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Mathematical Modeling of Channel Flows with Heat in an induced Magnetic Field

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Abstract: An analytical study of channel flow with heat in an induced magnetic field in cylindrical domain has been done. The flow is considered along the channel axis, and is taken to be axi-symmetric. No-slip boundary condition is considered for velocity while the temperature has non-zero positive constants on the boundary. The physics of the problem is termed by the usual MHD equations with suitable boundary conditions. The basic equation governing the flow is that of Partial Differential Equation which was later reduced using an appropriate transformation. The two coupled system were solved analytical using Frobenius method and method of undetermined coefficient to obtain two solutions of flow variables. The expressions of these flow variables were displayed graphically.
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1. Introduction:

Mathematical models grow out of equations of which fluid dynamics models cannot be left behind due to its numerous applications in our daily lives. Such applications include the human blood vessels, oil and water, filtration and purification process in chemical engineering, thermal insulation and underground energy transportation etc. However, channel flow related problems can be seen in the works of [1], [2], [5],[7],[8] and [9] etc .

More so, channel flows are very essential in real life applications because it is the medium in which the physical quantities are modeled for the purpose of practical findings. This is why, it is imperative to understand the dynamic nature of the physical problem to be solved and also know why such method is good. Physical problem of this nature needs analytic approach, which can give exact solutions for proper mathematical predictions. On the general note, model signifies an abstraction of naturally or technically occurring phenomena or complex systems. Models are aimed to understand the essential parameters of the phenomena and to assemble them into mathematical equations that can be solved analytically or numerically. In fluid dynamics transport models allow an integrating consideration of complex physical systems. Such models have proven to be indispensible tools in many Engineering problems, for example, for improving process understanding and identification, interpretations of measurement data etc, A model helps in explaining situations, for example ,the appearance or distribution of a contamination in a ground water system, or the origin/leakage path of Co_2 that is detected somewhere at the ground surface. Models can also simulate interventions into systems and predict their effects. In addition, analysis of parameter sensitivities is an important purpose of model applications.

It is essential to study channel flow related problems, well formulated and accurate analytical solutions in order to stimulate realistic flow results; therefore, the analytical solution is adopted based on the specific feature of the problem, even though fluid characteristics may depend on additional physical quantities in terms of modeling, [15].

More so, an aspect of fluid dynamics that is of great interest in this study is non-isothermal flow, which is a channel flow with a non- constant temperature. This condition can enhance some certain process taking place within the fluid; which may be useful in carrying out certain operations. For instance, when a fluid is subjected to a temperature change, its material properties, such as density and viscosity change accordingly. In some certain circumstances, these changes are large enough to have substantial influence on the flow field. So, in non-isothermal fluid flow interface, it sets the temperature in the model input section and defines the density in the fluid properties.

It is important to know that, magnetic field as one of the fluid properties in the model helps to engender mechanical forces which adjust the flow of the fluid. Such interface occurs with a non- constant temperature. The studies of [3], [6] are significant in this purpose. Secondly we cannot be ignorant to the fact that many transport procedures exist in its natural nature and many real-life applications where heat and mass transfer arises as a result of joint buoyancy effects on the flow. Heat is form of energy transferred by virtue of a difference in temperature. Heat exists everywhere to a greater or lesser degree. Then, on aspect of cooling parameter applied in the model measures the relative change of temperature. It is a term referring to lack of heat in an object and its rate depends upon the area of the surface through which heat is lost. Another fundamental issue is problem of mass concentration in the model.

These flows, such as those in human cardiovascular system takes place in cylindrical domain; the human blood vessel, for example. Fluid dynamics models in such domains are more challenging than those in Cartesian coordinate systems. The problem gets more difficult when the transport of heat and mass are incorporated into the system. To this end, we propose an accurate analytical solution in a cylindrical channel flow.

This paper investigated the problem of heat and mass transfer in cylindrical channel under non-isothermal flow. The two fundamental equations governing the flow are in the form of partial differential equations and besides have been reduced to set of non-linear ordinary differential equations by means of suitable transformations. The governing coupled equations are solved analytically using Frobenius method and method of undetermined coefficient to obtain two different solutions of flow variables. The expressions for velocity and temperature are obtained graphically. The effects of key parameters are examined, discussed, and presented graphically.

The paper is presented as follows: In Section 2, we present the physical and mathematical models of the problem, and a detailed analytical solution is derived in Section 3, the derived analytic solution in Section 4. The results are presented and discussed in Section 4. The paper is concluded in Section 5.

