Ultimate Shear Resistance of Plate Girders: Part 1- Cardiff Theory

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Abstract: Theoretical predictions of the ultimate shear resistance of slender plate girders can be made using two main theories Cardiff theory and Höglund's theory. This study will be published in two parts; the first part will be concerned with Cardiff tension-field theory; where the second part will be concerned with Höglund's rotating-stress-field theory and EC3. Cardiff theory is based on an assumed equilibrium stress field (tension-field) in the girder which satisfies the theoretical conditions for a lower-bound strength prediction provided the material possesses sufficient ductility for the stress field to develop. In most of the previous study, there isn't any constrain indicated for using such theory; consequently, in this paper theoretical analysis has been conducted to study the effect of shear panel aspect ratio (b/d) in the ultimate shear obtained using Cardiff theory. This study is concerned with the limits of b/d which can be applied for Cardiff theory and it's relation with c/b. The analysis based on ninety six test results of steel plate girders subjected to shear performed experimentally by Höglund, Nethercot and Byfield. From this study, it is pointed out the limits of panel aspect ratio which can be applicable for using Cardiff theory. New formula proposed to predict the percentage of the distance between the plastic hinges form in the flanges "c" to panel width "b". Conservative limits of (c/b) have been suggested to get consistent value of ultimate shear resistance using Cardiff theory.

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1. Introduction

Fabricated plate girders are normally designed to support vertical loads over long spans, where high bending moments and shearing forces are developed. The primary function of the flanges is to resist axial compressive and tensile forces arising from bending action, while the web resists the shear forces. Subject to adequate boundary constraint, web plates may exhibit a significant post-buckling reserve of strength and stiffness, which is taken into account in ultimatelimit-state design methods and codes of practice [1,2]. The ultimate shear resistance of steel plate girders has been studied extensively, both experimentally and theoretically, results used in the development of the well-established Cardiff tension-field [1] and Höglund's Rotating-stress fields [2] theories. Experimental studies of the ultimate shear resistance of steel plate girders have indicated that at failure, the girders exhibit the characteristic diagonal shear buckling of the web and the development of the plastic hinges in the flanges.

Cardiff tension-field theory is intended to produce more economical designs for a limited range of girder configurations. Herein, currently available test results for ultimate shear resistance of steel plate girders are reviewed and discussed. An improvement for the theoretical procedure to predict the ultimate shear resistance, using Cardiff tension field theory limits will be executed in this paper.

2. Shear strength of plate girder subjected to shear using Cardiff tension-field theory:

For a plate girder subjected to a small shear load, bending theory can be used to determine how the internal forces are carried by the web and the flanges. When the applied load is increased, the failure mode of a plate girder will depend largely on the panel aspect ratio (b/d) and the web slenderness ratio (d/t), where b is the clear distance between vertical stiffeners, and d and t are the clear depth and the thickness of the web panel, respectively. When the panel is stocky, the web will fail by yielding in shear, which is governed by the theoretical shear yield

strength
$$\tau_{yw} = \frac{\sigma_{yw}}{\sqrt{3}}$$
, where σ_{yw} is the uniaxial

tensile yield strength of the web. For most practical plate girders, however, web panels are generally thin and tend to buckle first before yielding. The overall behavior of a web panel is thus divided into three stages, (1) unbuckled, (2) post-buckled, and (3) collapsed stage.

Unbuckled stage

If a uniform shear stress is applied to the web, there will be a principal tensile stress of magnitude τ

acting throughout the whole web. This stress state will continue until the applied shear stress reaches the critical shear strength τ_{cr} which can be determined from classical stability theory for plates by Timoshenko and Gere [7]:

$$\tau_{cr} = K \left[\frac{\pi^2 E}{12(1-\mu^2)} \right] * \left(\frac{t}{d} \right)^2 \le \tau_{yw}$$
(1)

The buckling coefficient K is obtained from the following equation

$$K = 5.35 + 4 * \left(\frac{d}{b}\right)^2, \qquad \text{for } \frac{b}{d} > 1 \quad (2)$$
$$K = 5.35 * \left(\frac{d}{b}\right)^2 + 4, \qquad \text{for } \frac{b}{d} < 1$$

Where: *E* the modulus of elasticity, and μ is the Poisson's ratio.

