On Bipreordered Approximation Spaces

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Abstract: We used preordered relations to define a bipreordered space and hence bitopological space and introduced a condition (*) on these relations such that $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$, where $\overline{R}(A) = \overline{R}^1(A) \cap \overline{R}^2(A)$, and hence we get a topology $\tau_{R_{12}}$ on X satisfies

$$\overline{A} = \overline{R}(A) = \overline{R}^1(A) \cap \overline{R}^2(A) = \{x \in X: xR_1 \cap xR_2 \cap A \neq \phi\} = \overline{A}^1 \cap \overline{A}^2$$

and $\tau_{R_{12}} = \tau_{R_1 \cap R_2} = \tau_{R_1} \vee \tau_{R_2}$ .We deal with bitopological spaces (X, τ_1, τ_2) which satisfying a certain condition (**) and proved that the family of all such bitopological spaces BTS^{**} is equivalent to the family of all bipreordered spaces BPS^*.

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1. Introduction

A classic paper of Z. pawlak [17] is the Rough Sets (RS), published in 1982, which declared the birth of the RS theory. A lot of mathematicians, logicians, and researchers of computers have become interested in the RS theory and have done a lot of research work of RS in theory [6, 14, 15] and application. Its applications are showed in wide fields such as machine learning [5], data mining [4], decision- making support and analysis [16, 18, 23], process control [22] and expert system [26].

Different kinds of generalizations of pawlak RS model can be obtained by replacing the equivalence relation with an arbitrary binary relation [3, 19, 20, 25]. It was proved that, the pair of lower and upper approximation operators induced by reflexive and transitive relations is exactly a pair of interior and closure operators of a topology [27, 29]. Some surveys of RS theory and applications are presented in [21, 28]. Many properties of RS were obtained when the approximation space is finite. When the universe is infinite, the relationship between generalized RS induced by binary relation and topologies were investigated in [11] and [24]. In [11], a kind of compactness condition (comp) was proposed and it was proved that a topology which satisfies (comp) can determine the lower and upper approximation operators induced by reflexive and transitive relation. In [24], the topology induced by reflexive and transitive relation does not satisfy (comp) in general. Another kind of compactness condition (COMP) is proposed and it is proved that there exists a one-to-one correspondence between the set of all reflexive and transitive relations and the set of all topologies which satisfy condition (COMP).

The formation and progress of the theory of bitopological spaces introduced in [10]. The theory acquires special importance in the light of applications of its results. The theory of bitopological space has been developed in [1, 7, 8, 12].

the order relations used to define a topology or bitopologies on a set X were often equivalence relations (e.g.[2]). In this paper we used only preordered relations(i.e. reflexive and transitive) to define topologies, however, $\overline{A \cup B} = \overline{A} \cup \overline{B}$ still does not hold, where $\overline{A} = \overline{A}^1 \cap \overline{A}^2$, so, we introduced the condition (*) for preordered relations R_1 and R_2 making the preceding equality holds and hence we could generate a topology by two preordered relations, $\tau_{R_{12}}$ and proved that for all $A \subseteq X$,

$$\overline{A} = \overline{R}(A) = \overline{R}^1(A) \cap \overline{R}^2(A) = \{x \in X: xR_1 \cap xR_2 \cap A \neq \phi\} = \overline{A}^1 \cap \overline{A}^2$$

and many other properties are proved , especially, $\tau_{R_{12}} = \tau_{R_1 \cap R_2} = \tau_{R_1} \vee \tau_{R_2}$ and many examples on finite and infinite universes are given. If a bitopological space (X, τ_1, τ_2) is given, we introduced a condition (**) such that $C(A \cup B) = C(A) \cup C(B)$ becomes hold, and hence we obtained a topology $\tau_{12} := \{A \subseteq X: A^{o1} \cup A^{o2} = A\}$ and proved that there exists a one-to-one correspondence between the family of all bitopological space satisfying the condition (**) which denoted by BTS^{**} and the family of all bipreordered spaces satisfying (*) which denoted by BPS^*.