2. Mathematical Formulation:

The flow is assumed to be dominated along the channel axis, and is taken to be axi-symmetric. No-slip boundary condition is considered for velocity while the temperature and concentration have non-zero positive constants on the boundary. Let r be the distance from the channel centre and $\vec{u}, T'$ are the fluid velocity and temperature as shown in Figure 1.Let the fluid velocity be $\vec{u} = (0, 0, u')$

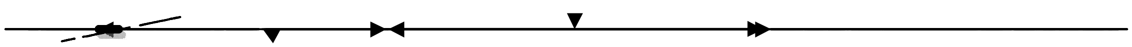

Figure 1: Physical Model

Since the flow is axisymmetric let (r', θ', z') be the cylindrical channel coordinates, where r is the radius of the channel, the directions of flow lies along horizontal axis z and flow is maintained at non-constant temperature (non-isothermal) the directions without flow lies along the vertical axis z see Figure 1.Based on the assumption of Boussinessq approximation of fluid model and taken consideration of induced magnetic field and steady flow; the equations governing the flow are:

$$\frac{\partial u'}{\partial z'} = 0 \tag{1}$$

$$-\frac{\partial p'}{\partial z'} + \mu\left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u'}{\partial r'}\right) + \rho g \beta_T \left(T' - T_\infty\right) - \alpha B_0'^2 u = 0 \tag{2}$$

$$\frac{k}{\rho C_P}\left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right) - \lambda^2\left(T' - T_\infty\right) = 0 \tag{3}$$

with following boundary conditions

$$u' = 0\ ,\ \theta' = \theta_a,\ \text{on}\ r' = a \tag{4}$$

$$u', \theta' < \infty\ \ \text{for all}\ r' \tag{5}$$

Where u' w' is the velocity components of fluid, ρ is the fluid density , B_0 is the magnetic field , k is the thermal conductivity, T' is the wall temperature , k is the thermal conductivity, g is the acceleration due to gravity and μ is the viscosity of the fluid, σ is the electrical conductivity , C_p is the heat capacity , q_r is constant magnetic field and β is the coefficient volumetric expansion.

2.1 Non-dimensionalization:

From equations (1)-(3) and boundary conditions (4) and (5) respectively the following non- dimensional variables were used:

$$\left.\begin{aligned} & r = \frac{r'}{a}, z = \frac{z'}{a}, \theta = \frac{T' - T_\infty}{T_w - T_\infty}, \mathrm{u} = \frac{u'a}{v}, t = \frac{t'v}{a^2}, \rho = \frac{\rho' a^2}{\mu v}, Pr = \frac{\mu C_\rho}{k} = 1, Gr = \frac{g\beta a^3}{v^2} T_\infty, \\ & \lambda^2 = \underline{\lambda}^2\, \mathrm{a}^2, M = \frac{\sigma B^2 a^2}{\mu} \end{aligned}\right\} \tag{6}$$

where, Gr is the Grashof number .Substituting (6) in (2)-(5) gives

$$-\frac{\partial p}{\partial z} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - M^2 u + Gr\theta = 0 \tag{7}$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} - \lambda^2 \theta = 0 \tag{8}$$

Subject to

$$u = 0, \theta = \theta_a,\ 0n\ r = 1 \tag{9}$$

$$u, \theta < \infty \tag{10}$$

Equation(1) implies that $u \neq u(z)$,hence u is a function of r alone and we arrived at the following system of ordinary differential equations:

$$\frac{d^2 u}{dr^2} + \frac{1}{r}\frac{du}{dr} - M^2 u = -p - Gr\theta \tag{11}$$

$$\frac{d^2 \theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - \lambda^2 \theta = 0 \tag{12}$$

Subject to the following boundary conditions

$$u = 0, \quad \theta = \theta_a, \quad on \quad r = 1 \tag{13}$$

$$u, \theta < \infty \ \text{ for } 0 \le r \le 1 \tag{14}$$

where p is constant term M represents magnetic field parameter, α represents injection term and λ represents cooling term.