Therefore, the shear load that causes the web plate to buckle is given by:

$$V_{cr} = \tau_{cr} dt. \tag{3}$$

Although the notion of real boundary condition at the juncture between web and flanges to be somewhere between simply supported and fixed has been recognized from early days. Recent studies show that the restraint provided by the flanges could enhance the buckling coefficient K, which would lead to an enhancement in shear strength [1,4,9]. In this paper the boundary condition has been arbitrarily and conservatively assumed to be fixed.

Post-buckled stage

Once the critical shear strength is reached, the web cannot carry any increase in shear load. Additional shear force will be supported by the mobilization of tensile membrane stress σ_t in the diagonal band of the web. For a web panel subjected to pure shear, the value of σ_t that causes the web to yield equal σ_t^y can be written as

$$\sigma_t^y = -\frac{3}{2}\tau_{cr}\sin 2\theta + \sqrt{\sigma_{yw}^2 + \tau_{cr}^2 \left[\left(\frac{3}{2}\sin 2\theta\right)^2 - 3\right]}$$
(4)

where the angle θ is the inclination of the membrane tensile yielding strength σ_t^y . It should be mentioned that, the above equation is applied only if τ_{cr} less than or equal τ_{yw} .

Collapsed stage

Failure of the plate girder occurs when sufficient numbers of hinges have been formed in the top and bottom flanges; together with the diagonal yield zone, the web panel forms a plastic sway mechanism. The additional shear load V_f sustained by

the web panel until it collapses is determined from a consideration of virtual work applied to the sway mechanism [9].

For the assumed failure mechanism, the ultimate shear resistance V_{f} of a transversely stiffened girder can be expressed as:

$$V_{f} = \sigma_{t}^{y} * t * \sin^{2} \theta * \left(d * \cot \theta - b + \frac{c_{c}}{2} + \frac{c_{t}}{2} \right) + \frac{2^{*}M_{pfc}}{c_{c}} + \frac{2^{*}M_{pft}}{c_{t}} \quad (5)$$

$$c = \frac{2}{\sin \theta} \sqrt{\frac{M_{Pf}}{\sigma_{t}^{y}t}} \quad \text{where}$$

$$M_{pf} = \frac{c^{2} * \sin^{2} \theta * \sigma_{t}^{y} * t}{4} \quad (6)$$

$$V_f = 2 * c * t * \sigma_t^y * \sin^2 \theta + \sigma_t^y * d * t * \sin^2 \theta * (\cot \theta - \cot \theta_d)$$
(7)

The first term on the right hand side of Eq. (7) represents the contribution of flanges to panel shear strength. The value of *c* is obtained by considering the equilibrium of the panel between the two plastic hinges in the flange, and θ_d is governed by the panel aspect ratio (*Cot* $\theta_d = b/d$).

$$V_{ult} = V_{cr} + V_f$$

$$V_{ult} = \tau_{cr} * d * t + 2 * c * t * \sigma_t^y * \sin^2 \theta + \sigma_t^y * d * t * \sin^2 \theta * (\cot\theta - \cot\theta_d)$$
(8)

In the above equations the ultimate shear load V_{ult} and the inclination of principal tensile stress θ are unknown. A parametric study shows that the variation of V_{ult} with θ is not abrupt [8,9]. It was suggested that the assumption of $\theta = 2 \theta_d / 3$, in order to maximize V_{ult} where $\cot \theta_d = b/d$ [6]. The BS 5950: Part 1 [3], provided a simplified version of the shear strength equations based on the assumption that $\theta = \theta_d/2$. The above calculation method based on Cardiff method is not applicable for unstiffened girders and for high web aspect ratios; since these procedures leads to $V_{ult} \rightarrow 0$ [6]. The Cardiff method preferred than Basler's method because it is more suitable for the reliability level, which has to be achieved with the needed uniformity; as recommended by Sulyok et al. [6]. Here au_{cr} is the critical or buckling shear stress of the web panel and σ_t^y the web tension-field stress developed in the post-buckling stage.

