Recursive Least Square (RLS) Based Channel Estimation for MIMO-OFDM System

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Abstract: Channel State information can be determined by adaptive filtering algorithms for wireless channels. For slow fading channels, simplified channel estimators can be exploited such as Least Square Error (LSE) and Linear Minimum Mean Square Error (LMMSE). But for fast fading channels, the matrix inversion required in case of LMMSE has to be taken recursively which increase the complexity. Under such conditions adaptive filtering algorithms are used to reduce the complexity with better performance. LMS, RLS and Kalman Filtering techniques can be used. But in wireless MIMO channels normally RLS and Kalman Filter are used at the cost of more complexity as compared to LMS which has better computational efficiency and feasibility. For initialization of adaptive filter, the channel can be estimated by LSE or LMMSE initially. In this paper the performance of RLS for both initially estimated LSE and LMMSE channel is compared in terms of Mean Square Error (MSE) and complexity is evaluated in terms of computational time. Optimization of LSE-RLS and LMMSE-RLS is performed as a function of wireless channel taps and Channel Impulse Response (CIR) samples. Monte-Carlo Simulations are carried for RLS channel estimation algorithm.
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1. Introduction

For high performance 4G wireless broadband internet and multimedia services, while providing a comparable quality of service (QoS) to that of existing wire line services, Multiple antennas has been proposed at both Base Transceiver Station (BTS) and subscriber ends. Multiple antennas are used for provision of high range of coverage in NLOS channel conditions (> 90% of the users in the given cell), data transmission with high reliability of 99.9%, high peak data rate for both UL and DL of greater than 1 Mb/s and high spectral efficiency of greater that 4 b/s/Hz/Sector [1]. These system requirements can be achieved by combining MIMO technology with Orthogonal Frequency Division Multiplexing (OFDM) modulation. For high data rates and high delay spreads of time-varying frequency selective channel, OFDM is preferred over single carrier modulation scheme with the advantage of low-complex adaptive equalizers [1].

Since the channel is time-varying, so the transmitter and receiver needs the channel statistics for optimization of the system parameters such as modulation and coding, signal bandwidth, transmission power, channel estimation etc. Channel can be estimated either in time-domain or frequency domain, which can be further categorized into pilot-assisted or decision directed channel estimation. Under fast time-varying channel, a lots of pilots need to be inserted for better channel estimation which comes at the cost of decreased system efficiency [2]. But these pilots can be used for determination of timing and frequency offset. In frequency domain channel estimators, the correlation of the channel parameters can be exploited which requires the inversion of a large matrix iteratively for decoupling of the inter-antenna interference. To reduce the complexity we use adaptive estimators as compared to simplified estimators i.e. LSE, LMMSE, DFT-CE and DCT-CE. Adaptive filters such as Wiener Filter, LMS and RLS Filters can be used for estimating the time-varying channel. Wiener Filter requires second order channel statistics but LMS and RLS do not require a priori knowledge of channel statistics. The performance of LMS and Kalman Filtering based channel estimator is evaluated in [2], [3], [4] as a function of channel taps and Channel Impulse Response (CIR) Samples. In this paper the performance and complexity of RLS-CE is evaluated for different MIMO techniques. The effect of varying Channel Taps and Channel Length is observed while using the initially estimated channel through LSE and LMMSE.

The rest of the paper is organized as: System Description of MIMO-OFDM is given in Section II, RLS algorithm is discussed in Section III with the simulation results given in Section IV and finally conclusion are drawn in last section.

2. MIMO-OFDM System Model

Suppose a MIMO-OFDM system with M_T transmit antennas and M_R receive antennas. The signal to be transmitted at k^{th} frequency is given as [5]

$$x_k = [x_k^1, x_k^2 \ldots x_k^{M_T}]$$

After passing through MIMO channel, the signal received will be [5]

$$y_k = \sum_{l=0}^{L} G_l x_{k-l} + n_k$$

Where G_l represents $M_R \times M_T$ channel matrix at the l^{th} delay.

In frequency domain the channel matrix is given by [5]

$$H(e^{j\theta}) = \sum_{l=0}^{L} h_l e^{-jl\theta}$$

Where $-\pi < \theta < \pi$. For L=0, the channel will be flat-fading and for L > 0 , the channel will be frequency-selective [7].

In case of frequency selective channel, the signals are distorted by Inter-Symbol Interference (ISI), which can be reduced with OFDM modulation. OFDM not only reduces ISI but also makes channel Memoryless [10].