3. Method of Solution: Equations (11) and (12) are coupled non-linear differential equations .We implements the method of Ferobenius in solving for $u(r,t)$ and $\theta(r,t)$ Firstly, we solve equation (12) and substituting the result in equation(11) then solving the resulting systems individually to obtain the following flow variables:

3.1 Solution of the Temperature Model: We assume $\theta(r) = \sum_{n=0}^{\infty} a_n r^{n+c}$, where $a_n, c \in \mathbb{R}$

$$\Rightarrow \theta = \sum_{n=0}^{\infty} a_n r^{n+c} = r^c \sum_{n=0}^{\infty} a_n r^n \tag{15}$$

$$\theta' = \sum_{n=0}^{\infty} a_n (n+c) r^{n+c-1} = r^{c-1} \sum_{n=0}^{\infty} a_n (n+c) r^n \tag{16}$$

$$\left. r\theta'' = r^{c-2} \sum_{n=0}^{\infty} a_n (n+c)(n+c-1) r^n = r^{c-1} \sum_{n=0}^{\infty} a_n (n+c)(n+c-1) r^n \right\} \tag{17}$$

Putting (15)-(17) into (12) and performing long algebraic expressions gives the solution below

$$\theta(r) = \frac{\theta_a}{y_1(\text{a})} \left\{ 1 + \frac{\lambda^2 r^2}{2^2} + \frac{\lambda^4 r^4}{2^2 \times 4^2} + \frac{\lambda^6 r^6}{2^2 \times 4^2 \times 6^2} + \frac{\lambda^8 r^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \cdots \right\} \tag{18}$$

3.2 Solution of Velocity Model: Also we assume a solution of the form:

$$u = \sum_{m=0}^{\infty} b_m r^{m+k} = r^k \sum_{m=0}^{\infty} b_m r^m \tag{19}$$

$$u' = \sum_{m=0}^{\infty} b_m (m+k) r^{m+k-1} = r^{k-1} \sum_{m=0}^{\infty} b_m (m+k) r^m \tag{20}$$

$$u'' = \sum_{n=0}^{\infty} am(m+k) r^{m+k-2} = r^{k-2} \sum_{n=0}^{\infty} am(m+k)(m+k-1) r^m$$

$$\Rightarrow ru'' = r^{k-1} \sum_{n=0}^{\infty} am(m+k)(n+k-1) r^m \tag{21}$$

Putting equations (19)-(21) into the left hand side of equation (11) gives

$$u = \frac{\theta_a}{y_1(1)} \left\{ 1 + \frac{M^2 r^2}{2^2} + \frac{M^4 r^4}{2^2 \times 4^2} + \frac{M^6 r^6}{2^2 \times 4^2 \times 6^2} + \frac{M^8 r^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \cdots \right\} \tag{22}$$

3.2.1 Non-homogenous part of Velocity Model: To get the particular solution of (11) which is the non-homogenous part of the problem, we solve using the method of undetermined coefficients.

$$y(r) = A \left\{ 1 + \frac{\lambda^2 r^2}{2^2} + \frac{\lambda^4 r^4}{2^2 \times 4^2} + \frac{\lambda^6 r^6}{2^2 \times 4^2 \times 6^2} + \frac{\lambda^8 r^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \cdots \right\}, A \in \mathbb{R} \tag{23}$$

The complementary solution for (11) is

$$u_c(r) = C_c\left[1 + \frac{(Mr)^2}{2^2} + \frac{(Mr)^4}{(2\times4)^2} + \frac{(Mr)^6}{(2\times4\times6)^2} + \frac{(Mr)^8}{(2\times4\times6\times8)^2} + \dots\right] \quad (24)$$

Consider the particular solution: $u_p(r) = A_0 + A_1r^2 + A_2r^4 + A_3r^6 + A_4r^8$ (25)

$$u_p'(r) = 2A_1r + 4A_2r^3 + 6A_3r^5 + 8A_4r^7 \quad (26)$$

$$u_p''(r) = 2A_1 + 12A_2r^2 + 30A_3r^4 + 56A_4r^6 \quad (27)$$

Putting (25)-(27) into (11) the RHS and performing some algebraic simplifications gives a solution of (11)

$$u(r) = c + A_0 + \left(\frac{cM^2}{4} + A_1\right)r^2 + \left(\frac{cM^4}{(2\times4)^2} + A_2\right)r^4 + \left(\frac{cM^6}{(2\times4\times6)^2} + A_3\right)r^6 + \left(\frac{cM^8}{(2\times4\times6\times8)^2} + A_4\right)r^8 \quad (28)$$

5 Results and Discussion:

However, the following parameter values were clearly used in the simulation study: Vary velocity with magnetic field $\Pr = 4,\ Gr = 1.0,\ \lambda = 180.0$,Vary velocity with Gr : $M = 1.5,\ p = 1,\ \lambda = 1000$,Vary velocity with Pr : $M = 2.5,\ Gr = 0.05,\ \lambda = 188$,Vary velocity with λ : $M = 2.5,\ \Pr = 6,\ Gr = 0.05$