3. Experimental results

Extensive Experimental studies have been conducted in Cardiff on the ultimate shear resistance of steel plate girders, a summary of which has been presented by Höglund, Nether-cot and by field and Newark [2]. However, the 96 test results listed in table1executed by Höglund, Nethercot and Byfield and collected by Davies *et al.* [2] are regarded and as being suitable for this study. Material and geometric properties were completed for the collected ninety six tests. A summary of the test results (girder dimensions, material properties and failure loads) is presented in table 1.

Table 1. Details of test girders and test results:

Girder	В	D	tw	$\mathbf{b}_{\mathbf{f}}$	t _f	Е	$\mathbf{f}_{\mathbf{yw}}$	f _{yf}	V_u	V_{exp}/V_S	V_{exp}/V_S
reference	254	256	1 47	41	61	210000	250	207	41	$0 - 2 0_{\rm d} / 3$	$0 - 0_{\rm d}/2$
C6 T1	1005	1270	1.47	209	10.4	210000	250	267	516	0.00	1 1 2 1
G0-11 C6 T2	052	1270	4.9	208	19.0	210000	253	201	662	0.04	0.021
G0-12 C6 T2	933 625	1270	4.9	208	19.0	210000	253	201	797	0.94	0.931
G0-13 C7-T1	1270	1270	4.9	310	19.0	210000	253	201	623	1.03	1.034
G7-T2	1270	1270	4.98	310	19.5	210000	253	259	645	1.05	1.034
G/-12 C9 T1	3810	1270	5.08	305	19.5	210000	255	233	375	1.07	1.070
G0-11 C8-T2	1905	1270	5.08	305	19.1	210000	203	284	445	0.90	0.906
C8-T3	1905	1270	5.08	305	19.1	210000	263	284	516	1.04	1.050
C0-T1	3810	1270	3 33	305	19.1	210000	203	204	213	1.04	1.050
G9-11 C9-T2	1905	1270	3 33	305	19.1	210000	307	288	334	0.97	0.999
G9-T2 G9-T3	1905	1270	3 33	305	19.1	210000	307	288	352	1.02	1.053
H1T2	1905	1270	9.95	459	24.8	210000	745	703	3421	1.02	1 141
H2T1	1270	1270	0.01	459	51.2	210000	760	750	4079	0.80	0.846
C-AC2	2/90	1270	3.1	102	97	210000	215	755	120	1.28	1 311
C-AC4	2515	457.2	4 3	102	16.3	210000	215	783	245	1.20	1.511
C-AC5	2515	457.2	43	127	19.1	210000	236	790	232	1.02	1.045
B	1200	1200	4 5	240	12	210000	490	491	760	0.98	0.955
<u>S-2</u>	581	319	3.2	100	10.5	210000	352	273	161	0.91	0.915
<u>S-3</u>	577	477	3.2	101	10.5	210000	317	272	198	0.99	0.979
2.2	1440	600	2.	175	6	210000	255	255	75	1.34	1.340
TG3	1000	1000	2.5	200	16.4	210000	200	281	190	0.97	1.004
TG3.1	1000	1000	2.5	200	16.4	210000	200	281	190	0.97	1.004
TG4	1000	1000	2.5	200	20.2	210000	200	281	219	1.02	1.072
TG4.1	1000	1000	2.5	200	20.1	210000	200	281	207	0.96	1.015
TG5	1000	1000	2.5	200	29.7	210000	200	281	308	1.17	1.275
TG5.1	1000	1000	2.5	200	29.7	210000	200	281	300	1.14	1.242
US2/5	788	359	3.17	97	12	210000	230	422	135	0.96	0.972
US3/5	788	359	2.7	96	12	210000	257	422	90	0.81	0.834
TG14	305	305	0.97	76	3.12	210000	219	305	25	1.12	1.110
TG15	305	305	0.97	76	5	210000	219	286	29	1.12	1.146
TG16	305	305	0.97	76	6.45	210000	219	337	32	1.07	1.125
TG17	305	305	0.97	76	9.32	210000	219	308	39	1.10	1.196
TG18	305	305	0.97	76	13	210000	219	304	51	1.19	1.326
TG19	305	305	0.97	76	15.5	210000	219	268	55	1.19	1.343
TG22	305	305	2.03	76	6.5	210000	229	337	79	1.03	1.036
TG23	305	305	2.03	76	9.2	210000	229	308	81	0.99	1.005
TG24	305	305	2.03	76	13	210000	229	304	96	1.06	1.099
TG25	305	305	2.03	76	15.5	210000	229	268	104	1.11	1.153
STG1	551	279	2	127	7.9	210000	255	275	60	0.95	0.984
STG2	502	253	1.6	127	6.4	210000	272	275	40	0.94	0.973
STG4	498	251	1.25	102	6.4	210000	246	275	35	1.28	1.352
RTG1	305	305	1.27	76	4.5	210000	244	275	40	1.09	1.091
RTG2	305	305	1.27	76	4.7	210000	244	275	41	1.11	1.107
RTG4	254	254	0.95	76	4.7	210000	259	275	24	0.93	0.952
TS1/3	700	813	4.06	209	12	210000	265	429	312	0.77	0.766
TS1/4	700	813	4.06	212	12	210000	265	429	387	0.95	0.948
MSO	947	608	2.01	102	10.1	210000	261	269	94	1.12	1.129
SD1 SD2	594	594	2	250	12	210000	276	212	129	0.92	0.964
SD3	594	594	2	250	12	210000	2/6	212	156	1.12	1.166
TGVI-I TGVI-2	1200	600	2.07	200	10	210000	211	247	83	1.17	1.222
1GV1-2 TCV2.2	600	600	2.07	200	10	210000	211	247	111	1.00	1.031
1GV2-2 TCV2-2	600	600	2.08	200	10	210000	211	247	115	1.03	1.062
1GV3-2 TCV4	507	500	2.01	200	10 1	210000	211	247	115	1.05	1.080
1674	391	398	1.97	201	10.1	210000	224	233	102	0.92	0.955

