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Univariate Coefficient of Variation Control Charts Based On Ranked Set Sampling Schemes in Phase I

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Abstract: Control chart is a very important tool used in Statistical Process Control. Monitoring variability is a vital part of modern statistical process control. In a situation where the in-control process has a constant mean and variance, the conventional Shewhart R and S charts are usually used to monitor the variation of the process. In cases where the mean and standard deviation are not constant, the coefficient of variation (CV) is often constant and is used to monitor variability. Improvement on the efficiency of these charts is often desirable especially with relatively small sample sizes. Moreover, the need of an efficient sampling scheme becomes more pronounced when the exact measurement of unit is difficult and expensive, but the visual ordering of units is possible and realizable. Consequently, in this paper, new CV charts based on ranked set sampling schemes are proposed to enhance the monitoring power of the classical CV chart. The charts are established based on ranked set sampling (RSS), median RSS (MRSS), extreme RSS (ERSS), systematic RSS (SRSS) and neoteric RSS (NRSS), and are evaluated in terms of their Probability to Signal. The efficiency of the proposed charts is compared with the existing classical CV chart under simple random sampling (SRS) scheme. The results, based on a simulation study, indicate that the newly developed rank-based CV charts show better detection of monitoring signals in process CV than the classical CV chart. In particular, the CV chart based on the NRSS and SRSS technique performs notably better. A real-life application, concerning the non-isothermal Continuous Stirred Tank Chemical Reactor (CSTR), is also provided to show the implementation of the proposed charts in phase I.
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I. INTRODUCTION

Control chart is one of the statistical process control (SPC) problem-solving tools, used to ensure process uniformity (Montgomery, 2009). The basic purpose of the implementation of control chart procedures is to detect abnormal/un-natural variations in process parameters. Control charts are the most sophisticated and commonly adopted SPC tools in the manufacturing and processing industries. One of the fundamental principles of Statistical Process Control (SPC) is that a normally distributed process cannot be claimed to be in-control until it has a constant mean and variance. This implies that a shift in the mean and / or the standard-deviation makes the process out-of-control.

However, there are many in-control processes where the mean μ is expected to fluctuate time to time and the standard deviation σ changes with the mean. In these cases, it is not possible to use $\bar{X}$ and R or S control charts to perform online monitoring. When the process standard deviation is a linear function of the process mean and the mean itself is expected to fluctuate time to time, control charts monitoring the sample coefficient of variation (CV) can be efficiently used to keep track of the process variability and to detect shifts in the process mean or standard deviation due to assignable causes. Control charts are mostly employed in a two phase procedure. In phase I (retrospective phase), they are used to study a historical reference sample, which involves establishing the in-control state and evaluating the process stability to ensure that the reference sample is representative of the process (Zhou *et al.,* 2007). Once the in-control reference sample is determined, the process parameters, if unknown, are estimated from phase I, and control chart limits are obtained for used in phase II. The phase II aspect involves online monitoring of the process. If there occur any shift in process parameters, it needs to be detected quickly so that corrective actions can be taken at an early stage.

Most of the works on control charts focus on monitoring either the process mean or the process variability and are based on the requirements that the process mean is stable and independent of the process standard deviation (Yeong *et al.*, 2016). However, in many real-life processes, the process standard deviation σ is dependent of the mean, and the mean is not constant. In such operations, monitoring the mean or the variability using X chart or S(R) chart, seem to be hindered. In such cases, it is more appropriate to monitor the coefficient of variation (CV).

Several univariate CV control charts have been adopted in many practical applications. A primary criterion for the usage of the univariate CV chart is that the standard deviation needs to be proportional to the mean so that the CV is constant. This is usually checked by plotting the rational group CV against the mean (Kang *et al.*, 2007). In most cases, the plot is supplemented with a regression line and is also followed by a formal test of the regression slope. If the CV is constant, the CV is independent of the mean. The univariate CV is the ratio of the standard deviation to the mean of a random variable. As a relative dispersion measure to the mean, it is useful for comparing the variability of populations with really diverse means, and considerably straightforward to interpret in practice (Aerts *et al.*, 2015). The use of the CV is prevalent in many applications in science, medicine, engineering, economics, etc. For example, it can be used to measure the reliability of an assay in medicine and chemistry (Reed *et al.,* 2002). Also, it can be used in clinical trials, to account for baseline variability of measurements (Pereira *et al.*, 2002), and in quality control, to seek production processes with minimal dispersion.

Monitoring of CV has found in literature, either classified as memory less-type, which is type using only information on current sample e.g. Shewhart chart or memory-type, which is type using information on both current and previous samples e.g. EWMA and CUSUM charts. Shewhart control charts are widely used statistical process control (SPC) tools for detecting changes in a quality characteristic of a process and triggering the search for assignable causes of variation. Charting has been used in manufacturing settings, but in recent times it has been extended to education, health care, and a variety of societal applications. The purpose of implementing control charts is to differentiate between random and special cause variations. A process working under random causes is considered as in control while if assignable causes are at work, the process is declared to be out of control. It is always desirable to detect the assignable cause variation at an early stage for quick implementation of corrective actions. Control charts act as a familiar tool for this purpose.

All the existing CV charts are based on the simple random sampling (SRS) scheme and are applicable in situations where the exact measurements of the process variable are available. Control charts, based on ranked set sampling schemes, and has been proposed recently for efficient monitoring of process location. All the proposals in the literature are based on the ideal assumption of normally distributed quality characteristics. The performance of CV charts under ranked-based sampling schemes in phase I has not been investigated in SPC literature. Hence, in the present study, we employed the CV charts under different ranked set sampling (RSS) plans to improve the detection power of the existing CV chart (based on SRS). The charts, named CV[R] charts, are proposed under various ranked sampling schemes, such as the RSS, MRSS, ERSS, SRSS and NRSS. The proposed enhancements in the deployment of the customary CV control chart are suitable in those real-life situations where the perfect measurement of the quality characteristic of concern is costly to obtain, but the rank ordering of its elements can be obtained at a negligible cost.

In section 2, a brief review of the mean estimator under RSS, MRSS and ERSS, SRSS and NRSS schemes is given. The description of the design structure of the CV charts is presented in Section 3, and the control structure of the proposed CV charts in Section 4. The performance evaluation of the proposed charts is presented in Section 5. The simulation study and results' discussion is presented in Section 6, A real data illustrative example is provided in Section 7, and conclusions and recommendations is given in Section 8.

II. RANK SET SAMPLING SCHEMES

In this section, we review the framework of the rank set sampling schemes under perfect ranking. The following subsections describe the different perfect ranked sampling schemes investigated in this study.