5.1 Results:

Consequently we present our graphical results and discussions:

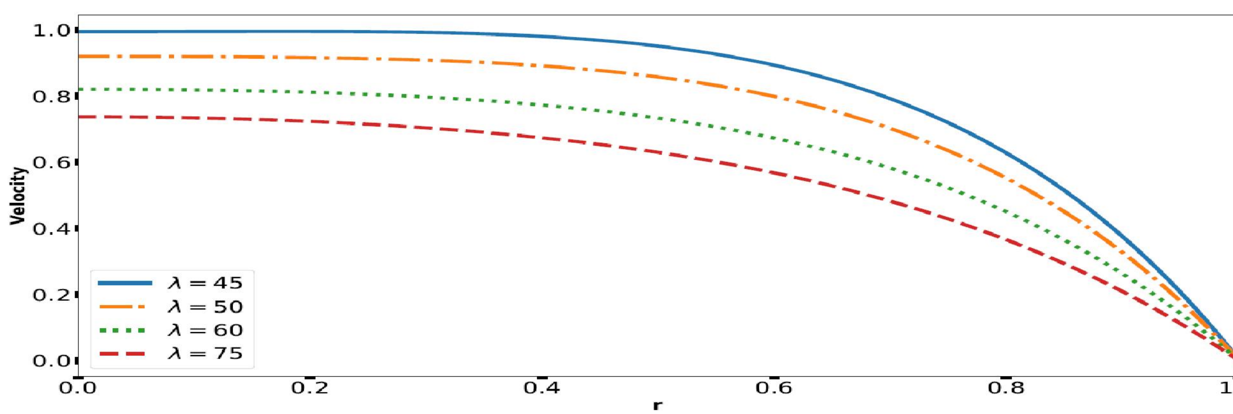

Figure .2: Velocity profiles for $\Pr = 4,\ Gr = 1.0,\ \lambda = 180.0$ and different values of velocity cooling parameter, λ

Figure 2 shows the various fluid velocity with the cooling parameter. It can be observed that the velocity diminishes with increasing cooling parameter. This is quite obvious since a rise in cooling parameter λ diminishes fluid temperature which of course rises the fluid viscosity, therefore diminishes the fluid velocity. This result is in consonance with the results of [3].

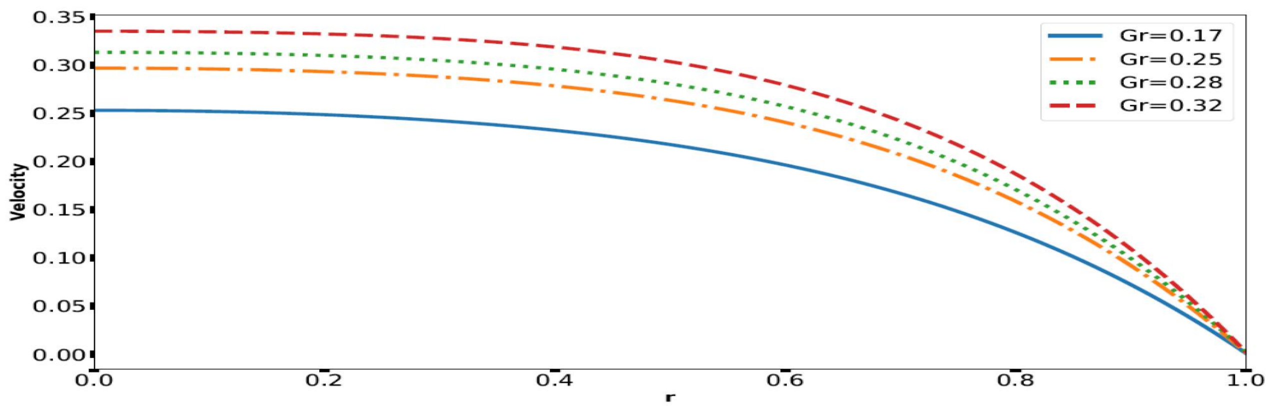

Figure 3: Velocity profiles for $\mathrm{M} = 1.5,\ \mathrm{p} = 1,\ \lambda = 1000$ and different values of Grashof, number Gr

In Figure 3 ,It is clear that an increase in Grashof number increases fluid velocity. This agrees with the results of [11] and [13].