TGV5	595	598	1.98	201	10	210000	232	252	105	0.93	0.955
TGV6	595	598	1.97	201	10.1	210000	228	254	102	0.91	0.940
TGV7-2	596	599	1.98	201	10.1	210000	221	250	106	0.96	0.995
TGV10-1	595	599	1.91	200	10	210000	219	284	102	0.95	0.984
TGV10-2	595	599	1.91	200	10	210000	219	284	106	0.98	1.023
TGV11-2	597	599	1.91	200	10	210000	220	211	102	1.00	1.032
S3/1	300	300	1.03	35	3.2	200000	169	295	19	1.05	1.026
S4/1	345	351	1.07	40	3.2	200000	169	295	21	1.02	0.987
S5/1	400	399	1.09	39	3.2	200000	169	295	23	1.05	1.009
S2/1.5	375	249	1.05	40	3.2	200000	169	295	16	1.13	1.123
S3/1.5	450	301	1.03	39	3.2	200000	169	295	16	1.12	1.104
S4/1.5	522	352	1.1	39	3.3	200000	169	295	13	0.78	0.768
lS1-BA	942	608	2.1	100	10	191000	183	269	76	1.10	1.119
LS3-BA	947	608	2.46	100	10.1	197000	201	283	103	1.13	1.136
MCS1-PB3	732	1000	4.4	300	15.1	210000	169.7	226.6	388	0.96	0.958
PA1	600	800	1	249	12	210000	216	206	81	0.94	1.003
PA2	600	800	1	249	12	210000	216	206	84	0.98	1.040
PA3	600	800	1	249	12	210000	216	206	85	0.99	1.053
PB1	500	800	1	249	12	210000	216	206	90	0.96	1.015
PB2	500	800	1	249	12	210000	216	206	91	0.97	1.026
PC1	1000	800	1	250	10	210000	216	262	54	0.89	0.953
PC2	1000	800	1	250	10	210000	216	262	54	0.89	0.953
PD1	750	800	1	250	10	210000	216	262	65	0.88	0.943
PD2	750	800	1	250	10	210000	216	262	65	0.88	0.943
PD3	750	800	1	250	10	210000	216	262	75	1.02	1.088
PC3	750	800	1	250	10	210000	216	262	79	1.08	1.146
PB3	732	1000	4.4	300	15.1	205000	169.7	226.6	388	0.96	0.963
PB4	732	1000	4.4	300	15.1	205000	169.7	226.6	388	0.96	0.963
B1	9000	600	2.86	226	9.9	210000	419	294	146	2.52	2.543
B4	9000	600	2	151	6.1	210000	280	304	71	3.34	3.350
K1	6000	600	2.86	226	9.9	210000	419	294	158	2.35	2.382
1A	8100	600	2.96	225	10	210000	243	251	146	2.56	2.585
1B	8100	600	2.97	225	10	210000	243	251	132	2.29	2.317
2A	8100	600	3	225	10	210000	243	251	132	2.23	2.260
2B	8100	600	2.94	225	10	210000	243	251	127	2.26	2.287
3A	8100	600	2	150	6	210000	292	286	59	2.65	2.652
3B	8100	600	2	150	6	210000	292	286	61	2.74	2.742
4A	8100	600	2.01	150	6	210000	292	286	71	3.16	3.158
4B	8100	600	2.03	150	6	210000	292	286	68	2.96	2.960
CP1/1	747	500	2.04	100	8	210000	246	256	88	1.19	1.193
RCP1/1	710	718	2.01	100	8.1	210000	271	288	127	1.07	1.048