A complete block diagram of MIMO-OFDM system is shown in Figure 1. AT a time, the transmitter takes N symbols in form of vectors and arranges them according to the channel matrix. After that IDFT operation is performed, followed by the addition of cyclic prefix. Now the signal is transmitted by M^{th} transmit antenna after Digital to Analog conversion.

At receiver side first the inserted cyclic prefix is removed before performing DFT operation.

3. RLS-Based Channel Estimation

Due to high convergence rate and fast steady-state adaptation, RLS channel estimator is used for time-varying mobile channels. Due to the poor convergence of LMS-CE, RLS is preferred for highly correlated data but for better performance the disadvantage comes in form of increased complexity.

As compared to Gradient Algorithms, RLS algorithm is used to implement simple LS-CE as daptive estimator. The cost function for LSE initially estimated channel case is given by [8]

$$J_{RLS}[N] = \sum_{i=1}^{N} \gamma^{N-i} . \left|E_{m,n}[i]\right|^2 + \delta . \gamma^N . \|w[N]\|^2$$

Where γ is forgetting factor whose exact value is difficult to be estimated and δ is regularization parameter.

The error vector for n^{th} OFDM symbol at m^{th} carrier is given by

$$E_{m,n}[i] = H_{m,n}[i] - w^H \hat{H}_{m,n}[i]$$

$\hat{H}_{m,n}[i]$ is the estimated channel, which is determined by LS method at initialization.

Channel up-dating is done by the following steps [9]

1- The value of correlation matrix $\hat{R}_{g_{r,t}g_{r,t}}$ at iteration n is given by

$$\hat{R}_{g_{r,t}g_{r,t}}[n] = \lambda \hat{R}_{g_{r,t}g_{r,t}}[n-1] + \hat{H}_{r,t}^{RLS}[n] \hat{H}_{r,t}^{H,RLS}[n]$$

2- Gain Matrix is given by

$$\hat{R}_{g_{r,t}g_{r,t}}[n] k[n] = \hat{H}_{r,t}^{RLS}[n]$$

3- Error vector is

$$E[n] = \hat{H}_{r,t}^{LS}[n] - \hat{W}^T[n-1] \hat{H}_{r,t}^{RLS}[n]$$

4- Conversion Factor at iteration n is

$$\alpha[n] = 1 - k[n] \hat{H}_{r,t}^{RLS}[n]$$

5- After which the error is given by

$$\varepsilon[n] = \alpha[n] E[n]$$

6- After n^{th} iteration, the up-dated co-efficients are

$$\hat{W}^T[n-1] = \hat{W}^T[n-1] + k[n-1] E^*[n]$$

Now the estimated channel becomes

$$\hat{H}_{r,t}^{RLS}[n] = \sum_{m=0}^{M-1} \hat{W}[m] \hat{H}_{r,t}^{RLS}[n-m]$$

The gain vector $k[n]$ is given by

$$k[n] = \frac{\lambda^{-1} Q[n-1] \hat{H}_{r,t}^{RLS}[n]}{1 + \lambda^{-1} \hat{H}_{r,t}^{RLS}[n] Q[n-1] \hat{H}_{r,t}^{RLS}[n]}$$

and

$$Q[n] = \frac{1}{\lambda}\left(Q[n-1] - k[n] \hat{H}_{r,t}^{RLS}[n] Q[n-1]\right)$$

Initially the parameter values are

$$Q[0] = [\hat{H}_{r,t}^{RLS}[0] . \hat{H}_{r,t}^{RLS}[0] + \delta I]^{-1}$$

and

$$k[0] = Q[0] \hat{H}_{r,t}^{RLS}[0] = \frac{1}{\left\|\hat{H}_{r,t}^{RLS}[0]\right\|^2 + \delta} . \hat{H}_{r,t}^{RLS}[0]$$