2. Material and Methods

2 Preliminaries

2.1 Definition[3]

Let R be any relation on X, $x \in X$ and $A \subseteq X$. The afterset(foreset) of x is defined respectively, by $xR := \{y \subset X : xRy\}$, $Rx := \{y \subset X : yRx\}$,
and the upper(lower) approximation of A is defined by

$$\overline{R}(A) := \{x \in X : xR \cap A \neq \phi\} \qquad (1)$$

$$\underline{R}(A) = (\overline{R}(A'))' \qquad (2)$$

.

2.2 Theorem[24]

If R is reflexive, then the operator $\overline{R}$ on $P(X)$, defined by (1), is Čech closure operator and hence it generates a topology on X given by

$$\tau_R = \{A \subseteq X : \underline{R}(A) = A\} \qquad (3)$$

Moreover, if R is a preorder relation on X, then $\overline{R}$ satisfies kuratowski's axioms i.e. for all $A \subseteq X$, $\overline{R}(A)$ represents the closure of A w.r.t. the induced topology τ_R and τ_R satisfies the following condition

$$COMP: \text{for all } x \in X \text{ and } A \subseteq X,\ x \in \overline{A} \Rightarrow \exists\ y \in A \text{ s.t. } x \in \overline{\{y\}}. \qquad (4)$$

Let (X, τ) be a topological space (TS) and $^{-}$ be its closure operator. We define a preorder relation on X by:

$$xRy \Leftrightarrow x \in \overline{\{y\}}\ \forall x, y \in X \qquad (5)$$

2.3 Theorem[24]

Let (X, τ) be a TS, $^{-}$ be its closure operator and R be as defined in (5). If (X, τ) satisfies the condition (4), then:

1. $\overline{R}(A) = \overline{A}\ \forall A \subseteq X$
2. $\tau_R = \tau$, where τ_R defined in (3)
3. $R_{\tau_R} = R$

2.4 Lemma

If R_1 and R_2 are two preorder relations on a non empty set X, then

$$x(R_1 \cap R_2) = xR_1 \cap xR_2$$

Proof. Straightforward.

2.5 Theorem[9]

Let (X, τ) be a TS. Then the following are equivalent:

1. (X, τ) satisfies the condition (4)
2. $\overline{\cup_{i \in J} A_i} = \cup_{i \in J} \overline{A_i}$
3. (X, τ) is an Alexandrov space.

2.6 Theorem[24]

There exists a one-to-one correspondence between the family of all preorder relations on X and the family of all topologies which satisfies (4).

3 Bipreordered Spaces

3.1 Definition

Let R_1 and R_2 be two preorder relations on a non empty set X. Then (X, R_1, R_2) is called bipreordered space (BPS).

3.2 Lemma

Let (X, R_1, R_2) be a BPS. Then pre-upper approximation operator $\overline{R} : P(X) \to P(X)$ given by:

$$\overline{R}(A) = \overline{R}^1(A) \cap \overline{R}^2(A) \qquad (6)$$

where $\overline{R}^j(A)$, $j = 1, 2$ be defined in (1), satisfies the following properties:

1. $A \subseteq \overline{R}(A)$
2. $\overline{R}(A) \cup \overline{R}(B) \subset \overline{R}(A \cup B)$
3. $\overline{R}(A \cap B) \subseteq \overline{R}(A) \cap \overline{R}(B)$
4. $\overline{R}(\overline{R}(A)) = \overline{R}(A)$
5. $\overline{R}(X) = X$
6. $\underline{R}(A) = (\overline{R}(A'))'$

Proof. Straightforward

.

The following example shows that $\overline{R}(A) \cup \overline{R}(B) \neq \overline{R}(A \cup B)$.

3.3 Example

Let
$X = \{a, b, c\}$, $R_1 = \Delta \cup \{(c, b), (b, c)\}$, $R_2 = \Delta \cup \{(c, a), (a, c)\}$, $A = \{a\}$
and $B = \{b\}$. Then $\overline{R}(A) \cup \overline{R}(B) \neq \overline{R}(A \cup B)$.

3.4 Definition

The BPS (X, R_1, R_2) is called BPS* if it satisfies the following condition
(*): If $(R_1 y \cap R_2 z) \setminus \{y, z\} \neq \phi$, then $yR_1 z$ or $zR_2 y$.

