Assessment of Internal Forces Induced due to Differential Shortening of Vertical Elements in Typical Medium- to High-Rise Buildings

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Abstract: Axial shorting of columns in a building structure due to long term creep and shrinkage causes axial force redistribution among columns and walls, and introduces additional forces in the horizontal members: beams and slabs. Thus, it needs to be considered in the design, especially for medium to high-rise buildings. Extensive research has been conducted to investigate this phenomenon, such issues were addressed through empirical equations and simplified models for individual vertical elements within the building. Meanwhile, no general conclusions appropriate for design practice have been drawn regarding differential column shortening behavior in typical medium- to high-rise buildings. General building codes do not give a specific guideline about when and how differential column shortening should be considered. Consequently, column shortening is usually left to the judgment of structural engineers. However, the combined causes for column shortening are not usually discussed either the type of statically system or time dependent material properties (creep and shrinkage) and inclusion of steel reinforcement into analysis are discussed. The aim of this study is to combine all these parameters. A parametric study is conducted and reported in this paper to investigate the influence of the variation of controlling parameters such as floor levels and type of statically system, using construction sequence analysis method. The results obtained in this research can serve as an aid to the structural engineers during schematic design. 3D finite element modeling has been performed, considering all the above causes using a reliable finite element analysis program MIDAS Gen.

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1. Introduction

In high-rise buildings, differential shortening of columns occurs due to three reasons under axial loads: Elastic shortening, shrinkage and creep. The effects of shrinkage and creep on a structure depend on several factors such as concrete strength, construction duration, concrete casting condition, and weather condition. The effect of shrinkage is not as extensive as the effects of elastic shortening and creep. In general, columns of a building are designed to be similar in their dimensions in spite of that their loaded areas are not similar to one another. This is for the purpose of maintaining simplicity of design and serviceability for occupants. Therefore, differences in loaded areas assigned for columns result in differential shortenings in columns. Also, differential shortenings may also occur due to unbalanced axial stiffness between closely spaced members such as columns and shear walls, which have considerably different axial stiffness. The effect of differential shortening is significant in tall buildings and may produce additional bending moments and shear forces to beam members. In order to avoid differential shortening effects, columns must be designed to proportion to their loaded areas, in other words, they should be equally stressed. However, this is not always possible for architectural reasons.

The elastic axial shortening has been calculated based on the following closed form equation:

$$\Delta = \frac{PL}{E\ \ A} \qquad (1)$$

Where: E is the Young's Modulus, L is member length, A is cross section area And P is the load.

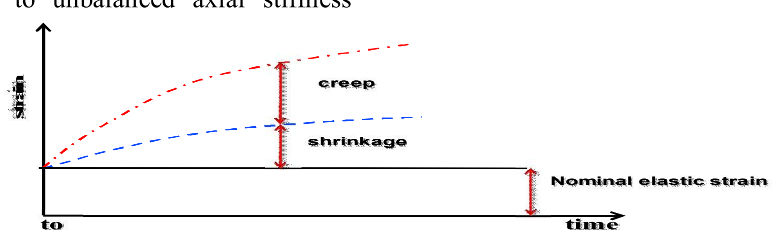


Figure 1: Time Dependant Concrete Deformation

The Creep Strain of concrete has been evaluated based on the following equation [1]:

$$\varepsilon_{cc}(t, t_0) = \frac{\sigma_c(t_0)}{E_c(28)} C_t \quad (2)$$

Where: $\varepsilon_{cc}(t,t_0)$ is the Creep strain at time t, $\sigma_c(t_0)$ is the stress applied at time t_0, C_t is the Creep coefficient as a function of time and $E_c(28)$ is the modulus of elasticity of concrete at 28 days.