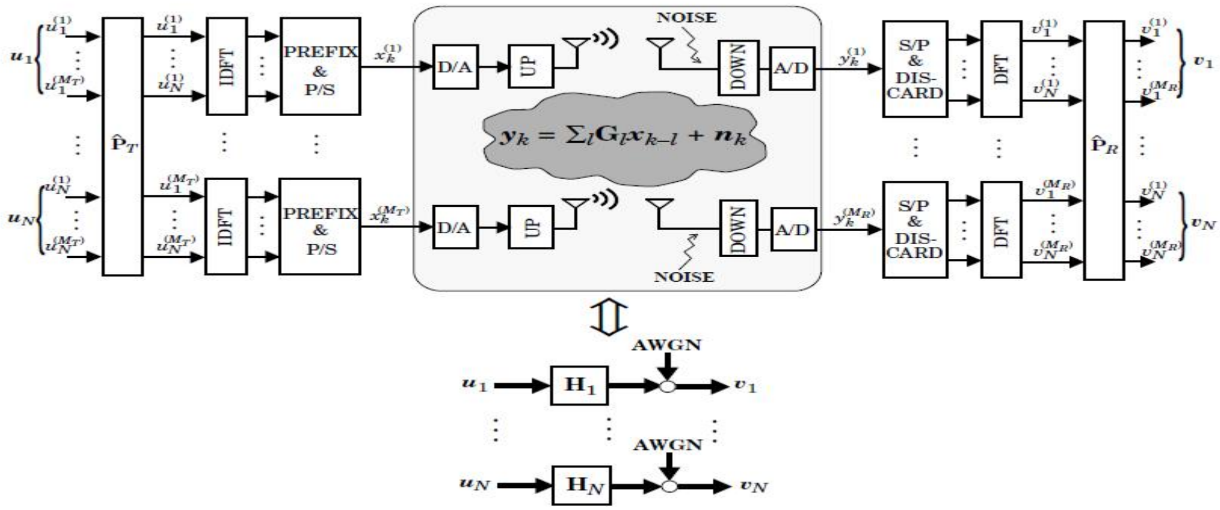

Figure 1. MIMO-OFDM System Model [6]

4. Simulation Results

Under different SNR operating conditions, the effect of varying the channel filter length on the performance of RLS estimator is shown in Figure 2. As we increase SNR value, the performance degrades for any channel filter length. For a specific SNR value, the performance degrades as larger length of channel filter is considered. So for better performance, less complexity and less power-consumption, less number of CIR samples are taken for low SNR values. The performance of RLS estimator as a function of SNR and CIR Samples is shown in Figure 3. The complexity of RLS estimator for different channel filter lengths is given in Table 1. By increasing the channel length from 10 CIR samples to 20, the complexity increases by 37%. Further increment of channel filter length to 40 increases the complexity by 93%.

MSE as a function of different channel filter lengths for different MIMO systems is given in Figure 4. Up to filter length of 5, the performance remains same for any MIMO system but as we increase the filter length beyond 5 CIR samples, the performance degrades almost as a linear function of increasing the channel filter length. Figure 4 also demonstrates that as the order of MIMO system is increased the performance also improves and this improvement is observed for all channel filter lengths under consideration. But higher order system gives better performance at the cost of more computational time. For RLS estimator, the initialized channel estimator can be either LSE or LMMSE. The performance comparison for both cases is given in Figure 5. LMMSE-RLS gives the better performance for all channel filter lengths as it exploits the prior knowledge of the channel statistics that is why it has more complexity as given in Table 2. From Table 2, we note that for 2×2 MIMO system the complexity of LMMSE-RLS is 113% greater than that of LS-RLS for channel filter length of 10 but as we increase the channel filter length to 40 CIR Samples then this increment is only 77%. For LS-RLS method, the complexity of 3×3 is 71% more than that of 2×2 system while for 4×4 this increment is about 233%. Similarly in case of LMMSE-RLS approach, as compared to 2×2 system the computational time of 3×3 is 151% greater while for 4×4 case this increment becomes 350%.

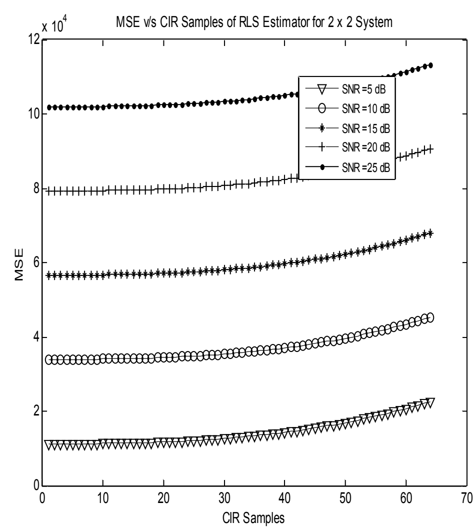

Figure 2. MSE vs CIR Samples of RLS Estimator for 2×2 System

The performance of RLS estimator in terms of Mean Square Error as a function of Channel Taps at different SNR operating conditions is shown in

Figure 6. The effect of channel taps is same as that of CIR samples. The performance is better for low SNR values and less number of multi-path channel taps. The effect of channel taps on complexity is shown in Table 3. By increasing the channel taps two time, the complexity increases by 14 % but if the channel taps are made four times, then increment in complexity is 28%.