Table 2. Mean, Standard deviation and coefficient of variation for 17 samples with $0.5 \le b/d \le 1.0$

Cardiff	Data	Sample Size (S.Size) = 17				
From $b/d \ge$	To b/d<	Maan	Standard deviation	Coefficient of variation		
0.5	1	Mean		Coefficient of variation		
$\theta = (2/3)^* \theta_d$		0.93	±0.09	0.10		
θ=(1/2)	* θ_d	0.96	±0.11	0.11		

4. Relation between panel aspect ratio (b/d) and inclination angle θ :

Extensive study has been conducted to calculate the ultimate shear resistance of 96 plate girders collected by Davies *et al.* [2]. Two different value of the inclination of principal tensile stress θ are suggested $\theta = 2 \theta_d /3$, and $\theta = \theta_d/2$. Comparisons between both cases of θ values with different ranges of b/d are shown from figures 1 to 4; where the mean, standard deviation and coefficient of variation listed from tables (2) to (5) for each test result. Figure 1 shows the relation between V_{exp} / V_s with panel aspect ratio b/d, in case of b/d varies from 0.5 to 0.94, these results related to 17 samples out of 96 samples. Table 2 also summarized the mean, standard deviation and coefficient of variation for the results of the ratio between ultimate shear resistances obtained experimentally compared with the value of ultimate shear resistance obtained by using Cardiff theory. From Fig. 1 and table 2, it can be summarized that the results obtained for ultimate shear resistance of plate girder using Cardiff theory with angle θ



equal $\theta = \theta_d/2$ more consistent than 2 θ_d /3 in this





Fig. 2. Comparison between V_{exp}/V_S for two different values of θ for $0.98 \le b/d \le 1$

