Custom-made Biomechanical Model of the Knee Joints

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Abstract: This paper obtained the data of custom-made tibia, femoral joints and meniscus morphology by studying relating researches and constructed several sagittal section of the knee joint. Based on the 2D bone morphology of femoral condyle and the contact radius relationship of the tibial plateau, it created the design of the sphere curvature of femoral condyle to conform to the demands of the patients. To simulate the various meshing curved surface between bones, this paper considered using sphere to fit femoral condyle, using femur image data to fit the femur into spheres and using the Hertz's elastic contact theory to construct a femur-tibia and femur-meniscus biomechanical model. Lastly, using the contact stress and contact deformation obtained by solving the numeric examples under specific cases can be used as a reference in the design and production of future prosthesis.
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1. Introduction

Recently, the quality of computer medical image for artificial prosthesis greatly improved. Due to these improvements, the patient who can only use crutches or canes to walk or even the patients who are amputated in the past can walk like normal people and this improved the quality of their life greatly. Not only that knee joint is the most complicated joint constructed and the biggest joint of a human body, it has high expectations in its movement function. Every year, a great number of patients have to undergo artificial knee arthroplasty. At present, the people that underwent artificial knee arthroplasty, the limbs of the patients can load, stretch, abduce and rotate in a very stable manner.

Most of the artificial knee prosthesis being clinically used should be imported because there are very few companies, such as United Orthopedic Corporation, who design and develop artificial knee products. Importing prosthesis is quite expensive and the products are designed based on knee joint parameters of Americans and Europeans which sometimes are incompatible to Asian knees. To recover the complex activities of the knees, the artificial knee prosthesis designed should be compatible in anatomy, biomechanical and kinematics. Stress analysis allows the stress morphology of the prosthesis and the human skeleton to be more compatible to improve the long-term stability of the prosthesis and avoid unreasonable stress peak and contact area morphology. Moreover, because the model of the reconstructed bones can be rotated and cut at any angles, it provides a good method to the complex surface geometric morphology researches and simulation surgeries of the knee. To design a better knee prosthesis compatible to Taiwanese, custom-made normal knees underwent bone morphology measurement and kinematic researches to create multi-segmented meshing curved surface and Hertz theory was used to derive the biomechanical model of the femur and tibia in different meshing sections. In addition, the model is used to derive the average contact stress, contact area in the meshing area. These results can serve as a reference to clinical diagnosis and custom-made prosthesis manufacture.

The studies relating to the femur and tibia contour morphology are as follows:

David Siu measured the medial femoral condyle and its distal condyle, radius of the patellar surface and arc angle and the lateral femoral condyle and its distal condyle, radius of the patellar surface and arc angle of 5 cadever's knee joints after CT scanning a 3D reconstruction[1]. The result of the study of Elias where 10 samples of cadavers with normal knees are dissected and compared to 6 normal adult knees showed patients of different races, different sizes or due to curve fitting results to different arcs for different femoral condyle joints[2]. Zhou's research shows that the width of the medial and lateral femoral condyle and the width of the medial and lateral tibia plateau are very similar which proves that the width of femoral condyle and tibia plateau is related. Greater femoral condyle width shows greater tibia plateau width. Thus, the value of femoral condyle and tibia plateau can be used as the index to represent the size of the knees. His simplified model shows that sagittal femoral condyle profile can be fitted into three sections arc curves. Moeinzadeh et al[3]. researched on the biomechanics of the femur tibia of knee joints where they are the first to create an artificial knee dynamic model human knee joint and fixed the femur. When the model is used to measure the fixed femur, the tibia is relative to the motion of the femur and can compute for the contact stress of the knee joints. Wongchaisuwat et al[4]. created another knee dynamic model where the femur is fixed and the tibia is simulated as a simple pendulum to analyze the control strategy which the femur is relative to tibia's sliding and rolling. Komistek et al[5]. created a model by applying Kane's method where the peak of the contact stress between femur and tibia was obtained by analyzing different walking speeds[6,7]. Wang released femur and created a two-dimensional biodynamic model of femur and tibia[8~11]. Tumer and Engin created a femur-tibia-patellar dynamic model of human knee. The model simulated that when the femur is fixed, tibia is relative to the relative motion of femur and patellar[12]. Wang released femur again and created a femur-tibia-patellar three knee joint 2D meshing motion mathematical model[13]. Jia created a lower limb 3D model to compute for the force and torque of the femur and tibia during gait cycle [14]. Please refer to [15,16,17] for other related researches.

