to objective. Saaty (2003a) proposed five techniques for combining these values. If W_j is final weight of each alternative. B_j,

O_j, C_j, and R_j respectively represent combined results of *j*th alternative with merits B, O, C, and R; b, o, c, and r also respectively represent normalized weights of B, O, C, and R merits with respect to the objective. These five methods will be in the form of equation (3) (Lee, 2009):

1. Additive

$$W_j = bB_j + oO_j + c\left[(1/C_j)_{Normalized}\right] + r\left[(1/R_j)_{Normalized}\right]$$

2. Probabilistic additive

$$W_j = bB_j + oO_j + c(1 - C_j) + r(1 - R_j) \qquad (3)$$

3. Subtractive

$$W_j = bB_j - oO_j + cC_j - rR_j$$

4. Multiplicative priority powers

$$W_j = B_j^b O_j^o \left[(1/C_j)_{Normalized}\right]^c \left[(1/R_j)_{Normalized}\right]^r$$

5. Multiplicative

$$W_j = B_j O_j / C_j R_J$$

4. max-min method

A general multi-objective model for the supplier selection problem can be stated as follows (Weber and Current, 1993; Amid et al., 2011):

$$\begin{aligned}
&Min\ \ Z_1, Z_2, \ldots, Z_k \\
&Max\ \ Z_{k+1}, Z_{k+2}, \ldots, Z_p \qquad (4) \\
&Subject\ \ to \\
&x \in X_d,\ \ X_d = \{x / g_s(x) \le b_s,\ \ s = 1,2,\ldots,m\}
\end{aligned}$$

in which the $Z_1, Z_2, \ldots Z_k$ are the negative objectives or criteria for minimization like cost, late delivery, etc. and $Z_{k+1}, Z_{k+2}, \ldots, Z_p$ are the positive objectives or criteria for maximization such as quality, on-time delivery, after sale service and so on. X_d is the set of feasible solutions that satisfy the set of system and policy constraints.

Zimmermann (1978) first used the max–min operator of Bellman and Zadeh (1970) to solve fuzzy multi-objective linear programming problems.

Values of objective function $Z_j, j = 1,2,\ldots,q$ change linearly in the interval Z_j^{min} to Z_j^{max}. Therefore, he held the opinion that these functions can be presumed as fuzzy numbers of linear membership functions (Fig.1).

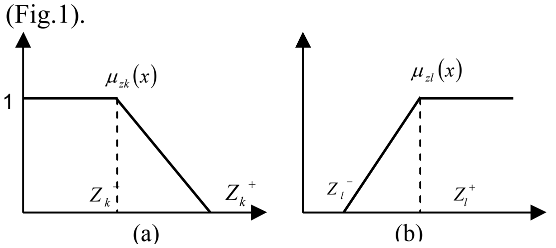

Fig. 1: Membership function: (a) minimum & (b) maximum objective

Linear membership function of each minimization function (Z_k) and maximization function (Z_l) are obtained as expressed in equations (5) and (6) using their minimal and maximal values.

$$\mu_{zk}(x) = \begin{cases} 1 & for Z_k \le Z_k^- \\ f_{\mu z_k} = \left(Z_k^+ - Z_k(x)\right)/\left(Z_k^+ - Z_k^-\right) & for Z_k^- \le Z_k(x) \le Z_k^+ \\ 0 & for Z_k \ge Z_k^+ \end{cases} \quad (k = 1,2,\ldots,p) \quad (5)$$

$$\mu_{zl}(x) = \begin{cases} 1 & for Z_l \ge Z_l^+ \\ f_{\mu z_l} = \left(Z_l(x) - Z_l^-\right)/\left(Z_l^+ - Z_l^-\right) & for Z_l^- \le Z_l(x) \le Z_l^+ \\ 0 & for Z_l \le Z_l^- \end{cases} \quad (l = p+1, p+2, \ldots q) \quad (6)$$

Where: Z_k^+ and Z_k^- are respectively maximum and minimum values of minimization function Z_k, and also Z_l^+ and Z_l^- respectively represent maximum and minimum values of maximization function Z_l. These values are obtained by solving each of objective functions separately and in two maximization and minimization states.