Ranked Set Sampling (RSS)

Ranked set sampling (RSS) was first suggested by McIntyre (1950) to estimate the mean pasture and forage yields. It is a cost-efficient sampling procedure alternative to the simple random sampling (SRS) for the population mean estimation in situations where visual ordering of a set units can be done easily, but the exact measurement of the units is difficult and expensive. The variance of RSS mean estimator is less than the variance of SRS mean estimator regardless of errors in ranking or the parent distribution. The ranked set sampling (RSS) method can be summarized as follows: Select n random samples of size n units and rank the units within each sample with respect to a variable of interest by a visual inspection. Then select for actual measurement the smallest unit from the first sample. From the second sample, select for actual measurement the second smallest unit. The procedure is continued until the largest from the nth sample is selected for measurement. In this way, we obtain a total of n measured units, one from each sample. The cycle may be repeated r times until nr units have been measured. These nr units form the RSS data.

The mean and the variance of the RSS are given by:

$$\mu_{\bar{Z}_{rssk}} = \mu = \frac{1}{n}\sum_{i=1}^{n} Z_{(i:n)k}, k = 1, \dots, r$$

$$\sigma^2_{\bar{Z}_{rssk}} = \frac{\sigma^2}{n} - \frac{1}{n^2}\sum_{i=1}^{n}\left(\mu_{(i:n)} - \mu_Z\right)^2$$

Median Ranked Set Sampling (MRSS)

The method of median ranked set sampling (MRSS) proposed by Muttlak (1997) can be summarized as follows: Randomly select a sample of size n^2 units from target population and partition the sample into n sets each of size n and rank the units of each set with respect to a variable of interest. The n measurements are then obtained depending on whether the set size is even or odd. For odd set sizes, select the median value for measurement from each ranked set (i.e. the $((n+1)/2)th$ smallest rank). And for the even set sizes, select the $(\frac{n}{2})^{th}$ smallest element from the first n/2 sets and select $((n+2)/2)^{th}$ smallest element from the remaining n/2 sets. The cycle may be repeated m times until nm units have been measured. Thus, the nm units form the MRSS sample data. The mean and variance of the MRSS are given as:

$$\mu_{\bar{Z}^{even}_{mrssk}} = \frac{1}{k}\sum_{j=1}^{m}\left(\sum_{i=1}^{n/2} Z_{i((n/2):n)j} + Z_{i((n/2)+1:n)j}\right)$$

$$+\sigma^{2\,even}_{\bar{Z}_{rssk}} = E(\bar{Z}^{even}_{mrssk} - \mu_Z)^2 - \frac{1}{kn}\sum_{i=1}^{n/2}\sigma^2{}_{Z((n/2):n)} + \sigma^2{}_{Z((n/2)+1:n)}$$

Extreme Ranked Set Sampling (ERSS)

Extreme Ranked Set Sampling was proposed by Samawi *et al.* (1996). The extreme ranked set sampling (ERSS) procedure, select n random samples of size n units from the population and rank the units within each sample with respect to a variable of interest by visual inspection. If the sample size n is even, select from n/2 samples the smallest unit and from the other n/2 samples the largest unit for actual measurement. If the sample size is odd, select from (n-1)/2 samples the smallest unit, from the other (n-1)/2 the largest unit and from one sample the median of the sample for actual measurement. The cycle may be repeated r times to get nr units. These nr units form the ERSS data. We can see that the ERSS in practical applications can be performed with fewer errors in ranking the units since all we have to do is find the largest or the smallest of the sample and measure it. The ERSS method is very easy to apply in the field and will save time in performing the ranking of the units with respect to the variable of interest. In addition, this method will reduce the errors in ranking and hence increase the efficiency of the ERSS when compared to RSS.

The mean and variance are given as:

$$\mu_{\bar{Z}_{erssk}} = \mu = \frac{1}{n}\sum_{i=1}^{n} Z_{(i:e)k}, k = 1, \dots, r$$

$$\sigma^2_{\bar{Z}_{erssk}} = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2_{(i:e)}$$

Systematic Ranked Set Sampling (SRSS)

Systematic ranked set sampling was developed by Khan *et al.* (2019). Systematic ranked set sampling procedure (SRSS) was developed for application in situations where ranking of the sample observations is much easier than obtaining their actual magnitudes. Let Y_{ij}, $Y_{2j,}$ …, Ym2j be a simple random sample of size m^2 units selected from target population. Also, let $Y_{(1)j}$, $Y_{(2)j}$, …, $Y_{(m2)j}$ be the order statistics of Y_{ij}, Y_{2j}, …, Y_{m^2j}, (j = 1, 2, …, r). The SRSS procedure can be summarized as follows:

Step 1: Select m^2 sample units randomly from the population.

Step 2: Rank the m^2 selected units in an increasing order of magnitude based on a concomitant variable.

Step 3: Select the *((m + 1)i + 1)th* ranked units for (*i = 0, 1, 2, …, m − 1*).

Step 4: Repeat steps 1 through 3 for *r* cycles to obtain an SRSS of size *n = mr*.

We select m^2 units in both SRSS and RSS methods, but we only measure *m* units. In addition, in SRSS, we rank all the m^2 selected units at the same time, while in RSS, we rank m units in each of the *m* sets. Unlike RSS, SRSS measured units $Y_{((m+1)\,i+1)\,j}$, *i = 0, 1, 2, …, m − 1; j = 1, 2, …, r* are dependent, and they follow the distribution of the *((m + 1)i + 1)th*-order statistics of a sample of size m^2. The mean and variance for SRSS are given below:

$$\bar{Y} = \frac{1}{mr}\sum_{j=1}^{r}\sum_{i=0}^{m-1} Y_{((m+1)i+1)j}$$

$$var(\bar{Y}_{SRSS}) = \frac{1}{m^2}\sum_{i=0}^{m-1} var(Y_{(m+1)i+1}) + \frac{2}{m^2}\sum_{i<j}^{m-1} cov(Y_{(m+1)i+1,} Y_{(m+1)j+1})$$

Neoteric Ranked Set Sampling (NRSS)

Neoteric ranked set sampling was proposed by Zamanzade and Al-Omari (2016). It is a recently developed sampling plan, derived from the well-known ranked set sampling (RSS) scheme. Neoteric ranked set sampling (NRSS) was suggested to be applied in situations where the ranking of the sample observations is much easier than obtaining their precise values. The NRSS scheme can be described as follows:

Step 1: Select a simple random sample of size k^2 units from the target population.

Step 2: Rank the k^2 selected units in an increasing magnitude based on a concomitant variable, personal judgment or any inexpensive method.

Step 3: If k is an odd, then select the $\{(k+1)/2 + (i-1)\ k\}th$ ranked unit for $i = 1, \ldots, k$. But if k is an even, then select the $\{l + (i-1)\}th$ ranked unit, where $l = k/2$ if i is an even and $l = (k+2)/2$ if i is an odd for $i = 1, \ldots, k$.

Step 4: Repeat steps 1 through 3 n times (cycles) if needed to obtain a neoteric ranked set sample of size $N = nk$.