Figure 2 shows the relation between V_{exp} / V_S with panel aspect ratio b/d, in case of b/d varies from 0.98 to 1, these results related to 39 samples out of 96 samples. Table 3 also summarized the mean, standard deviation and coefficient of variation for the results of the ratio between ultimate shear resistances obtained experimentally compared with the value of ultimate shear resistance obtained from Cardiff theory. From Fig. 2 and table 3, it can be summarized that the results obtained for ultimate shear resistance of plate girder by applying Cardiff theory with angle θ equal 2 θ_d /3 more consistent than $\theta = \theta_d/2$ in this range of b/d.

Table 3.Mean, Standard deviation and coefficient of variation for 39 samples with $0.98 \le b/d \le 1$

Cardiff Data		Sample Size (S.Size) = 39				
From $b/d \ge$	To $b/d \leq$	Mean	±Standard deviation	Coefficient of variation		
$\theta = (2/3)^* \theta_d$	1	1.03	±0.08	0.08		
$\theta = (1/2)^* \theta_d$		1.06	±0.11	0.10		

Figure 3 shows the relation between V_{exp} / V_s with panel aspect ratio b/d, in case of b/d varies from 1.2 to 3, these results related to 26 samples out of 96 samples. Table 4 also summarized the mean, standard deviation and coefficient of variation for the results of the ratio between ultimate shear resistances obtained experimentally compared with the value of ultimate shear resistance obtained from Cardiff theory. From Fig. 3 and table 4, it can be summarized that the results obtained for ultimate shear resistance of plate girder using Cardiff theory with angle θ equal 2 θ_d /3 more consistent than $\theta = \theta_d/2$ in this range of b/d.

Figure 4 shows the relation between V_{exp} / V_s with panel aspect ratio b/d, in case of b/d varies from 3 to 15, these results related to 14 samples out of 96 samples. Table 5 also summarized the mean, standard deviation and coefficient of variation for the results of the ratio between ultimate shear resistances obtained experimentally compared with the value of ultimate shear resistance obtained from Cardiff theory. From Fig. 4 and table 5, it can be summarized that the results obtained for ultimate shear resistance of plate girder using Cardiff theory with angle θ equal 2 θ_d /3 or $\theta = \theta_d/2$ are inconsistent.



Fig. 3. Comparison between V_{exp} , V_S for two different values of θ for $1.2 \le b/d \le 3$

Cardiff Data		Sample Size (S.Size) = 26				
From $b/d \ge$	To $b/d \leq$	Moon	±Standard	Coefficient of		
1.2	3	Mean	deviation	variation		
$\theta = (2/3) \cdot \theta_d$		1.04	±0.14	0.13		
$\theta = (1/2) * \theta_d$		1.06	±0.14	0.13		

Table 4. Mean, Standard deviation and coefficient of variation for 26 samples with $1.2 \le b/d \le 1.6$

Table 5. Mean	Standard deviation and coefficient of	variation for 14 sam	ples with $3 \le b/d \le 15$

Cardiff Data		Sample Size (S.Size) = 14			
From b/d	To <i>b/d</i>	Moon	±Standard	Coefficient of	
More than 3 up to 15		Mean	deviation	variation	
$\theta = (2/3)^* \theta_d$		2.32	±0.70	0.30	
$\theta = (1/2)^* \theta_d$		2.34	±0.69	0.30	



Fig. 4. Comparison between V_{exp}/V_S for two different values of θ for $3 \le b/d \le 15$

5. Relation between panel aspect ratio (b/d) and (c/b):

Table 6 shows the relation between panel aspect ratio "(b/d)" and the corresponding value of the ratio of "c" the distance between the plastic hinges form in

the flanges to panel width "b". This relation is represented in the second term in equation No. 7 $[d^{*}(\cot \theta - (b/d) + (c/d)]$. Parametric analysis has been conducted using the geometric and material properties of G7-T1.