Max-min model was proposed by Zimmerman (1978, 1993) for solving MOLP problems as shown by equation (7):

$$\begin{aligned}
&Max\lambda \\
&\text{Subject to} \\
&\lambda \le f_{\mu z_j}(x), J = 1,\ldots,q\ (for\ all\ objective\ functions) \qquad (7) \\
&g_r(x) \le b_r,\ r = 1,\ldots,m \\
&x_i \ge 0, i = 1,\ldots,n,\ \lambda \in [0,1]
\end{aligned}$$

5. Integrated FAHP-FMOMILP model for supplier selection and order lot-sizing

The proposed algorithm is illustrated in Fig. 2. This algorithm consists of two major stages: evaluation stage and order allocation stage.

5.1. Evaluation Stage

In the evaluation stage, the suppliers are evaluated and weighted through FAHP technique and the obtained weights enter the allocation stage as coefficients of TVP objective function. In the following section, this stage is analyzed in four main steps:

Determination of decision-making criteria, control criteria, and construction of AHP model:

Determination of criteria is one of the most important parts of decision-making process. The criteria indicate level of attaining the objectives and strategies of organization. Taking into account the kind of respective product in this stage, evaluation criteria are primarily determined according to BOCR merits.

Following determination of criteria, hierarchical structure is constructed. The important point in hierarchical structures with BOCR merits is the fact that if significance degrees of B, O, C and R are not equal, they cannot be mutually compared with the objective in order to achieve the weights. To obtain