The main task of this paper is to discuss the sagittal plane section near femoral condyle and tibial plateau under the 3D geometry of the obtained femur and tibia through geometric analysis software to obtain the bone contour shape of the knee joint. Numerical method was applied on the contour to fit the arc with different curvature and the contour is also fitted into several spheres with different diameter by setting the length of an appropriate Z axle. Similar cross section was also applied to proximal tibia to explore the contact stress of the femoral and tibial meshing.

The results include:

1. applying the elastic theory to simplify the biomechanical model of femur, tibia and meniscus can compute the contact stress of the femur, tibia and meniscus.
2. through image processing, use the femoral condyle and tibial plateau meshing to fit different spheres and calculate the contact stress and contact area according to different patients.

Through the sagittal section of the femoral condyle and tibial plateau stated above, we can create sphere radius of contact morphology in any section to use as the knee stress analysis foundation which can be used as the design and production of custom-made artificial knee joints.

2. Research Methodology

The research method of this paper include construction and analysis of the contact morphology of profile fitting of the femur and tibia under the same section and solve the problem by using the meshing surface as a multiple sphere radius according to the bone morphology of the different patients.

2.1 Femur fitting calculation

Figure 1 shows the femur/tibia meshing 3D geometry. By the use of the software, we can obtain the bone morphology of the sagittal plane and the bone meshing surface as shown in Figure 1 and 2.

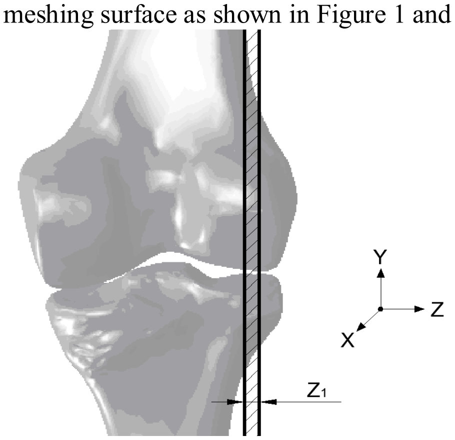

Figure 1. Geometric model operation

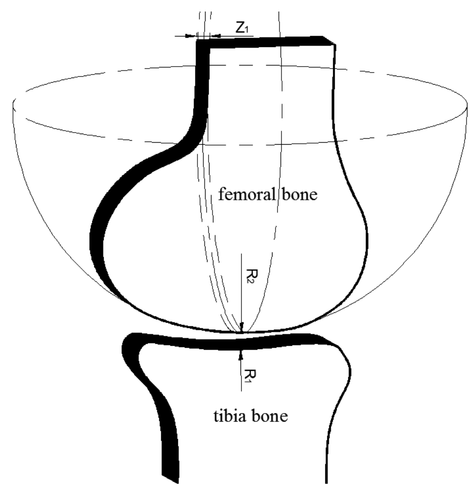

Figure 2. Sagittal Skeletal Profile

2.2 The sphere fitting explanation using femur is as follows

Consider the Z-direction length on Figure 1 and with C given, the sphere equation is established.

$$R^2 = (x-A)^2 + (y-B)^2 + (z-C)^2 \quad (1)$$

$$R^2 = x^2 - 2Ax + A^2 - 2By + B^2 + z^2 - 2Cz + C^2 \quad (2)$$

where (A,B,C) are the coordinates of the center and R is the radius of the sphere.