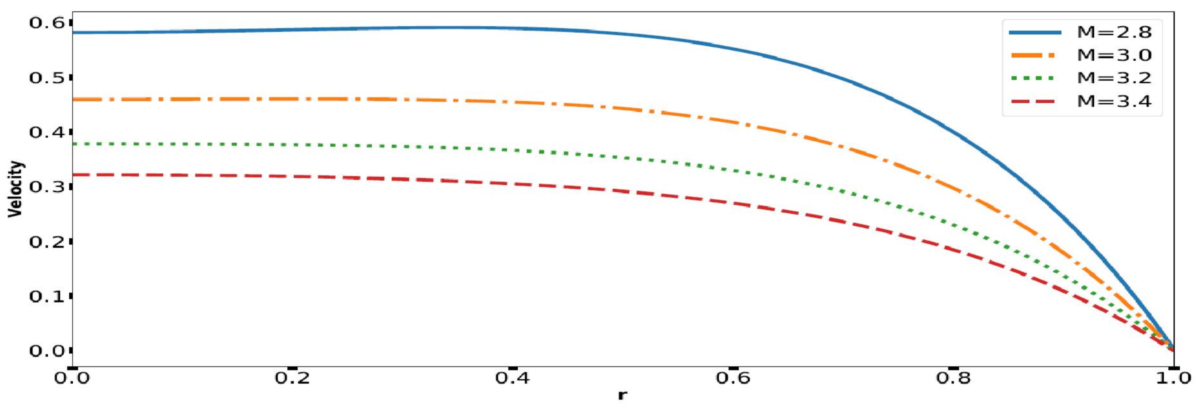

Figure 4: Velocity profiles for $\mathrm{M} = 2.5,\ Gr = 0.05,\ \lambda = 188$ and different values of Magnetic field parameter, M

In Figure 4, It can be seen that increase in the magnetic field parameter decreases the fluid velocity. This is in line with the results of [3] , [4],[9],[10] ,[12] and [14].

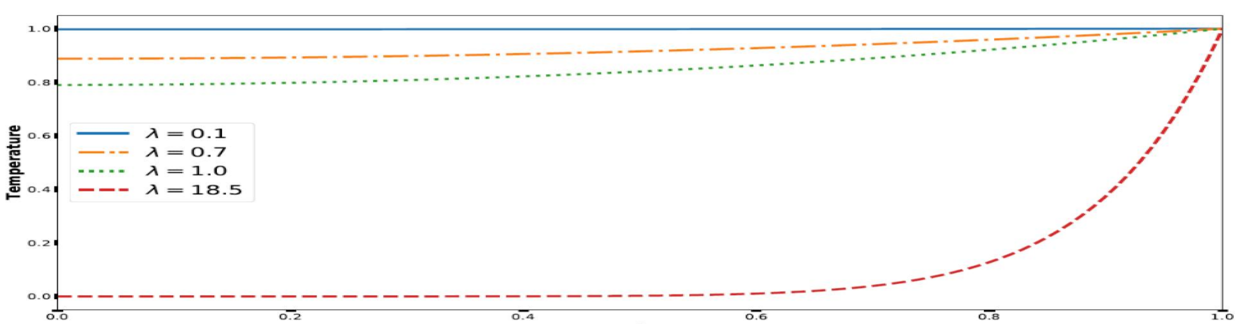

Figure 5: Temperature profiles for with $\theta_a = 1.0$ and different values of temperature cooling parameter, λ

Figure 5, It is observed that the temperature decreases as the cooling parameter increases. These agree with the result of [10].

6 Conclusions:

This paper considered the problem of heat transfer for simulation of non-constant flow in a cylindrical channel. A coupled system of two differential equations was judiciously framed. Complete analytical solutions were obtained; the graphical solutions displayed (i) a rise in cooling parameter, λ drops the fluid velocity (ii) temperature drops as the cooling parameter rises λ (iii) a rise in Grashof Gr number also leads to a rise in fluid velocity.

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Appendix: $A_4 = -\frac{\lambda^8}{M^2}\frac{Gr\theta}{(2\times4\times6\times8)^2}, \Rightarrow A_3 = -\frac{1}{M^2}\left[64A_4 - \frac{\lambda^6 Gr\theta}{(2\times4\times6)^2}\right], \Rightarrow A_2 = -\frac{1}{M^2}\left[36A_3 - \frac{\lambda^4 Gr\theta}{(2\times4)^2}\right],$

$\Rightarrow A_1 = -\frac{1}{M^2}\left[16A_2 - \frac{\lambda^2 Gr\theta}{4}\right], \Rightarrow A_0 = -\frac{1}{M^2}\left[4A_1 + p - Gr\theta\right]$

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