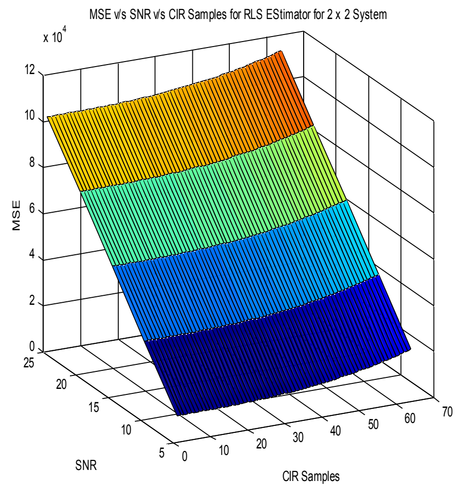

Figure 3. MSE vs SNR vs CIR Samples of RLS Estimator

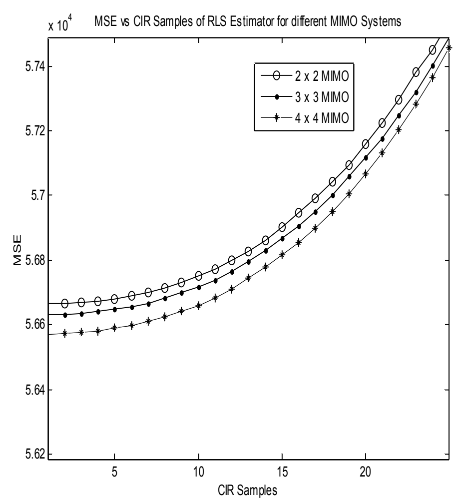

Figure 4. MSE vs CIR Samples of RLS Estimator different MIMO Systems

Table 1. Complexity of RLS as a function of CIR Samples for 4×4 MIMO

CIR Samples	Time (μsec)
10	403.2
20	553.15
40	779.34

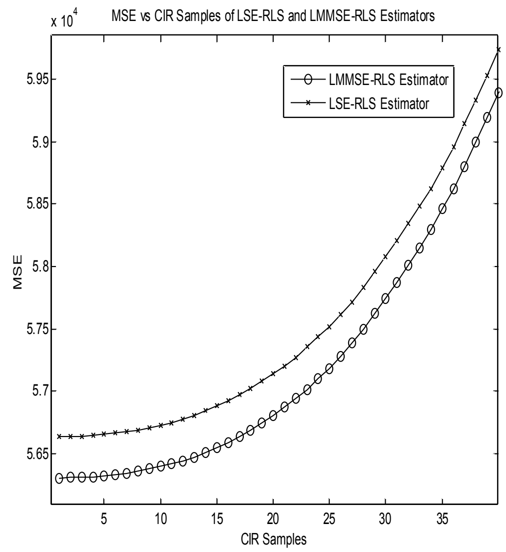

Figure 5. MSE vs CIR Samples of LS-RLS and LMMSE- for RLS Estimator

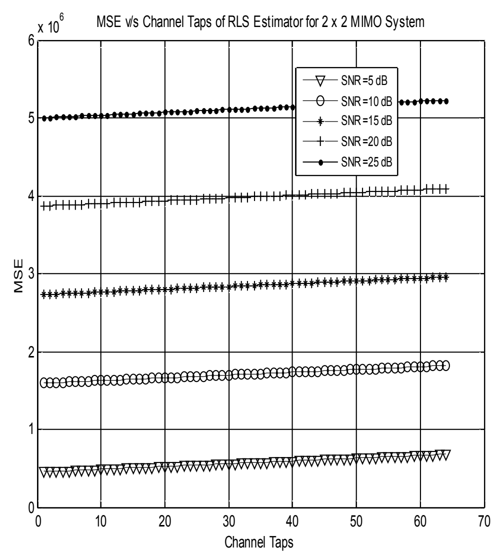

Figure 6. MSE vs Channel Taps of RLS Estimator for 2×2 System

The effect of increasing the channel taps is not so significant in case of high order MIMO e.g. 4×4, but for low order MIMO systems the effect of increasing channel taps results in degraded performance. For better performance under any channel tap number, higher order MIMO is preferred which gives improved performance at the cost of more computational time. The complexity behavior of both LS-RLS and LMMSE-RLS is given in Table 4. For all values of channel taps, LMMSE-RLS takes 6-7 times more computational time than that of LS-RLS method. The performance comparison of LS-

RLS and LMMSE-RLS is shown in Figure 8. We note that the effect of increasing the channel taps is more significant in case of LSE-RLS than LMMSE-RLS. We also observe that the performance of LSE-RLS for less number of channel taps is same to that of LMMSE-RLS at large number of channel taps. So we can optimize the complexity by considering appropriate value of channel taps for LMMSE-RLS estimator. The combined effect of SNR and channel taps on performance is shown in Figure 9.