When a=-2A, B=-2B, c=-2C, d=A2+B2+C2-R2, another form of sphere formula can be derived as shown in equation (3)

$$x^2+y^2+c^2+ax+by+cz+d=0 \quad (3)$$

As long as a, b,c and d are derived, we can obtain the center and radius parameters:

$$\begin{cases} A = -\dfrac{a}{2} \\ B = -\dfrac{b}{2} \\ C = -\dfrac{c}{2} \\ D = \dfrac{1}{2}\sqrt{a^2+b^2+c^2-4d} \end{cases} \quad (4)$$

In ball point set (X_i, Y_i, Z_i), $i \in (1,2,3\ldots N)$, D_i is the distance between midpoint and the center.

$$D_i^2 = (X_i - A)^2 + (Y_i - B)^2 + (Z_i - C)^2 \quad (5)$$

Thus, we can obtain the distance between the center to points (X_i, Y_i, Z_i) and square difference of the length of radius (δ):

$$\delta = D_i^2 - R^2 \quad (6)$$

$\Phi(a,b,c)$ can be obtained as the square least of δ_i:

$$\Phi(a,b,c,d) = \sum \delta_i^2 = \sum (X_i^2 + Y_i^2 + Z_i^2 + aX_i + bY_i + cZ_i + d)^2$$

(7)

Use partial derivatives to obtain a,b,c,d values and the value of $\Phi(a,b,c,d)$ is the smallest, thus

$$\begin{cases} \dfrac{\partial\Phi(a,b,c,d)}{\partial a} = \sum 2(X_i^2+Y_i^2+Z_i^2+aX_i+bY_i+cZ_i+d)X_i = 0 \\ \dfrac{\partial\Phi(a,b,c,d)}{\partial b} = \sum 2(X_i^2+Y_i^2+Z_i^2+aX_i+bY_i+cZ_i+d)Y_i = 0 \\ \dfrac{\partial\Phi(a,b,c,d)}{\partial c} = \sum 2(X_i^2+Y_i^2+Z_i^2+aX_i+bY_i+cZ_i+d)Z_i = 0 \\ \dfrac{\partial\Phi(a,b,c,d)}{\partial d} = \sum 2(X_i^2+Y_i^2+Z_i^2+aX_i+bY_i+cZ_i+d) = 0 \end{cases} \quad (8)$$

From equation (8), the values of a,b,c,d can be derived. Equation (4) derived the center and radius which completed the feature extraction of the spatial sphere.

2.3 Axis line alignment

The design method of the tibial plateau is similar as the one above. The difference is that the diameter of the tibial plateau is bigger and it can be flat, concave or convex after fitting according to the different elastic model. In obtaining tibia and the tibia cross-section, several important axis need to be fit into one straight line, thus the vertical fitting algorithm is needed.

The axis line equation is

$$Ax+by+1=0 \quad (9)$$

In using the least square fit, the deviation (ℓ) of equation (10) should be the smallest.

$$\ell = \sum (ax_i + by_i + 1)^2 \quad (10)$$

Equation (10) separately solves for the partial derivatives of a and b setting the partial derivatives to 0.

$$\begin{cases} \dfrac{\partial \ell}{\partial a} = 2\sum (ax_i + by_i + 1)x_i = 0 \\ \dfrac{\partial \ell}{\partial b} = 2\sum (ax_i + by_i + 1)y_i = 0 \end{cases} \quad (11)$$

Equation (11) can obtained the smallest a and b values of the deviation (ℓ) to fit the required straight line.

2.4 Bone profile morphology related authentication

Several researchers conducted studies regarding the curvature design of distal femur femoral condyle among which Zhang measured three fitting image as shown in Figure 3 and 4. Similar study[18], Chou has the curvature design of the 3 blocks of the femoral condyle. Table 1 describes the measurement results and errors of the sample patients in the statistics[3].

Figure 3. The X-ray shows the 3 arcs of the distal femur articular surface (ref. 16)

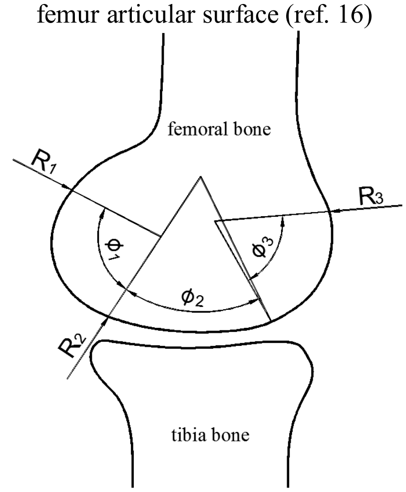

Figure 4. The measured parameters of distal femur

Table 1. Parameters of distal femur (ref. 4)