The mean and variance are given as:

$$\bar{Y}_{NRSS} = \frac{1}{nk}\sum_{j=1}^{n}\sum_{i=1}^{k} Y_{[(i-1)k+l\}j}$$

$$Var\ (\bar{Y}_{NRSS}) = \frac{1}{k^2}\sum_{i=1}^{k} var\ (Y_{[(i-1)k+l]j}) + \frac{2}{k^2}\sum_{i<j}^{k} Cov\ (Y_{[(i-1)k+l],}\ Y_{[(j-1)k+l]})$$

III. RANKED SET BASED COEFFICIENT OF VARIATION (CV) CHARTS

Here, we give the general form of the CV chart that can be used with any of the sampling schemes considered in this study. Assuming that X has a mean (μ) and standard deviation (σ), by definition, the CV, γ, for X is defined as:

$$\gamma = \frac{\sigma}{\mu} \quad (1)$$

The CV given above is a standardized measure of the dispersion of X and has several benefits over the other measures of the dispersion, such as σ, which is always understood in the context of μ. However, since the coefficient of variation depends on the process mean and the process standard deviation which are usually unknown; it is inapplicable from the practical perspective. Consequently, it is more appropriate to estimate γ from the preliminary reference sample of the process. Now, consider a historical in-control reference sample observations of size n (X_1, X_2, ..., Xn) from the process with underlying parameters: σ and μ. We assume that the underlying distribution of X_j (j = 1, 2...n) is a standard normal distribution with μ and σ. Let $\bar{X}$ and s be the sample statistics representing mean and standard deviation, respectively, which is used for the estimation of the parameters μ and σ, respectively. The estimated γ of X denoted by Z, is defined as:

$$Z = \frac{s}{\bar{X}} \quad (2)$$

Let $\bar{X}_{[R](n)}$ and $s_{[R](n)}$ symbolizing the sample mean and sample standard deviation attained from the nth sample, taking into account sampling scheme (R), where R represents any sampling schemes. We define $CV_{[R],}$ given in equation 3.13 are the sample coefficient of variation from the different sampling schemes, considered in this study. Here, $Z_{[R](n)}$ is the statistic estimated for CV that are computed using the subgroup of size n taken from the underlying process that has been scaled to estimate coefficient of variation γ, based on the different sampling schemes considered.

$$Z_{[R](n)} = {}^{S_{[R]}}/_{\bar{x}_{[R]}}$$

$$\forall R = \text{RSS, MRSS, ERSS, SRSS, NRSS} \quad (3)$$

IV. THE PROPOSED CONTROL CHART STRUCTURE

Here, we present the CV chart structure that can be used with any of the Ranked set sampling schemes.

Let $A_{[R]}(n)$ be a pivotal parameter that describes a link between the estimated CV statistics $Z_{[R]}(n)$ and the process parameter value γ, given as:

$$A_{[R](n)} = (Z_{[R](n)})/\gamma$$

The expected value of $A_{[R]}$ (n) is given as:

$$E(A_{[R](n)}) = E(Z_{[R](n)}/\gamma) = E\ (Z_{[R](n)})/\gamma$$

Where R is the sampling schemes from RSS, ERSS, MRSS, SRSS, NRSS

Let E [$A_{[R]}$(n)] = $d_{2,n,m}$ where for a specific sampling scheme, d_2 depends on n. E ($Z_{[R]}$(n) can be estimated as the average of [$Z_{[R]}$(n)] i.e $\bar{\boldsymbol{Z}}_{R(n)} = E\left(Z_{R(n)}\right)$, computed from an appropriate size n obtained from a normal operating process conditions. Hence, we can define the unbiased estimator of γ for the n[th] samples as:

$$\hat{\gamma} = \frac{\bar{Z}_{[R](n)}}{d_{2,n,m}} \quad (4)$$

For all the sampling schemes R, the probability limit is designed for the $CV_{[R]}$ chart with respect to the sampling scheme under consideration by using the quantile points of distribution of $A_{[R]j}$ (n). Let us define α as the false alarm probability (α), and $A_{[R]}$ (n) to be the α-quantile point of the underlying distribution of $A_{[R]}$ (n). Then the probability limit for the $CV_{[R]}$ chart established on $Z_{[R](n)}$ are given as

$$UCL_{[R](n)} = A_{[R](n)(1-\frac{\alpha}{2})} \frac{Z_{[R]j(n)}}{d_{2,n,m}},$$

With

$$p(A_{[R](n)} \geq A_{[R]}(n)_{(1-\frac{\alpha}{2})}) = 1-\frac{\alpha}{2} \quad (5)$$

Where $UCL_{[R](n)}$ is the upper control limit for any of the sampling scheme

V. PERFORMANCE EVALUATION

This chapter provides a comprehensive study of the phase I analysis of the univariate CV control charts considered in this study. For a fixed False Alarm Probability (FAP) α =0.01, the chart that yields the highest probability of signaling is considered better than the other control charts. We aim to recommend the choice of the univariate CV control chart that gives the best phase I performance. The effects of the shift size (δ), the sample size (n), and m_l on the proposed charts are also studied. The PTS values are presented in tables 4.1A-4.1C. The significant findings of the phase I analysis are summarized as follows.

1. Only the control chart based on MRSS is performing poorly than the CV chart based on SRS, while all the other charts lead to enhanced detection power in comparison with the classical CV chart when n is 5. For example, when n = 5, shift = 2, m_l =3, the PTS values are 0.3964 and 0.4076 for MRSS and SRS respectively. However, when n is 7 and 10, all the other charts lead to enhanced detection power in comparison with the classical CV chart. For example, when n = 7, shift = 2.25, m_l =3, the PTS values are 0.9576, 0.9570, 0.9446, 0.8979, 0.7938,0.7868 for NRSS, SRSS, ERSS, RSS, MRSS and SRS respectively.
2. The UCV_{NRSS} chart performed better than all the other charts. The UCV_{SRSS} chart performed better than the UCV_{ERSS} chart. The UCV_{ERSS} chart performed better than the UCV_{RSS} chart. The UCV_{RSS} chart dominantly performed better than the UCV_{SRS} and UCV_{MRSS} charts especially for small to moderate shift in the in-control CV. For example, when n = 5, shift = 2.25, m_l = 9, the PTS values are 0.9923, 0.9736, 0.8364, 0.7247, 0.5815, and 0.5712, for the UCV_{NRSS}, UCV_{SRSS}, UCV_{ERSS}, UCV_{RSS}, UCV_{SRS} and UCV_{MRSS} respectively.
3. It is observed that the PTS values approaches 1, as the size of the shift increases and m_l changes for all values of n.
4. Greater PTS values are obtained when n is large for any value of m_l at moderate or high shift level

The probability value for NRSS and SRSS equals 1 when m_l = 9 and 12 at a moderate or high shift level for all values of n. d The probability value for NRSS, SRS and ERSS equals 1 when m_l = 9 and 12 at a moderate or high shift level when n = 7. When n = 10, the probability value for NRSS, SRSS, ERSS and RSS equals 1 at a moderate or high level of shifts when m_l = 9 and 12