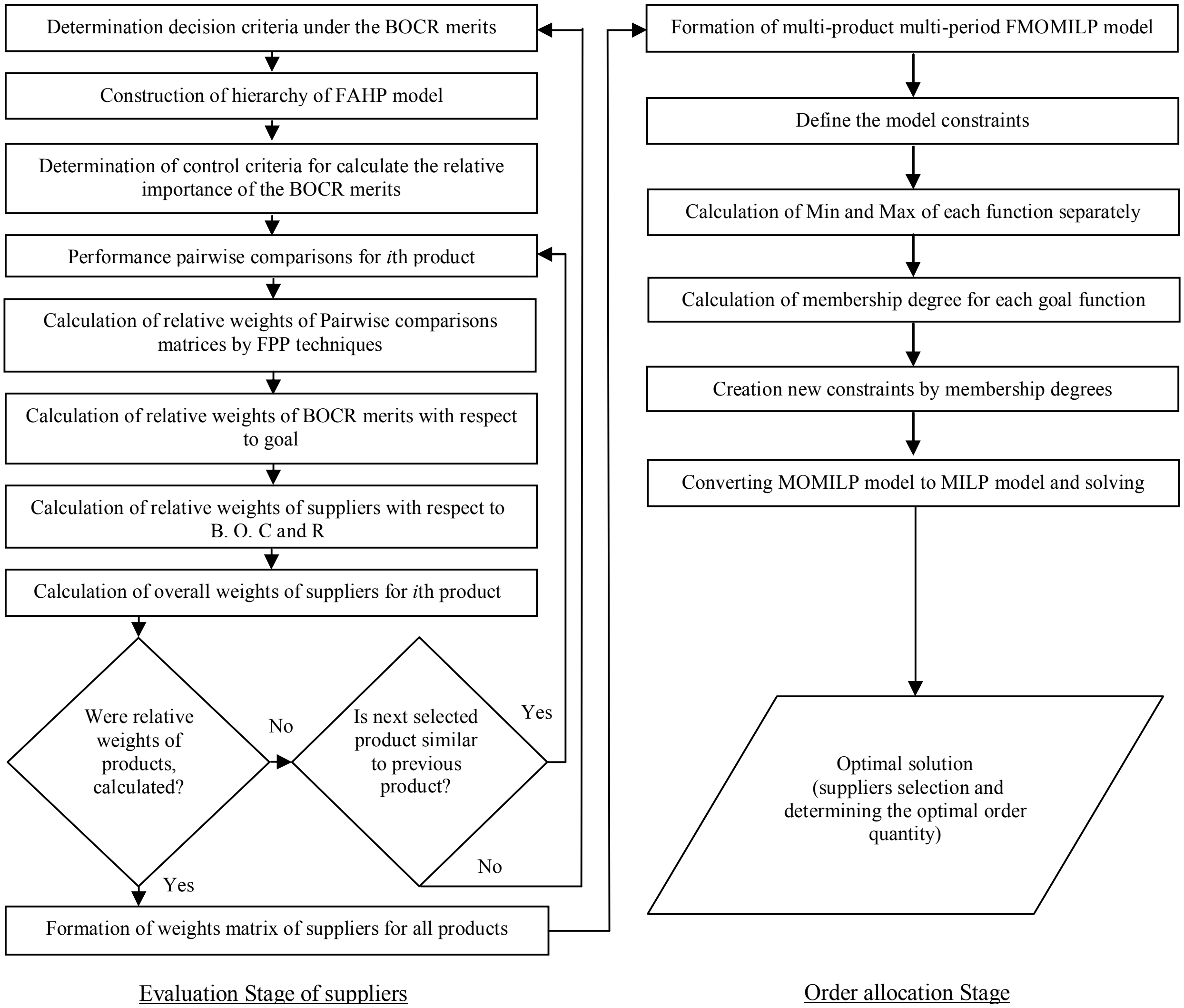

Fig 2: The algorithms of the Evaluation and allocation Stages

weight of each priority, a series of control criteria are defined with respect to the objective. So, the relative weights of these priorities can be derived through pairwise comparisons between control criteria compared to objective and BCOR merits compared to control criteria (Lee, 2009).

Performing Pairwise Comparisons and Calculation of Relative Weights:

In this step, elements of each hierarchy level are compared with the corresponding element in the upper level in pairs.

For benefits (B) and opportunities (O), the question is to ask what gives the most benefit or presents the greatest opportunity to influence fulfillment of the sub-criterion (detailed criterion). For costs (C) and risks (R), the question is to ask what incurs the most cost or faces the greatest risk (Lee, 2009).

In the respective calculations, the corresponding values of linguistic variables are introduced by triangular fuzzy numbers in the pairwise comparison matrices as demonstrated in table 1 (Lee, 2009; Cakir & Canbolat, 2008). FPP method is used in this step to calculate relative weights of pairwise comparison matrices.

Table 1. Membership function of the Linguistic variable

Linguistic variable	Membership function
Equally important	(1,1,2)
Moderately important	(2,3,4)
More important	(4,5,6)
Strongly important	(6,7,8)
Extremely important	(8,9,9)
Preferences between the above intervals	(x-1,x,x+1) for x= 2,4,6,8

Calculation of Relative Weights B, O, C, and R with respect to objective, and suppliers with respect to B, O, C, and R:

Relative weight of each alternative with respect to each of B, O, C, and R merits are obtained via sum of multiplication of relative weights of sub-criteria of the same priority by relative weight of that alternative with respect to these sub-criteria; these values are respectively designated B_i, O_i, C_j, and R_j. Relative weights of B, O, C and R with respect to objective are also determined through sum of multiplication of relative weight of each merits by relative weight of each control criteria. These values are respectively represented by b, o, c, and r.

Calculation of final weights of suppliers and creation of weight matrix:

In this step, the relative weights obtained for B, O, C, and R merits with respect to objective are combined with the relative weights of suppliers with respect to B, O, C, and R merits using a method proposed by Saaty in section 3 so as to derive the total weight of suppliers. Saaty (2003b) has investigated advantages of each of these methods in his research.