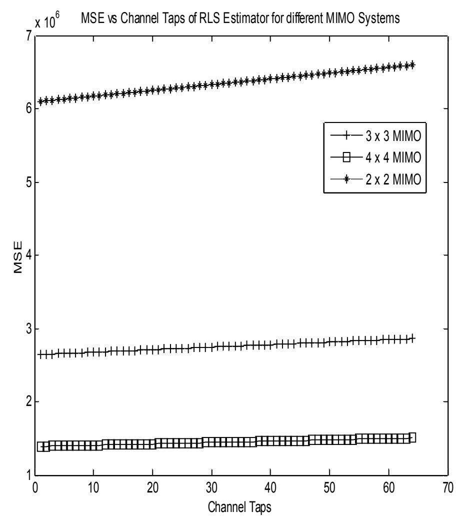

Figure 7. MSE vs Channel Taps of RLS Estimator for different MIMO Systems

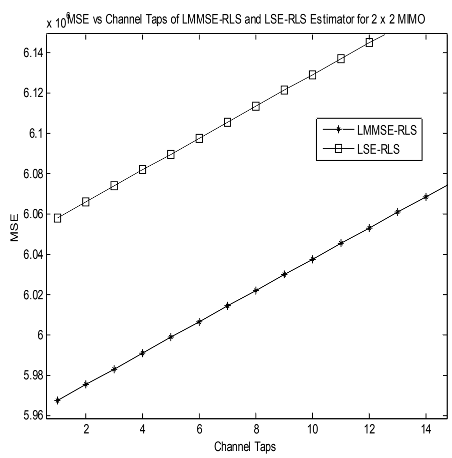

Figure 8. MSE vs Channel Taps of LS-RLS and LMMSE-RLS Estimator

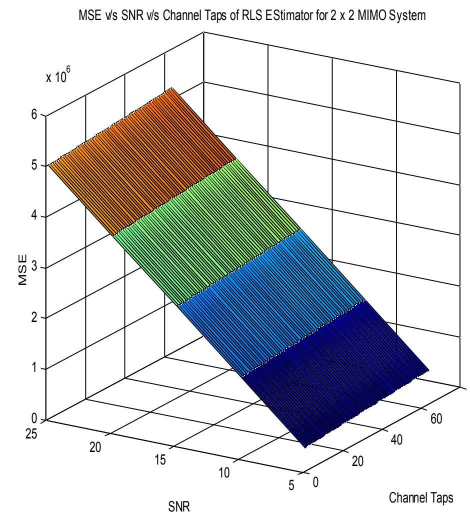

Figure 9. MSE vs SNR vs Channel Taps of RLS Estimator

5. Conclusion

In this paper adaptive filtering based channel estimation algorithm, Recursive Least Square (RLS), is optimized for performance in terms of Mean Square Error (MSE) and complexity in terms of computational time. These two parameters are compared for different CIR samples and multi-path channel taps. To make power-efficient communication with better performance less number of CIR samples are used under low SNR values. When the initialized channel estimation is by LMMSE, then RLS gives better performance but with high complexity as LMMSE exploits the second order channel statistics. Higher the order of MIMO system, better will be performance and for any MIMO system, channel filter length of 5 CIR samples is preferred for optimized performance and complexity. Similar behavior is also observed for channel taps as that of channel filter length. For higher order MIMO system, the effect of varying channel taps on performance goes on diminishing so for reduced computational time less number of channel taps are preferred. The optimized channel estimator can also be implemented by using other adaptive filtering techniques such as LMS and Kalman Filtering based channel estimation.

Table 2. Complexity of RLS for different MIMO Schemes

CIR Samples	2 × 2		3 × 3		4 × 4	
	LS-RLS	LMMSE-RLS	LS-RLS	LMMSE-RLS	LS-RLS	LMMSE-RLS
10	104.33	222.62	178.84	559.85	348.15	1000
20	113.74	252.68	236.3	640.61	464	1200
30	170.73	302.92	356.17	700.44	765.6	1400

Table 3. Complexity of RLS vs Channel Taps for 2×2 System

Channel Taps	Time (μsec)
5	252.5
10	850.25
20	323.41

Table 4. Complexity of RLS vs Channel Taps for different MIMO Systems

Channel Taps	2×2 (μsec)		4×4 (μsec)	
	LS-RLS	LMMSE-RLS	LS-RLS	LMMSE-RLS
5	175.53	1400	328	2900
10	201.9	1400	365.5	3100
20	223.4	1600	394.57	3800

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