Table 4.1A: The PTS values of CV model when n = 5, m = 30, α = 0.01 at different levels of m_l

				n = 5, m = 30			
m_l		SRS	RSS	MRSS	ERSS	SRSS	NRSS
	d2	0.942578	1.033578	0.503314	1.289163	1.480282	0.978386
	d3	0.344942	0.309002	0.184346	0.332126	0.254883	0.167241
	L	3.872	3.838	3.902	3.798	4.081	3.669
	δ						
0	1	0.0101	0.0101	0.0101	0.0103	0.0102	0.0102
3	1.25	0.0288	0.0300	0.0266	0.0369	0.0531	0.0877
	1.5	0.1050	0.1378	0.1003	0.1816	0.3234	0.4546
	1.75	0.2392	0.3318	0.2357	0.4322	0.6740	0.7775
	2	0.4076	0.5357	0.3964	0.6530	0.8593	0.8975
	2.25	0.5514	0.6879	0.5423	0.7864	0.9230	0.9355

	2.5	0.6573	0.7844	0.6500	0.8570	0.9448	0.9490
	2.75	0.7372	0.8428	0.7304	0.8979	0.9526	0.9542
	3	0.7908	0.8782	0.7881	0.9185	0.9555	0.9562
	3.5	0.8566	0.9156	0.8544	0.9390	0.9572	0.9573
	4	0.8923	0.9325	0.8902	0.9478	0.9575	0.9575
	4.5	0.9124	0.9416	0.9104	0.9519	0.9576	0.9576
	5	0.9241	0.9470	0.9233	0.9539	0.9576	0.9576
	6	0.9375	0.9519	0.9371	0.9559	0.9576	0.9576
	7	0.9444	0.9540	0.9437	0.9567	0.9576	0.9576
6	1.25	0.0357	0.0399	0.0350	0.0453	0.0668	0.1081
	1.5	0.1360	0.1661	0.1227	0.2250	0.3741	0.5351
	1.75	0.2986	0.3908	0.2847	0.5066	0.7534	0.8686
	2	0.4770	0.6133	0.4648	0.7389	0.9292	0.9681
	2.25	0.6329	0.7721	0.6245	0.8720	0.9796	0.9901
	2.5	0.7476	0.8688	0.7382	0.9359	0.9923	0.9955
	2.75	0.8241	0.9220	0.8175	0.9652	0.9961	0.9973
	3	0.8748	0.9511	0.8716	0.9797	0.9975	0.9980
	3.5	0.9332	0.9774	0.9304	0.9909	0.9984	0.9985
	4	0.9599	0.9875	0.9593	0.9947	0.9986	0.9987
	4.5	0.9736	0.9918	0.9732	0.9963	0.9987	0.9987
	5	0.9810	0.9940	0.9809	0.9971	0.9987	0.9987
	6	0.9886	0.9961	0.9884	0.9979	0.9988	0.9988
	7	0.9919	0.9970	0.9918	0.9982	0.9988	0.9988
9	1.25	0.0378	0.0386	0.0349	0.0456	0.0640	0.1049
	1.5	0.1255	0.1563	0.1180	0.2067	0.3261	0.4921
	1.75	0.2727	0.3562	0.2596	0.4589	0.6919	0.8436
	2	0.4360	0.5594	0.4212	0.6853	0.9025	0.9654
	2.25	0.5815	0.7247	0.5712	0.8364	0.9736	0.9923
	2.5	0.6981	0.8342	0.6912	0.9179	0.9929	0.9979
	2.75	0.7846	0.8989	0.7769	0.9577	0.9977	0.9993
	3	0.8457	0.9383	0.8399	0.9776	0.9991	0.9997
	3.5	0.9160	0.9743	0.9116	0.9925	0.9998	0.9999
	4	0.9508	0.9877	0.9478	0.9969	0.9999	0.9999
	4.5	0.9688	0.9934	0.9671	0.9984	0.9999	1.0000
	5	0.9791	0.9959	0.9778	0.9991	1.0000	1.0000
	6	0.9890	0.9981	0.9884	0.9996	1.0000	1.0000
	7	0.9932	0.9989	0.9929	0.9997	1.0000	1.0000
12	1.25	0.0348	0.0362	0.0339	0.0403	0.0532	0.0863
	1.5	0.1061	0.1308	0.1013	0.1621	0.2503	0.3940
	1.75	0.2178	0.2771	0.2078	0.3556	0.5468	0.7355
	2	0.3493	0.4470	0.3296	0.5610	0.7921	0.9179
	2.25	0.4723	0.5916	0.4533	0.7218	0.9193	0.9775
	2.5	0.5783	0.7106	0.5610	0.8271	0.9714	0.9938
	2.75	0.6656	0.7996	0.6508	0.8969	0.9897	0.9981
	3	0.7354	0.8594	0.7233	0.9366	0.9961	0.9994
	3.5	0.8288	0.9280	0.8229	0.9752	0.9993	0.9999
	4	0.8860	0.9603	0.8808	0.9888	0.9998	1.0000
	4.5	0.9200	0.9765	0.9164	0.9944	0.9999	1.0000
	5	0.9416	0.9853	0.9391	0.9970	1.0000	1.0000
	6	0.9653	0.9931	0.9629	0.9988	1.0000	1.0000
	7	0.9771	0.9962	0.9752	0.9994	1.0000	1.0000

Table 4.1B: The PTS values of the CV model when n = 7, m = 30, α = 0.01 at different levels of m_1

			n = 7, m = 30				
m_1		SRS	RSS	MRSS	ERSS	SRSS	NRSS
	d2	0.960999	1.033145	0.439712	1.459326	1.43675	0.98498
	d3	0.284996	0.234545	0.130347	0.260965	0.187558	0.114736
	L	3.846	3.838	3.787	3.716	4.106	3.502
	δ						
0	1	0.0101	0.0102	0.0102	0.0103	0.0102	0.0104
3	1.25	0.0304	0.0441	0.0313	0.0694	0.0887	0.2421
	1.5	0.1399	0.2228	0.1481	0.4029	0.5478	0.8150
	1.75	0.3426	0.5163	0.3551	0.7360	0.8691	0.9384
	2	0.5414	0.7332	0.5558	0.8777	0.9413	0.9550
	2.25	0.6915	0.8459	0.7030	0.9272	0.9547	0.9573
	2.5	0.7868	0.8979	0.7938	0.9446	0.9570	0.9576
	2.75	0.8439	0.9239	0.8489	0.9518	0.9575	0.9576
	3	0.8800	0.9379	0.8840	0.9548	0.9576	0.9576
	3.5	0.9167	0.9496	0.9191	0.9569	0.9576	0.9576
	4	0.9336	0.9540	0.9353	0.9574	0.9576	0.9576
	4.5	0.9426	0.9557	0.9433	0.9575	0.9576	0.9576
	5	0.9475	0.9567	0.9476	0.9576	0.9576	0.9576
	6	0.9524	0.9572	0.9525	0.9576	0.9576	0.9576
	7	0.9545	0.9574	0.9546	0.9576	0.9576	0.9576
6	1.25	0.0401	0.0574	0.0434	0.0888	0.1095	0.2961
	1.5	0.1740	0.2705	0.1875	0.4699	0.6172	0.9074
	1.75	0.4054	0.5906	0.4219	0.8303	0.9397	0.9920
	2	0.6248	0.8181	0.6438	0.9545	0.9907	0.9977
	2.25	0.7784	0.9236	0.7938	0.9856	0.9971	0.9985
	2.5	0.8713	0.9663	0.8820	0.9938	0.9983	0.9987
	2.75	0.9235	0.9830	0.9293	0.9965	0.9986	0.9987
	3	0.9517	0.9900	0.9560	0.9976	0.9987	0.9988
	3.5	0.9776	0.9953	0.9795	0.9983	0.9988	0.9988
	4	0.9875	0.9971	0.9884	0.9986	0.9988	0.9988