Now, it is necessary to realize that whether weights of suppliers have been obtained for all products or not. If the answer is “yes” the procedure is directed to the second stage of algorithm otherwise weights of suppliers shall be calculated for all products. There is another fundamental question here: is the selected product similar to the former one in terms of performance features and selection criteria? If not, the first stage of the proposed algorithm should be executed for evaluating the suppliers. The main reason lies in the fact that control criteria and sub-criteria affecting the selection will vary if the features of a product change. Therefore, a new hierarchy with decision-related criteria shall be defined to precise acquire the preferences of suppliers. Conversely, if the answer to the question is “yes”, there will be no need to define a new hierarchy and algorithm procedures will be repeated after its fourth stage. These comparisons are only performed for suppliers and control criteria. The reason is the fact that sub-criteria and control criteria have not changed in the previous stage.

Algorithm of evaluation stage is iterated as many times as the number of products until weights of all suppliers are achieved for all products. In the last stage, the weights obtained in a $I*J$ matrix are categorized and denoted by W_{ij}. Accordingly, result of evaluation stage of W_{ij} weight matrix will be indicative of score of “j” supplier for the “i” product. It must be vitally noted that if the *j*th supplier cannot supply the *i*th product, then W_{ij} will equal zero. The calculated weight matrices are introduced as input to the allocation stage to enter the decision-making space as coefficients of objective function of TVP.

5.2. Order allocation Stage

In this stage, the suppliers are selected via six objective functions ad optimal order quantity for each of suppliers will be determined. Steps of allocation stage are as below:

Creation of multi-product multi-period FMOMILP model in quantity discount environments:

The notations used in MOMILP model are as follows:

- **Notations**

Indices:

“i”: Index of products
“j”: Index of suppliers
“t”: index of time periods
“k”: index of price level

Parameters:

I: number of products
J: Number of suppliers
T: Number of time periods
n_{ijt}: Number of price levels of *j*th supplier for *i*th product and *t*th period
D_{it}: Demand of *i*th product in the *t*th period
C_{ijt}: Capacity of *j*th supplier for *i*th product in *t*th period
P_{ijkt}: Purchase price of the *i*th product from the *j*th supplier in *k*th price level and for *t*th period
S_{ijkt}: The *k*th price level of *j*th supplier for *i*th product in *t*th period ($0=S_{ij0t}<S_{ij1t}<\ldots<S_{ijn(ijt)t}=C_{ijt}$)
O_{it}: ordering cost for the *j*th supplier in *t*th period
H_{it}: Holding cost of *i*th product in *t*th period
W_{ij}: Overall weight of *j*th supplier for *i*th product
q_{ijt}: Defect rate of *i*th product by *j*th supplier in *t*th period
t_{ijt}: On-time delivery rate of *i*th product by the *j*th supplier in *t*th period
Lt_{ijt}: Lead time for a unit of *i*th product by *j*th supplier in *t*th period
g_{ijt}: Guarantee provided for a unit of *i*th product by *j*th supplier in *t*th period

Decision variables:

X_{ijkt}: Value of *i*th product ordered to *j*th supplier in *k*th price level for *t*th period
$Y_{ijkt:}$ It value is 1, if *i*th product is ordered to *j*th supplier in *k*th price level for *t*th period, and equals 0 otherwise.

Intermediate Variable:

I_{it}: Inventory of *i*th product at the end of *t*th period (Inventory of *i*th product transferred from *t*th to (t+1) th period) is assumed: $I_{i0} = 0$ and $I_{iT}=0$.

- **Objective Functions**

Total Cost: The expenses related to supply of product includes three items: purchase cost, ordering cost, and holding cost. Sum of these three costs equals total cost which shall be minimized.

$$\min f_1(X,Y)=\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{n_{ijt}}\sum_{t=1}^{T}P_{ijkt}X_{ijt}+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{n_{ijt}}\sum_{t=1}^{T}O_{jt}\,Y_{ijkt}+\sum_{i=1}^{I}\sum_{t=1}^{T}h_{it}I_{it} \quad (8)$$

TVP: The weights measured in suppliers' evaluation stage are transferred to this stage as input data so that Sum of multiplication of order quantities to suppliers and their weights will yield the total purchase amount defined via equation (9):

$$\max\ f_2(X)=\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{n_{ijt}}\sum_{t=1}^{T}W_{ij}\ X_{ijkt} \quad (9)$$