3.5 Examples

Let X be a non empty set, $a \in X$ and $A \subseteq X$. Then the following spaces (X, R_1, R_2) are examples for BPS*

1. $R_1 = \Delta \cup \{(x, a) : x \in X\}$,
$R_2 = \Delta \cup \{(a, y) : y \in X\}$
2. $R_1 = \Delta \cup \{(x, y) : y \in A\}$,
$R_2 = \Delta \cup \{(x, y) : x \in A'\}$

3.6 Theorem

If (X, R_1, R_2) is BPS*, then

1. $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$, where $\overline{R}(A)$ as defined in(1)
2. $\overline{R}(A) = \{x \in X : xR_1 \cap xR_2 \cap A \neq \phi\}$
3. If we define $\tau_{R_{12}} := \{A \subseteq X : \underline{R}(A) = A\}$ then $\tau_{R_{12}}$ is a topology on X. Moreover, $\overline{A} = \overline{R}(A) = C(A) = \overline{A}^1 \cap \overline{A}^2$, where $\overline{A}^j$ is the closure of A w.r.t. τ_{R_j}, $j = 1, 2$

Proof.

1. By Lemma (3.2).2

$$\overline{R}(A) \cup \overline{R}(B) \subseteq \overline{R}(A \cup B) \quad (7)$$

Let $x \in \overline{R}(A \cup B)$. Then $x \in \overline{R}^1(A \cup B)$ and $x \in \overline{R}^2(A \cup B)$ i.e. $xR_1 \cap (A \cup B) \neq \phi$ and $xR_2 \cap (A \cup B) \neq \phi$, i.e. there exists $y \in xR_1 \cap (A \cup B)$ and $z \in xR_2 \cap (A \cup B)$.

We have the following cases:

- If $y, z \in A$ then $xR_1 \cap A \neq \phi$ and $xR_2 \cap A \neq \phi$ which implies that $x \in \overline{R}(A)$ and then $\overline{R}(A) \cup \overline{R}(B) = \overline{R}(A \cup B)$.

- Similarly if $y, z \in B$.
- If $y \in A$, $z \in B$ and $y \in xR_1$, $z \in xR_2$, hence by (*) yR_1z or zR_2y. Since R_1, R_2 are transitive we have xR_1z or xR_2y, and hence $(xR_1 \cap B \neq \phi,\ xR_2 \cap B \neq \phi)$ or $(xR_1 \cap A \neq \phi,\ xR_2 \cap A \neq \phi)$. Hence $x \in \overline{R}(B)$ or $x \in \overline{R}(A)$, accordingly,

$$\overline{R}(A \cup B) \subseteq \overline{R}(A) \cup \overline{R}(B) \quad (8)$$

From (6) and (7) we get $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$.

- Similarly if $y \in B$, $z \in A$.

2. Let $x \in \overline{R}(A)$. Then $x\overline{R}^1(A)$ and $x \in \overline{R}^2(A)$, i.e. $xR_1 \cap A \neq \phi$ and $xR_2 \cap A \neq \phi$, i.e. there exists $y \in xR_1 \cap A$ and $z \in xR_2 \cap A$, hence by (*) yR_1z or zR_2y. Since R_1, R_2 are transitive we have xR_1z or xR_1y and hence $xR_1 \cap xR_2 \cap A \neq \phi$, i.e.$\overline{R}(A) \subseteq \{x \in X: xR_1 \cap xR_2 \cap A \neq \phi\}$. $\{x \in X: xR_1 \cap xR_2 \cap A \neq \phi\} \subseteq \overline{R}(A)$ is trivial. 3. Straightforward.

3.7 Theorem

Let (X, R_1, R_2) be a BPS^*. Then $\tau_{R_{12}}$ satisfies condition (4).

Proof. Let $x \in C(A)$. It follows that $x \in \overline{R}(A)$ and hence $xR_1 \cap xR_2 \cap A \neq \phi$, i.e. $x \in \overline{R}\{y\} = C(\{y\})$.

3.8 Theorem

Let (X, R_1, R_2) be a BPS^*. Then the family $\{xR_1 \cap xR_2 : x \in X\}$ is a basis for $\tau_{R_{12}}$.