	4.5	0.9918	0.9977	0.9924	0.9987	0.9988	0.9988
	5	0.9940	0.9981	0.9945	0.9987	0.9988	0.9988
	6	0.9961	0.9984	0.9964	0.9987	0.9988	0.9988
	7	0.9970	0.9986	0.9972	0.9988	0.9988	0.9988
9	1.25	0.0435	0.0594	0.0447	0.0883	0.1023	0.2796
	1.5	0.1636	0.2491	0.1721	0.4252	0.5485	0.8932
	1.75	0.3653	0.5341	0.3827	0.7931	0.9125	0.9951
	2	0.5727	0.7714	0.5915	0.9462	0.9906	0.9996
	2.25	0.7294	0.9015	0.7502	0.9867	0.9988	0.9999
	2.5	0.8377	0.9584	0.8513	0.9963	0.9997	1.0000
	2.75	0.9003	0.9816	0.9108	0.9987	0.9999	1.0000
	3	0.9389	0.9913	0.9458	0.9994	0.9999	1.0000
	3.5	0.9747	0.9975	0.9775	0.9998	1.0000	1.0000
	4	0.9878	0.9990	0.9893	0.9999	1.0000	1.0000
	4.5	0.9933	0.9995	0.9942	0.9999	1.0000	1.0000
	5	0.9959	0.9997	0.9965	1.0000	1.0000	1.0000
	6	0.9981	0.9998	0.9984	1.0000	1.0000	1.0000
	7	0.9989	0.9999	0.9990	1.0000	1.0000	1.0000
12	1.25	0.0373	0.0522	0.0400	0.0749	0.0858	0.2311
	1.5	0.1364	0.2013	0.1446	0.3357	0.4237	0.8111
	1.75	0.2869	0.4226	0.3000	0.6678	0.8079	0.9867
	2	0.4573	0.6414	0.4761	0.8794	0.9633	0.9993
	2.25	0.6069	0.7994	0.6283	0.9616	0.9947	0.9999
	2.5	0.7204	0.8933	0.7426	0.9879	0.9992	1.0000
	2.75	0.8048	0.9438	0.8229	0.9960	0.9999	1.0000
	3	0.8621	0.9706	0.8781	0.9986	1.0000	1.0000
	3.5	0.9306	0.9906	0.9391	0.9997	1.0000	1.0000
	4	0.9618	0.9965	0.9666	0.9999	1.0000	1.0000
	4.5	0.9776	0.9985	0.9808	1.0000	1.0000	1.0000
	5	0.9859	0.9992	0.9878	1.0000	1.0000	1.0000
	6	0.9933	0.9997	0.9944	1.0000	1.0000	1.0000
	7	0.9962	0.9999	0.9968	1.0000	1.0000	1.0000

Table 4.1C: The PTS values of the CV model when n = 10, m = 30, α = 0.01 at different levels of m_1

		n = 10, m = 30					
m_1		SRS	RSS	MRSS	ERSS	SRSS	NRSS
	d2	0.974315	1.029015	0.361855	1.714042	1.383358	0.976043
	d3	0.234549	0.175752	0.086924	0.204471	0.136237	0.076911
	L	3.736	3.882	3.745	3.652	4.137	3.499
	δ						
0	1	0.0101	0.0101	0.0101	0.0101	0.0104	0.0101
3	1.25	0.0404	0.0627	0.0415	0.1854	0.1774	0.5562
	1.5	0.2220	0.3778	0.2255	0.7646	0.8067	0.9451
	1.75	0.5013	0.7309	0.5132	0.9299	0.9476	0.9574
	2	0.7149	0.8815	0.7190	0.9536	0.9572	0.9576
	2.25	0.8283	0.9304	0.8292	0.9570	0.9576	0.9576
	2.5	0.8863	0.9467	0.8865	0.9575	0.9576	0.9576
	2.75	0.9157	0.9530	0.9157	0.9576	0.9576	0.9576
	3	0.9320	0.9555	0.9318	0.9576	0.9576	0.9576
	3.5	0.9464	0.9572	0.9465	0.9576	0.9576	0.9576
	4	0.9523	0.9575	0.9519	0.9576	0.9576	0.9576
	4.5	0.9547	0.9576	0.9544	0.9576	0.9576	0.9576
	5	0.9559	0.9576	0.9557	0.9576	0.9576	0.9576
	6	0.9570	0.9576	0.9569	0.9576	0.9576	0.9576
	7	0.9574	0.9576	0.9573	0.9576	0.9576	0.9576
6	1.25	0.0529	0.0771	0.0546	0.2318	0.2106	0.6535
	1.5	0.2712	0.4327	0.2645	0.8593	0.8828	0.9946
	1.75	0.5849	0.8175	0.5827	0.9876	0.9940	0.9986
	2	0.8047	0.9536	0.8071	0.9972	0.9984	0.9988
	2.25	0.9129	0.9863	0.9143	0.9984	0.9987	0.9988
	2.5	0.9583	0.9944	0.9595	0.9987	0.9988	0.9988
	2.75	0.9784	0.9969	0.9788	0.9987	0.9988	0.9988
	3	0.9874	0.9978	0.9875	0.9988	0.9988	0.9988
	3.5	0.9942	0.9984	0.9941	0.9988	0.9988	0.9988
	4	0.9964	0.9986	0.9964	0.9988	0.9988	0.9988
	4.5	0.9974	0.9987	0.9973	0.9988	0.9988	0.9988
	5	0.9978	0.9987	0.9978	0.9988	0.9988	0.9988