Total defect rate: Sum of multiplication of defect rate of purchase unit and total purchase amount signifies total defect rate of purchase as defined through equation (10):

$$\min\ f_3(x)=\sum_{i=1}^{I}\sum_{J=1}^{J}\sum_{k=1}^{n_{ijt}}\sum_{t=1}^{T}q_{ijt}\ X_{ijkt} \quad (10)$$

Total on-time delivery rate: Sum of multiplication of on-time delivery rate of purchase unit and total purchase amount gives the total on-time delivery rate of purchase as expressed by equation (11):

$$\max\ f_4(x)=\sum_{i=1}^{I}\sum_{J=1}^{J}\sum_{k=1}^{n_{ijt}}\sum_{t=1}^{T}t_{ijt}\ X_{ijkt} \quad (11)$$

Total lead time: Sum of multiplication of lead times of unit ordered product and total purchase amount yields the total lead time of purchase as defined by equation (12):

$$\min\ f_5(x)=\sum_{i=1}^{I}\sum_{J=1}^{J}\sum_{k=1}^{n_{ijt}}\sum_{t=1}^{T}Lt_{ijt}\ X_{ijkt} \quad (12)$$

Total guarantee: Sum of multiplication of guarantee of unit purchased products and total purchase amount gives the total guarantee as expressed by equation (13).

$$\max\ f_6(x)=\sum_{i=1}^{I}\sum_{J=1}^{J}\sum_{k=1}^{n_{ijt}}\sum_{t=1}^{T}g_{ijt}\ X_{ijkt} \quad (13)$$

- Model constraints

The constraints of the problem are formulated as follows:

Capacity constraints:

$$\sum_{k=1}^{n_{ijt}}X_{ijkt}\le C_{ijt}Y_{ijkt} \quad i=1,2,\dots I \quad j=1,2,\dots,J \quad t=1,2,\dots T \quad (14)$$

Demand constraint:

$$I_{i(t-1)}+\sum_{j=1}^{J}\sum_{k=1}^{n_{ijt}}X_{ijkt}\ge D_{it} \quad i=1,2,\dots,I \quad t=1,2,\dots,T \quad (15)$$

Material balance equations:

$$I_{it}=\sum_{j=1}^{J}\sum_{k=1}^{n_{ijt}}X_{ijkt}-D_{it}+I_{i(t-1)} \quad i=1,2,\dots,I \quad t=1,2,\dots,T \quad (16)$$

Discount intervals constraints:

$$S_{ij(k-1)t}Y_{ijkt}\le X_{ijkt}\le S_{ijkt}Y_{ijkt} \quad i=1,\dots I \quad j=1,\dots J \quad t=1,\dots T \quad k=1,\dots n_{ijt} \quad (17)$$

$$\sum_{k=1}^{n_{ijt}}Y_{ijkt}\le 1 \qquad k=1,2,\dots,n_{ijt} \quad (18)$$

Inventory constraints:

$$I_{iT}=0 \quad i=1,2,\dots I \quad (19)$$

Non negativity and binary constraints:

$$X_{ijkt}\ge 0$$
$$I_{it}\ge 0 \quad (20)$$
$$Y_{ijkt}=0 \quad or \quad 1 \quad Integer$$

Set of (14) constraints guarantees sum of orders from a supplier shall be less than its capacity. In one period of time, demand is supplied from two quantity; its order quantity and the inventory remaining from the previous period, which according to (15), sum of two values shall be always greater than or equal to the required demand value. Sum of order quantity of *i*th product in *t*th period and the quantity of inventory remaining from previous period is always equal to demand quantity of that product and the quantity transferred to the next period. This constraint is considered in (16). It is noteworthy that due to comprehensiveness of constraint of material balance equations (16) compared to demand constraints (15), there is no need to demand constraints for performing the model and their presence is mainly for more profound comprehension of model constraints. Set of constraints of (17) will guarantee that order quantity in each price level is in its discount domain. Constraint (18) is taken into account to select the supplier only in one discount level. Constraint (19) is imposed to have zero for inventory of each product at the end of programming period.