Proof. Let $x \in G$ be an open subset of X. It follows that $x \in G = \underline{R}(G)$ and hence $x \in xR_1 \cap xR_2 \subseteq G$.

3.9 Lemma

Let (X, R_1, R_2) be a BPS^*. Then

1. Since $xR_1 \cap xR_2$ is the smallest possible neighborhood of x
2. A subset A of X is open if and only if $A = \cup_{x \in A}(xR_1 \cap xR_2)$.

Proof.

1. Since R_1 and R_2 are reflexive relations. Then $x \in xR_1 \cap xR_2 \quad \forall x \in X$, hence $xR_1 \cap xR_2$ is a neighborhood of x.

Let A be any neighborhood of x. It follows that $x \in i(A) = \underline{R}(A)$,

hence $xR_1 \cap xR_2 \subseteq A$, i.e. $xR_1 \cap xR_2$ is the smallest possible neighborhood of x.

2. By Theorem 3.8. the result follows immediately.

3.10 Theorem

If (X, R_1, R_2) is BPS^*, then

$\tau_{R_{12}} = \tau_{R_1 \cap R_2}$

Proof. For simplicity put $R_1 \cap R_2 = Q$. $A \in \tau_Q \Leftrightarrow \underline{Q}(A) = A \Rightarrow \{x: xQ \subseteq A\} = A \Leftrightarrow \{x: xR_1 \cap xR_2 \subseteq A\} = A \Leftrightarrow \underline{R}(A) = A \Leftrightarrow A \in \tau_{R_{12}}$, for all $A \subseteq X$. Then the result.

3.11 Theorem

If (X, R_1, R_2) is BPS^*, then

$\tau_{R_{12}} = \tau_{R_1} \vee \tau_{R_2}$

i.e. $\tau_{R_{12}}$ is the least upper bound topology containing τ_{R_1}, τ_{R_2}.

Proof. We want to show that$\tau_{R_1} \vee \tau_{R_2} \subseteq \tau_{R_{12}}$ and the other inclusion is clear.

Let $A \in \tau_{R_1} \vee \tau_{R_2}$. Then

$A = \overline{R}(\cap_{i,j}(B_i^c \cup B_j^c))$

$\subseteq \cap_{i,j} \overline{R}(B_i^c \cup B_j^c)) = \cap_{i,j}(\overline{R}(B_i^c) \cup \overline{R}(B_j^c))$, by theorem 3.6(1)

$= \cap_{i,j}(B_i^c \cup B_j^c)$, by (1)

$= A^c$. Hence $\overline{R}(A^c)$, and then $A \in \tau_{R_{12}}$.

4 Special Kinds of Bitopological Spaces

4.1 Definition

The bitopological space (BTS) (X, τ_1, τ_2) is called BTS^{**} if it satisfies the following condition

(**): $(\overline{\{y\}}^1 \cap \overline{\{z\}}^2) \setminus \{y, z\} \neq \phi \Rightarrow y \in \overline{\{z\}}^1$ or $z \in \overline{\{y\}}^2$, where τ_1 and τ_2 satisfy the condition (4).

4.2 Example

Let X be a non empty set, $a \in X$ and $A \subseteq X$ Then the following spaces (X, τ_1, τ_2) are examples for BTS^{**}

1. $\tau(a) = \{A \subseteq X: a \in A\} \cup \{\phi\}$, $\tau_a = \{A \subseteq X: a \notin A\} \cup \{X\}$

2. $\tau_A = \{B \subseteq X: A \subseteq B\} \cup \{\phi\}$, $\tau^A = \{B \subseteq X: B \subseteq A\} \cup \{X\}$

The following example shows that the two topologies satisfy (**) but one of them does not satisfy (4)

4.3 Example

Let X be an infinite set and $a \in X$. The BTS (X, τ_{co}, τ_a) where
$\tau_{co} = \{A \subseteq X : A' \ \ finite\} \cup \{\phi\}$,
$\tau_a = \{A \subseteq X : a \notin A\} \cup \{X\}$. Then each of τ_{co}, τ_a satisfies (**) and (X, τ_{co}) does not satisfy (4).
The following example shows that the two topologies satisfy (**) but neither of them is COMP.