	6	0.9983	0.9988	0.9983	0.9988	0.9988	0.9988
	7	0.9985	0.9988	0.9984	0.9988	0.9988	0.9988
9	1.25	0.0557	0.0758	0.0556	0.2096	0.1891	0.6149
	1.5	0.2514	0.3950	0.2457	0.8317	0.8373	0.9976
	1.75	0.5357	0.7679	0.5300	0.9897	0.9948	0.9999
	2	0.7659	0.9403	0.7640	0.9993	0.9998	1.0000
	2.25	0.8925	0.9860	0.8925	0.9999	1.0000	1.0000
	2.5	0.9514	0.9963	0.9524	0.9999	1.0000	1.0000
	2.75	0.9773	0.9988	0.9776	1.0000	1.0000	1.0000
	3	0.9885	0.9995	0.9887	1.0000	1.0000	1.0000
	3.5	0.9963	0.9999	0.9964	1.0000	1.0000	1.0000
	4	0.9985	0.9999	0.9985	1.0000	1.0000	1.0000
	4.5	0.9992	1.0000	0.9992	1.0000	1.0000	1.0000
	5	0.9996	1.0000	0.9995	1.0000	1.0000	1.0000
	6	0.9998	1.0000	0.9998	1.0000	1.0000	1.0000
	7	0.9999	1.0000	0.9999	1.0000	1.0000	1.0000
12	1.25	0.0514	0.0699	0.0522	0.1679	0.1477	0.5103
	1.5	0.2018	0.3067	0.1956	0.7228	0.7028	0.9936
	1.75	0.4285	0.6309	0.4194	0.9706	0.9778	1.0000
	2	0.6451	0.8594	0.6353	0.9981	0.9993	1.0000
	2.25	0.7957	0.9549	0.7941	0.9998	1.0000	1.0000
	2.5	0.8881	0.9865	0.8872	1.0000	1.0000	1.0000
	2.75	0.9390	0.9957	0.9392	1.0000	1.0000	1.0000
	3	0.9663	0.9985	0.9662	1.0000	1.0000	1.0000
	3.5	0.9883	0.9998	0.9883	1.0000	1.0000	1.0000
	4	0.9953	0.9999	0.9953	1.0000	1.0000	1.0000
	4.5	0.9978	1.0000	0.9978	1.0000	1.0000	1.0000
	5	0.9988	1.0000	0.9988	1.0000	1.0000	1.0000
	6	0.9996	1.0000	0.9996	1.0000	1.0000	1.0000
	7	0.9998	1.0000	0.9998	1.0000	1.0000	1.0000

Comparison of the Proposed Charts

We present the comparisons of the proposed charts. A chart with a higher probability to signal (PTS) value is considered better than others. We compare the performance of the proposed charts based on the various ranked set sampling schemes (RSS, MRSS, ERSS, SRSS and NRSS) with the classical CV chart based on simple random sampling (SRS). The PTS of each univariate CV chart is plotted against shift δ. The PTS comparison of the charts is shown in figures 4.3A to 4.3C for the values of n (5, 7 and 10), m =30, and α=0.01 at different levels of $m_{1.}$ As clearly seen in the figures 4.3A - 4.3C, the proposed charts predominantly performed better than the classical CV chart. The PTS values of all the charts approaches 1, as the size of the shift increases (Figure 4.3A – 4.3C).

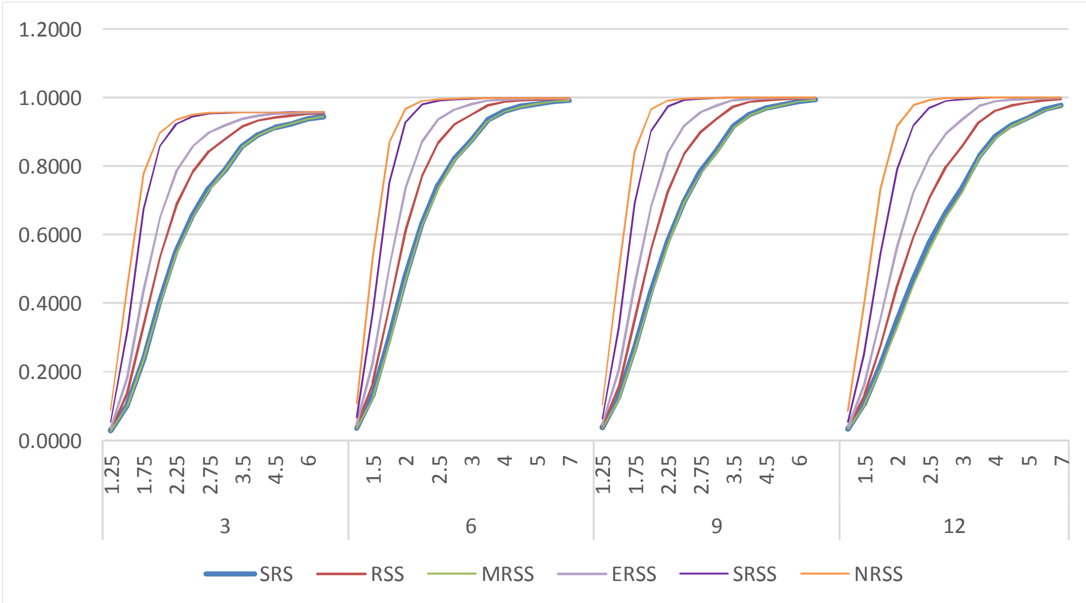

Figure 4.3A: PTS comparison of the univariate CV charts, when n=5, m =30, and α=0.01 at different levels of m_1

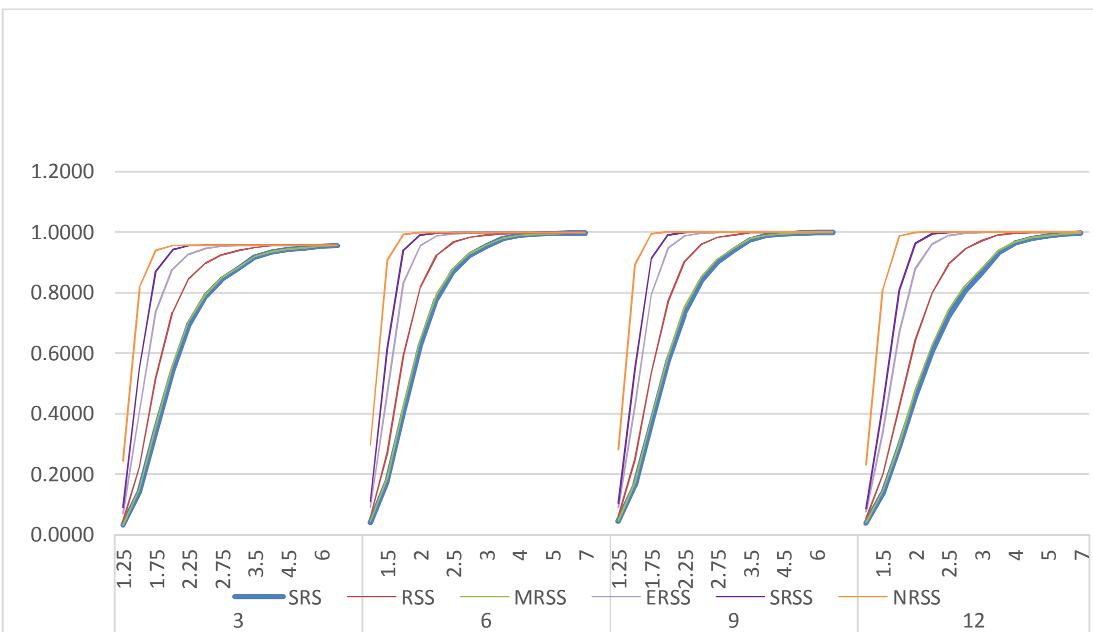

Figure 4.3B: PTS comparison of the univariate CV charts, when n=7, m =30, and α=0.01 at different levels of m_1