Using max-min technique for solving FMOMILP model

The last general stage of the proposed model comprises model solution and determination of superior suppliers. Max-min technique in the current paper for solving the model so that suitable suppliers are selected and their optimal order quantity were determined.

6- Case Study

To have a better insight of the model, a numerical example is presented in this part. The information used in this example has been prepared from purchase sector of a steel industry company which has three suppliers and two required products of steel bar kind. Purchase programming is done for all three periods.

6-1- Evaluation Stage

Determination of decision-making criteria, control criteria, and construction of AHP model

This proposed model consists of 11 criteria and five control criteria used in Lee's research (2009). Their hierarchical structures are shown in figures 3 and 4.

Performing the pairwise comparisons and calculation of relative weights

In this stage, elements of each hierarchy level are compared in pairs with respect to the element in the upper level and pairwise comparison matrices are formed. By applying FPP technique for each of matrices and with the aid of LINGO 80 software, weight values of each hierarchy element are summarized in table 2 and 3.

Calculation of B, O, C, and R relative weights with respect to objective and suppliers with respect to B, O, C, and R

To calculate relative weight of each of B, O, C, and R merits with respect to objective, sum of multiplication of relative weight of each priority and relative weight of each control criteria, which yield the values of b, o, c, and r as shown in table 2.

To calculate relative weight of each supplier with respect to each of B, O, C, and R merits, sum of multiplication of relative weight of sub-criteria of the same priority and relative weight of that supplier with respect to this sub-criterion. The respective values of B_i, O_i, C_i, and R_i are included in table 3.

Calculation of final weight of suppliers and creation of weights matrices:

To obtain the final weight of suppliers, b, o, c, and r values obtained for merits B, O, C, and R are combined with values of R_j, C_j, O_j, B_j obtained for three suppliers by probabilistic additive method and the results are normalized. As such, final weights of suppliers are obtained for a product. The same procedures are repeated to have the weights of suppliers for all other products. Accordingly, total weights matrices of suppliers are obtained for two ordered products in the form of following matrices:

$$W_{ij} = \begin{bmatrix} 0.307 & 0.301 & 0.392 \\ 0.217 & 0.287 & 0.496 \end{bmatrix}$$

6-2- Order allocation Stage

Creation of multi-product multi-period FMOMILP in quantity discount environments:

The needed values for formulating the model are expressed in tables (4) to (7). Values of purchasing cost of product unit from three suppliers are shown in table (4) for different price levels, which are assumed constant for all periods. Purchasing cost unit is expressed in dollars for one ton of product. Values of h_{it} and O_{jt} are expressed dollar for each ton and D_{it} values are expressed in tons as included in table (5). The values of C_{ijt}, q_{ijt}, Lt_{ijt}, g_{ijt} in table (6) are taken constant for all periods. Units of C_{ijt}, q_{ijt}, Lt_{ijt}, g_{ijt} values are respectively expressed in tons, quantity in product unit, days, and months. t_{ijt} values are shown for three periods in table (7) expressed in percents. Having applied parameter values in functions (8) through (13) and constraints (14) through (20), FMOMILP model is formulated.