Geometric dimension	Values
r_1 (RPMFC)	20.62 ± 3.59
φ_1 (APMFC)	114.72 ± 6.17
r_2 (RDMFC)	37.94 ± 6.79
φ_2 (ADMFC)	52.62 ± 7.68
r_3 (RPaMFC)	25.71 ± 2.45
φ_3 (APaMFC)	42.59 ± 5.43

2.5 Joint elastic contact model

The organizational form of the joint of femur and tibia is shown in Figure 5. The distal femur and tibial plateau don't have a pure rolling meshing relationship which affects the flexing posture and stretching ligaments and because the meniscus roll buffer is usually a sliding movement. The motion range of every bone has its limit. Thus, the curvature geometric design and the construction of elastic mechanical model helps simulate contact stress and contact area morphology through different flexing needs.

The E_f, μ_f is set as the elastic modulus and Poisson's ratio of knee femoral elastic meshing, respectively. E_t, μ_t is the occlusion of tibial elastic modulus and Poisson's ratio, respectively. R_f, R'_f is the two main curvature radius of the elastic femur contact surface, respectively. R_t, R'_t is the two main curvature radius of the elastic tibia contact surface, respectively. a and b are the long and short axis the tibial-femoral joint contact spnere surface. δ is the relative displacement of the femur and tibia elastic center . q_0 is the average stress of the tibial-femoral joint contact elastic surface. P is the axial static pressure. Based from Hertz theory, the maximum stress, elastic occlusal contact area and the deformation of the solution between the meshing contact surface of the knee, shown below, among which the average stress is $q_0 = \frac{3P}{2\pi ab}$ and a and b separately represents the two circle axis.

$$a = \alpha \cdot \sqrt[3]{\frac{3}{4}\frac{P}{A}\left(\frac{1-\mu_f^2}{E_f}+\frac{1-\mu_t^2}{E_t}\right)} \quad (12)$$

$$b = \beta \cdot \sqrt[3]{\frac{3}{4}\frac{P}{A}\left(\frac{1-\mu_f^2}{E_f}+\frac{1-\mu_t^2}{E_t}\right)} \quad (13)$$

For sphere,the coefficient a=b.

$$q_0 = \frac{3P}{2\pi ab} \quad (14)$$

$$q = q_0\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}} \quad (15)$$

$$\delta = \lambda \sqrt[3]{\frac{9}{128}AP^3\left(\frac{1-\mu_f^2}{E_f}+\frac{1-\mu_t^2}{E_t}\right)} \quad (16)$$

$$A = \frac{1}{2}\left(\frac{1}{R_f}+\frac{1}{R'_f}+\frac{1}{R_t}+\frac{1}{R'_t}\right) \quad (17)$$

$$B = \frac{1}{2}\left[\left(\frac{1}{R_f}-\frac{1}{R'_f}\right)^2+\left(\frac{1}{R_t}-\frac{1}{R'_t}\right)^2+2\left(\frac{1}{R_f}-\frac{1}{R'_f}\right)\left(\frac{1}{R_t}-\frac{1}{R'_t}\right)\cos 2\Phi\right]^{\frac{1}{2}} \quad (18)$$

$$\cos\theta = \frac{B}{A} \quad (19)$$

In the equation, Φ is the two curvature angle of R_f and R_t in the between planes.

Moreover, α, β, λ is used as the parameters where the values can be based on the θ value derived from equation (19) where the results is shown in Table 2. If the two equations of the meshing surface of the knee joint is known, the maximum contact stress, contact area and meshing deformation of the tibial-femoral surface of the knee can be solved according to equations (12)~(19).