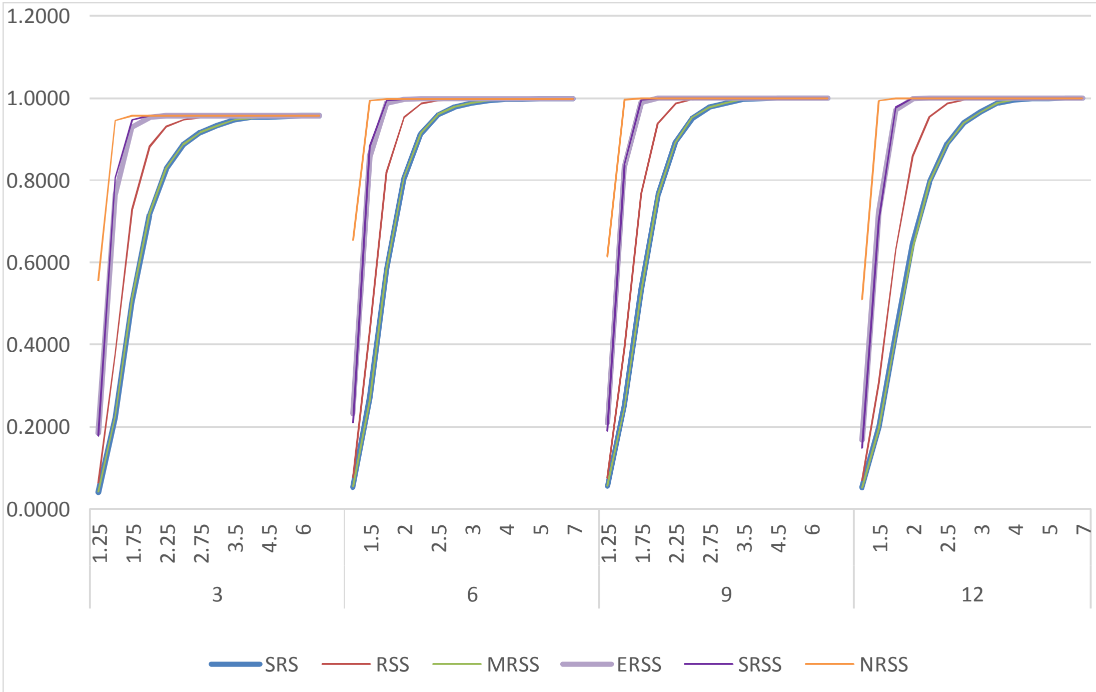

Figure 4.3C: PTS comparison of the univariate CV charts, when n=10, m =30, and α=0.01 at different levels of m_1

VII. INDUSTRIAL APPLICATION

This section provides an example based on real data to better illustrate the significance and application of proposed CV control charts designed under different sampling schemes (i.e., RSS, MRSS, ERSS, SRSS and NRSS). The data set considered here is the non-isothermal Continuous Stirred Tank Chemical Reactor (CSTR) obtained from Yoon and MacGregor (2001) and also used by Mahmood and Abbasi (2021). The outlet temperature (X) variable is selected as a study variable from the CSTR dataset. We obtained m = 40 ranked set samples of size n = 5 considering different perfect ranked set schemes.

To check the constancy of the proposed CV control charts statistics, we plotted the means under ranked set schemes against the square of the computed CV statistics of SRS, RSS, MRSS, ERSS, SRSS and NRSS of different sample subgroups. The results in Figure 4.4A show that the plotted CV statistics based on SRS, RSS, MRSS, and ERSS against $\bar{Y}$ SRS, $\bar{Y}$ RSS , $\bar{Y}$ MRSS , and $\bar{Y}$ ERSS , respectively, are constant.

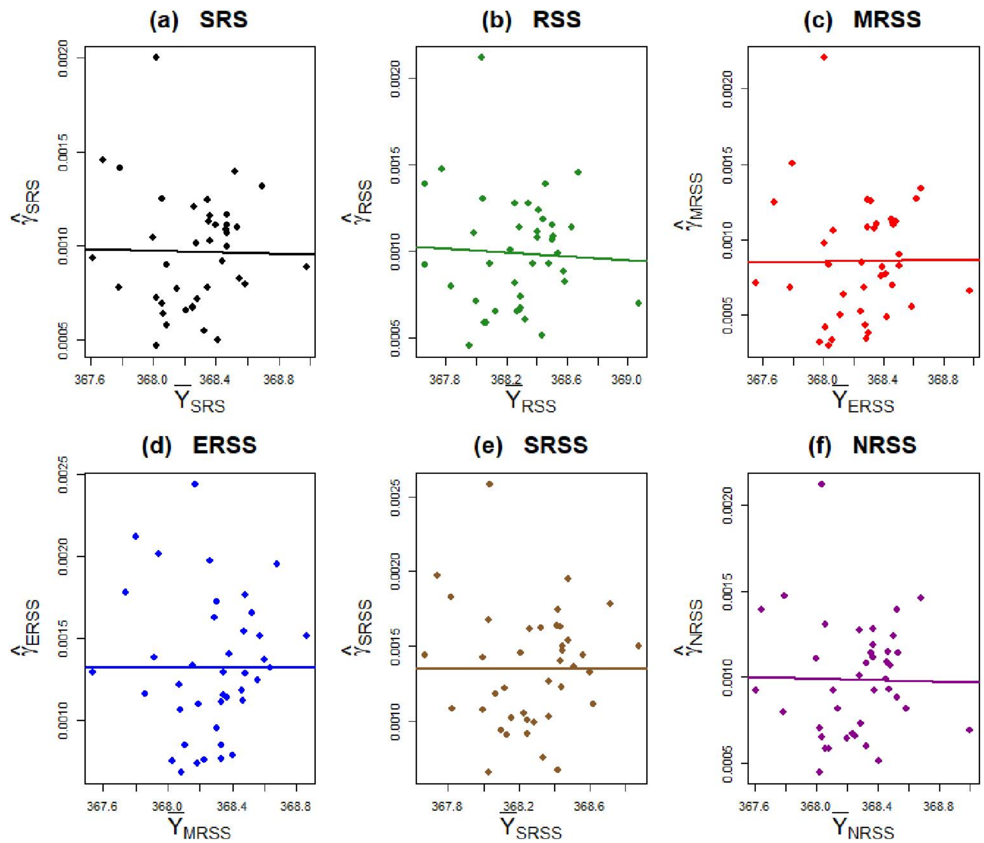

Figure 4.4A: A chart showing CV constancy at different ranked set sampling schemes

As shown in Figure 4.4A, there is no apparent correlation between the estimated CV statistics with their respective averages. Hence, the idea of using the CV charts is justified. The hypothesis of constant CV is further tested by running a regression model with the null hypothesis that CV statistics are constant with respect to their respective means. The regression supports the null hypothesis as the p-value is greater than 0.05 level of significance.