b/d	$\theta = \frac{2}{3} * \theta_d$		$\theta = \frac{1}{2}$	$2 \cdot \theta_d$	Notes	
	c/b	$d*(cot \ \theta - (b/d)+(c/d))$	c/b	$d^{*}(\cot \theta - (b/d) + (c/d))$		
0.50	0.50	1080.251	0.627++	1818.859	⁺⁺ c/b More than 0.55	
0.60	0.40	1089.59	0.500	1863.235		
0.75	0.32	1140.955	0.410	1979.488		
1.00	0.27	1268.135	0.343	2232.527		
1.10	0.25	1328.292	0.328	2347.37		
1.20	0.24	1391.695	0.316	2467.302		
1.30	0.24	1457.812	0.307	2591.53		
1.40	0.23	1526.222	0.300	2719.413		
1.50	0.22	1596.584	0.293	2850.43		
1.60	0.22	1668.622	0.288	2984.148		
1.70	0.22	1742.107	0.284	3120.209		
1.80	0.21	1816.849	0.280	3258.311		
1.90	0.21	1892.687	0.277	3398.2		
2.00	0.21	1969.486	0.275	3539.659		
2.20	0.21	2125.531	0.270	3826.581		
2.50	0.20	2364.443	0.265	4264.924		
2.75	0.20	2566.907	0.262	4635.765		
3.00	0.20	2771.699	0.260	5010.453		

Table 6.	Relation	between	b/d	value	and	c/b

Using the data of G7-T1 the only variable was the panel width *b* and using the rest of the data as constant values (*web thickness, flange thickness, web depth,*). By comparing the results obtained from (*b/d*) equal 0.5 to (*b/d*) equal 2.5 in both cases of θ with the relevant value of *c/b*, it can be summarized that, convenient value of ultimate shear resistance can be obtained in case of (c/b) ranged between 0.2 and 0.55. Using these limits two charts plotted to represent the relationship between b/d and c/b as shown in Fig.5 for each case of θ .



Fig. 5 Relation between c/b and panel aspect ratio b/d

From figure 5 two formulas proposed to predict the value of c/b using the assumed panel aspect ratio b/d. these formulas can be expressed as follow:

In case of
$$\theta = \frac{2}{3} * \theta_d$$

 $\frac{c}{b} = 0.034 * \left(\frac{b}{d}\right)^4 - 0.294 * \left(\frac{b}{d}\right)^3 + 0.915 * \left(\frac{b}{d}\right)^2 - 1.268 * \left(\frac{b}{d}\right) + 0.88$
In case of $\theta = \frac{1}{2} * \theta_d$
 $\frac{c}{b} = 0.041 * \left(\frac{b}{d}\right)^4 - 0.353 * \left(\frac{b}{d}\right)^3 + 1.102 * \left(\frac{b}{d}\right)^2 - 1.526 * \left(\frac{b}{d}\right) + 1.082$

6. Conclusion

1. Cardiff theory can be used to predict the ultimate shear resistance of plate girder having intermediate transverse stiffeners in case of web panel aspect ratios b/d (width of web panel/depth of web panel) between 0.5 to 3.

- 2. In case of *b/d* varies from 0.5 to 1, the ultimate shear resistance of plate girder using Cardiff theory with angle θ "*the inclination of the membrane tensile yielding strength* σ_t^y " equal $\theta = \theta_d/2$ more consistent than 2 $\theta_d/3$.
- 3. When panel aspect ratio b/d varies from 1.0 to 3, the results obtained for ultimate shear resistance of plate girder using Cardiff theory with angle θ equal 2 θ_d /3 more consistent than $\theta = \theta_d/2$ in this range of b/d.
- 4. Two formulas proposed to predict the value of c/b using the assumed panel aspect ratio b/d which will be used in the design.
- 5. The consistent values of (c/b) ranged between 0.2 and 0.55 to predict a convenient value of ultimate shear resistance using Cardiff theory.

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