4.4 Example

The BTS $(\mathbb{R}, \tau_{co}, \tau_N)$ where
$\tau_{co} = \{A \subseteq X : A' \ \ finite\} \cup \{\phi\}$,
$\tau_N = \{G \subseteq \mathbb{R} : \forall x \in G \ \exists \ \varepsilon > 0 \ \ s.t. \ (x - \varepsilon, \ x + \varepsilon) \subseteq G\}$
satisfy (**), but neither $(\mathbb{R}, \tau_{co})$ nor $(\mathbb{R}, \tau_N)$ satisfies (4).

4.5 Theorem

Let (X, τ_1, τ_2) be a BTS**. Then

1. $C(A \cup B) = C(A) \cup C(B)$, where

$$C(A) = \overline{A}^1 \cap \overline{A}^2, \ \forall \ A \in P(X), \qquad (9)$$

$\overline{A}^j$ denotes the closure of A w.r.t τ_j, $j = 1,2$;

2. $C(A)$ defined in (8) satisfies kuratowski's axioms and hence it generates a topology $\tau_{12} = \{A \subseteq X : i(A) = A\}$, where the interior of A

$$i(A) = (C(A'))' \quad (10)$$

3. τ_{12} satisfies condition COMP

Proof.

1. it's clear that

$$C(A) \cup C(B) \subseteq C(A \cup B) \qquad (11)$$

Now, we want to prove the another inclusion
Let $x \in C(A \cup B)$. Then $x \in \overline{A \cup B}^1$ and $x \in \overline{A \cup B}^2$. Hence by condition COMP, there exists $y \in A \cup B$ such that $x \in \overline{\{y\}}^1$ and $z \in A \cup B$ such that $x \in \{z\}^2$.

we have the following cases:
- if $y, z \in A$ then $x \in \overline{\{y\}}^1 \subseteq \overline{A}^1$ and $x \in \overline{\{z\}}^2 \subseteq \overline{A}^2$ and hence $x \in \overline{A}^1 \cap \overline{A}^2 = C(A)$. It follows that $C(A \cup B) = C(A) \cup C(B)$.
- Similarly if $y, z \in B$.
- If $y \in A$, $z \in B$ and $x \in \overline{\{y\}}^1 \cap \overline{\{z\}}^2$. Hence by (**) $y \in \overline{\{z\}}^1$ or $z \in \overline{\{y\}}^2$. It follows that $x \in \overline{\{z\}}^1$ or $x \in \overline{\{y\}}^2$, and hence $x \in C(z)$ or $x \in C(y)$. it implies that $x \in C(B)$ or $x \in C(A)$, accordingly,

$$C(A \cup B) \subseteq C(A) \cup C(B) \qquad (12)$$

From (10) and (11) we get $C(A \cup B) = C(A) \cup C(B)$.
- Similarly if $y \in B$, $z \in A$.

2. Straightforward

3. Let $x \in C(A)$. Hence $x \in \overline{A}^1 \cap \overline{A}^2$. It implies that there exists $y, z \in A$ such that $x \in \overline{\{y\}}^1 \cap \overline{\{z\}}^2$. Hence by (**) $y \in \overline{\{z\}}^1$ or $z \in \overline{\{y\}}^2$, and hence $x \in \overline{\{z\}}^1$ or $x \in \overline{\{y\}}^2$. It follows that $x \in C(z)$ or $x \in C(y)$.

4.6 Theorem
There exists one-to-one correspondence between the family of all BPS* and family of all BTS*.
Proof. It suffices to prove that

$$(*) \Leftrightarrow (**) \qquad (13)$$

Let $(\overline{\{y\}}^1 \cap \overline{\{z\}}^2) \backslash \{y, z\} \neq \phi$. Then there exists $x \in X$ such that $x \in (\overline{\{y\}}^1 \cap \overline{\{z\}}^2) \backslash \{y, z\} \neq \phi$. Hence xR_1y and xR_2z, and hence by (*) yR_1z or zR_2y. It implies that $y \in \overline{\{z\}}^1$ or $z \in \overline{\{y\}}^2$.
Necessity of (12) is similar

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