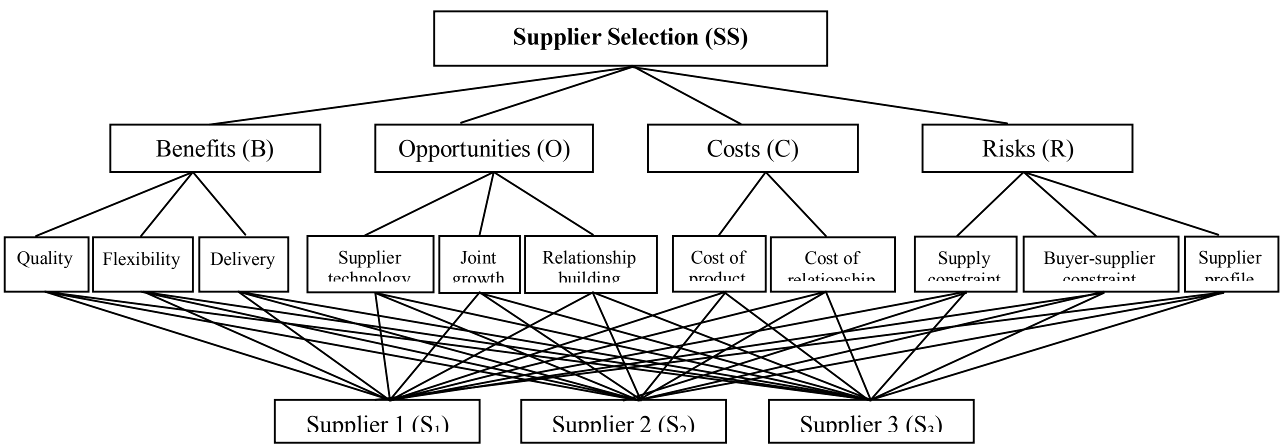

Fig.3. BOCR hierarchy

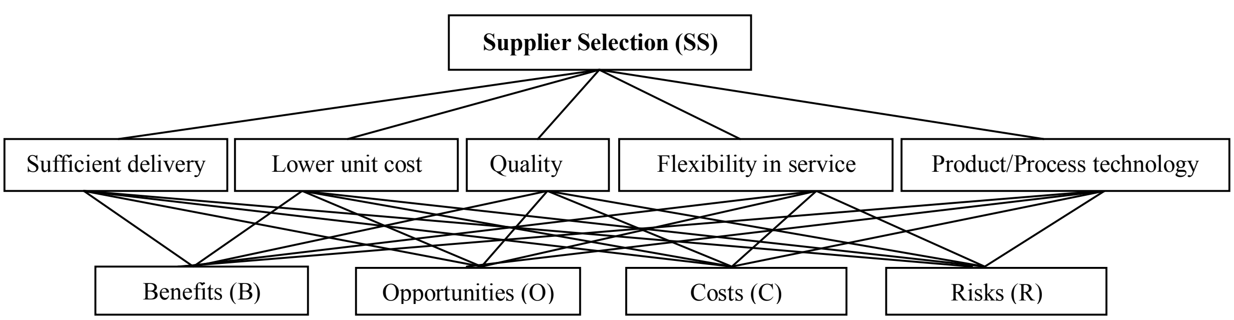

Fig.4. control hierarchy

Table 2. Priorities of merits

Merits	Sufficient delivery (0.128)	Lower unit cost (0.217)	Quality (0.276)	Flexibility in service (0.218)	Product/Process technology (0.161)	Normalized Weights (b, o, c, r)
B	0.335	0.385	0.531	0.641	0.047	0.420
O	0.106	0.301	0.348	0.079	0.508	0.274
C	0.473	0.097	0.094	0.226	0.106	0.174
R	0.086	0.217	0.027	0.054	0.339	0.132

Table 3. Priorities of alternatives under four merits

	Merits								
	Benefits					Opportunities			
	Quality (0.379)	Flexibility (0.208)	Delivery (0.413)	Normalized Weights (B_j)		Supplier technology (0.463)	Joint growth (0.128)	Relationship building (0.409)	Normalized Weights (O_j)
S1	0.508	0.313	0.114	0.305	S1	0.508	0.313	0.114	0.322
S2	0.137	0.068	0.294	0.187	S2	0.137	0.068	0.294	0.192
S3	0.355	0.619	0.592	0.508	S3	0.355	0.619	0.592	0.486
	Merits								
	Costs					Risks			
	Cost of product (0.694)	Cost of relationship (0.306)		Normalized Weights (C_j)		Supply constraint (0.561)	Buyer-supplier constraint (0.083)	Supplier profile (0.356)	Normalized Weights (R_j)
S1	0.508	0.313		0.448	S1	0.438	0.313	0.114	0.312
S2	0.137	0.068		0.116	S2	0.152	0.068	0.294	0.196
S3	0.355	0.619		0.436	S3	0.41	0.619	0.592	0.492