Response: CV_SRS

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
mean_SRS	1	1.5000e-09	1.5230e-09	0.0154	0.9019
Residuals	38	3.7561e-06	9.8846e-08		

Response: CV_RSS

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
mean_RSS	1	1.1100e-08	1.113e-08	0.0983	0.7556
Residuals	38	4.3014e-06	1.132e-07		

Response: CV_MRSS

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
mean_MRSS	1	8.0000e-10	7.6800e-10	0.0048	0.9448
Residuals	38	6.0182e-06	1.5837e-07		

Response: CV_ERSS

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
mean_ERSS	1	0.0000e+00	1.0000e-12	0	0.9979
Residuals	38	7.0213e-06	1.8477e-07		

Response: CV_SRSS

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
mean_SRSS	1	1.00e-10	7.3000e-11	5e-04	0.983
Residuals	38	6.05e-06	1.5921e-07		

Response: CV_NRSS

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
mean_NRSS	1	1.2000e-09	1.1710e-09	0.0103	0.9196
Residuals	38	4.3114e-06	1.1346e-07		

To investigate the detection ability of the charts, a shift of size δ = 1.5 was applied to the last 20 samples. The results given in Figure 4.4B show that the CV control charts based on SRSS and NRSS detect more out-of-control samples than the other charts. This indicates that the CV control chart based on NRSS and SRSS offer the best detection ability. This superiority of the NRSS and SRSS based CV chart for real data validates the findings in the simulation study.

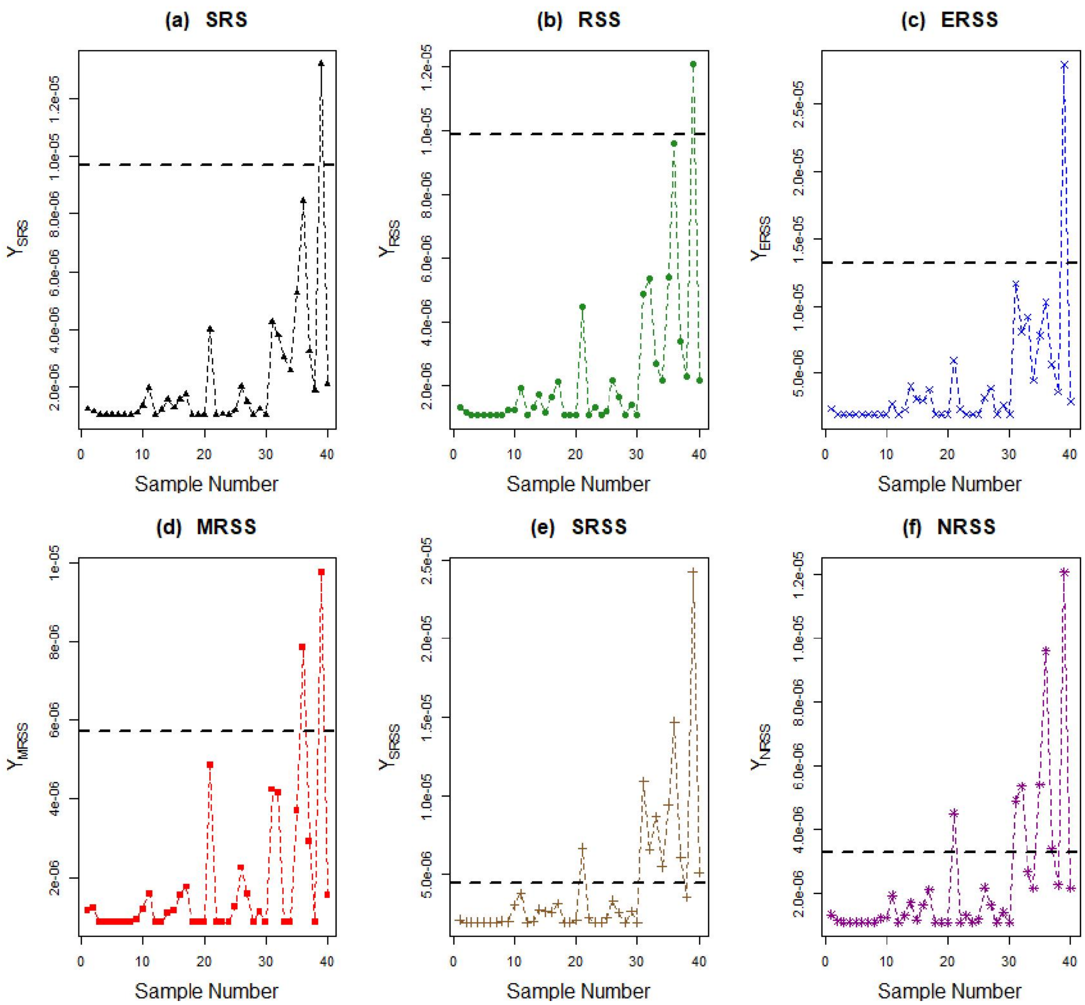

Figure 4.4B: A univariate coefficient of variation control chart based on different ranked set sampling schemes

VIII. CONCLUSION AND RECOMMENDATIONS

In this study, a new set of CV control charts, designed under different rank set sampling schemes; RSS, MRSS, ERSS, SRSS and NRSS were proposed. The CV control charts were evaluated and compared with each other. Our simulation results showed that CV control charts, based on SRSS and NRSS schemes, outperformed the SRS, RSS, ERSS and MRSS CV control charts. The CV control chart based on SRSS and NRSS schemes are the best performing CV chart for detecting shifts in process CV. Based on the findings and conclusion of this study, the use of NRSS and SRSS-based CV chart in phase I of SPC is performing best for the detection of process variability. This study will help quality practitioners to choose an efficient coefficient of variation CV control chart in phase I. Further research can be carried out on univariate coefficient of variation CV control chart based on different ranked set sampling schemes in phase II of statistical process control (SPC).

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