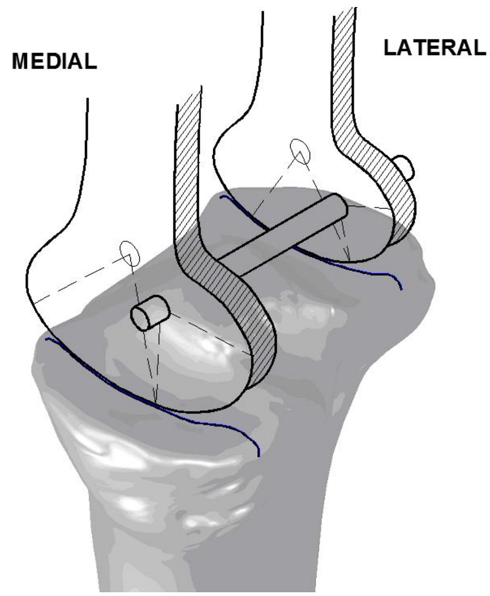

Figure 5. The intra-articular organizational profile

Table 2. The coefficient of equations

θ	0°	10°	20°	30°	35°	40°	45°	50°
α	∞	6.612	3.778	2.731	2.397	2.135	1.926	1.754
β	0	0.319	0.408	0.493	0.530	0.567	0.604	0.604
λ		0.851	1.220	1.453	1.550	1.637	1.709	1.772
θ	55°	60°	65°	70°	75°	80°	85°	90°
α	1.611	1.486	1.378	1.284	1.202	1.128	1.061	1.000
β	0.678	0.717	0.759	0.802	0.846	0.893	0.944	1.000
λ	1.828	1.875	1.912	1.944	1.967	1.985	1.996	2.000

3. Results and Discussion

Combining the results of literatures based on femoral condylar morphology, the hypothesize tibia and femoral condyle software contact of this study separately considered tibia plateau radius as -400mm and plateau as 400mm where the contact radius (R) and average pressure (q_0) are shown in Figure 6 and the parameters are shown in Table 3.

If the contact between meniscus and femur is considered, the meniscus coefficient is E1=59MPa and ν1=0.49. The meniscus and femur contact radius considers the meniscus as -400mm, plane as +400mm and the femur condyle changes 2° each time from 20° to 40°. The contact radius and average pressure is shown in Figure 7.

Table 3. The coefficient of two cases

Case1	Case2
$E_t = E_f = 5MPa$	$E_f = 59Mpa$; $E_t = 5Mpa$
$\nu_t = \nu_f = 0.46$	$\nu_f = 0.46$; $\nu_t = 0.49$
$h_t = h_f$ (cartilage thickness) = 1mm	$h_f = 1mm$
F: bearing capacity	F: bearing capacity

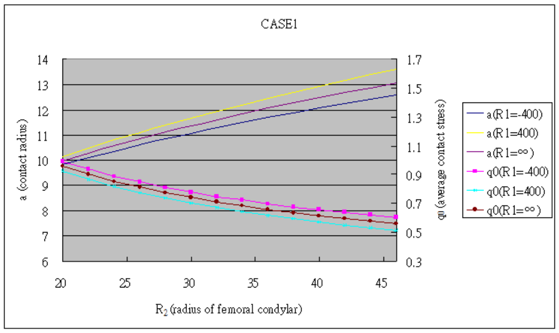

Figure 6. Contact of femur and tibia

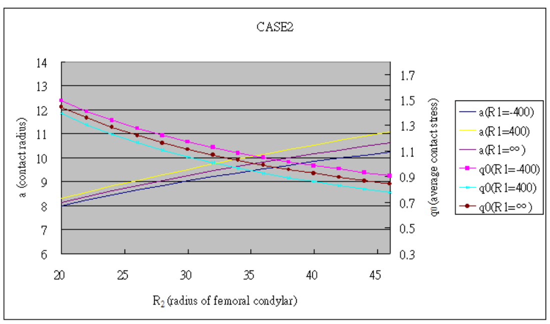

Figure 7. Contact of femur and meniscus

4. Conclusion

The demand for artificial knee in the market and the quality of its demand rapidly works toward the direction of custom-made. According to the human skeleton contour modeling techniques and then according to the patient's weight, different lifestyle and habits and movement patterns of the knee, designing and constructing knee prosthesis is quite complex and challenging. With a main direction of custom-made and with the help of the results medical images, this paper designed a femur condyle sphere and sphere or concave sphere of tibia plateau biomechanical model through change curvature value of the bone processing technique and with the elastic theory of Hertz between balls to calculate the contact stress, contact area and deformation in the meshing of the joints to serve as a reference in the design, production and analysis of custom-made prosthesis.

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