Table 4. Suppliers quantity discount

	Product 1					Product 2				
	P_{1i1t}	S_{1i1t}	P_{1i2t}	S_{1i2t}	P_{1i3t}	P_{2i1t}	S_{2i1t}	P_{2i2t}	S_{2i2t}	P_{2i3t}
S_1	330	80	315	230	305	440	120	410	260	390
S_2	375	110	340	-	-	390	85	370	240	340
S_3	315	90	305	250	295	435	120	415	-	-

Table 5. Supplier's quantitative information (D_{it}, h_{it}, O_{it})

	D_{it}			h_{it}				O_{it}		
	t=1	t=2	t=3	t=1	t=2	t=3		t=1	t=2	t=3
Product 1	540	570	480	25	30	35	S1	3600	3500	3400
Product 2	630	740	450	40	43	45	S2	4000	3100	3900
							S3	3400	3400	3200

Table 6. Supplier's quantitative information (C_{ijt}, q_{ijt}, Lt_{ijt}, g_{ijt})

	C_{ijt}			q_{ijt}			Lt_{ijt}			g_{ijt}		
	S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3
Product 1	360	370	420	0.0035	0.0040	0.0020	6	7	4	18	20	24
Product 2	440	420	360	0.0035	0.0020	0.0040	8	5	7	15	20	18

Table 7. Supplier's quantitative information (t_{ijt})

	Period 1			Period 2			Period 3		
	S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3
Product 1	0.97	0.95	0.99	0.95	0.96	0.98	0.96	0.94	0.99
Product 2	0.96	0.99	0.93	0.96	0.98	0.94	0.95	0.99	0.95

Table 8. Supplier selected, optimal order quantity and Inventory of product

	Y_{iikt}			X_{iikt}			h_{it}		
	t=1	t=2	t=3	t=1	t=2	t=3	t=1	t=2	t=3
Product 1	$Y_{1131}=1$ $Y_{1331}=1$	$Y_{1332}=1$	$Y_{1333}=1$	$X_{1131}=330$ $X_{1331}=420$	$X_{1332}=420$	$X_{1333}=420$	$I_{11}=210$	$I_{12}=60$	I13=0
Product 2	$Y_{2231}=1$ $Y_{2321}=1$	$Y_{2232}=1$ $Y_{2322}=1$	$Y_{2233}=1$ $Y_{2323}=1$	$X_{2231}=420$ $X_{2321}=210$	$X_{2232}=420$ $X_{2322}=320$	$X_{2233}=242.8$ $X_{2323}=207.2$	$I_{21}=0$	$I_{22}=0$	I23=0

Using max-min technique for solving FMOMILP model:

To solve model through this model, maximal and minimal values of each objective function are initially obtained disregarding other objective functions and by imposing all constraints. LINDO 6.1 software is used for this purpose.

Subsequently, membership degrees of objective functions are obtained according to (5) and (6). These membership degrees are fixed greater than or equal to λ variable and new constraint is imposed equal to number of objective functions. Through adding these constraints to the former constraints and conversion of objective function to Max Z=λ, multi-objective linear programming model is transformed into a single-objective linear programming model; optimal answers are gained as shown in table (8) through solving this model by means of LINDO 6.1 software.

7. Conclusions

In the current research, an integrated two-stage model was proposed for selection of suppliers in general purchase state i.e. multi-products, multi-suppliers, multi- periods and in quantity discount envinments. In the first stage, the suppliers were evaluated and selected in the second stage using a mathematical programming model. One of problems of the mathematical models is the fact that qualitative criteria are not brought under consideration. TO overcome problem, a two-stage approach was taken where suppliers were first evaluated in the first stage by means of qualitative and quantitative criteria affecting the decision-making. From another aspect, one of most important features of mathematical models is presence of variables by obtaining which optimal answers can be achieved. Set of the variables in the proposed model consists of three different groups of variables: variable "Y" which determines the superior supplier for products and different periods; variable "X" which is representative of order quantity of each product to supplier in programming periods, and variable "I" which denotes the inventory of each product at the end of each period. Through solving this model and obtaining these variables, superior suppliers are identified and optimal order quantity for each one is also determined.

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07/20/2012