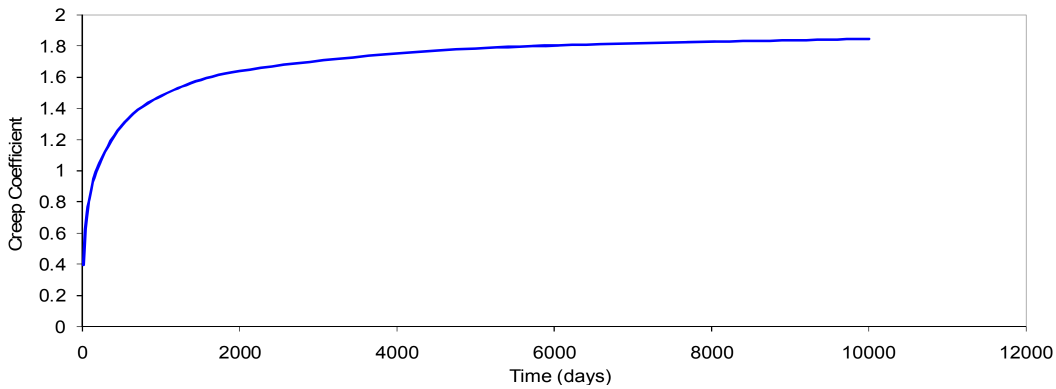


Figure 2: Time Dependant Creep function based on ACI standards

The Shrinkage Strain of concrete has been evaluated based on the following equation [1]:

$$(\varepsilon_{sh}) = \frac{t}{35 + t}(\varepsilon_{sh})_u$$

For 7 days moist cured duration (3)

$$(\varepsilon_{sh}) = \frac{t}{55 + t}(\varepsilon_{sh})_u$$

For 1-3 days steam cured duration (4)

Where: (ε_{sh}) is the Shrinkage strain at time t, $(\varepsilon_{sh})_u$ is the Ultimate Shrinkage strain

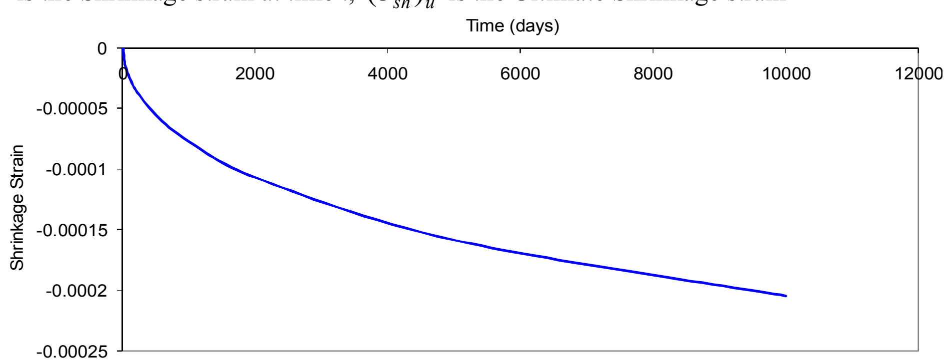


Figure 3: Time Dependant Shrinkage function based on ACI standards

The Compressive Strength of concrete has been evaluated based on the following equation [1]

$$(f'_c)_t = \frac{t}{4 + 0.85\,t}(f'_c)_{28} \quad (5)$$

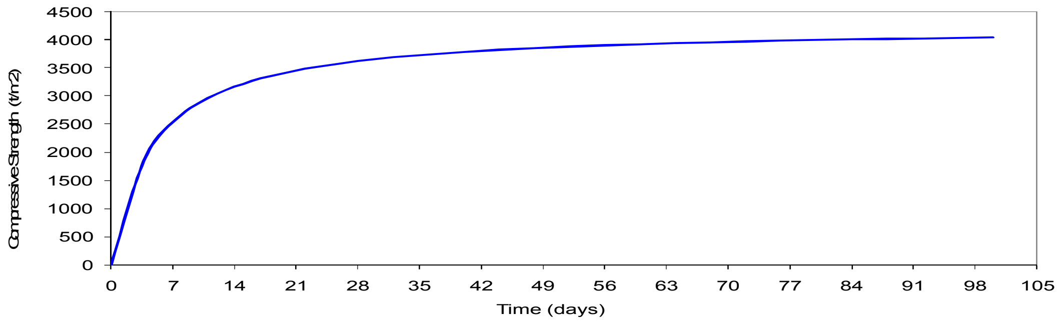


Figure 4: Time Dependant Compressive Strength function based on ACI standards

Where: $(f'_c)_t$ is the Compressive Strength of concrete at time t, $(f'_c)_{28}$ is the Compressive Strength of concrete at time 28-day and t in days is the age of concrete.

2. Theoretical Background

Axial shortening is cumulative over the height of a structure so that detrimental effects due to differential axial shortening become more pronounced with increasing building height. For example, in an 80 storey concrete building, it has been reported that the elastic shortenings of columns is 65mm and that due to shrinkage and creep is 180 to 230mm **[Fintel et al. 1987].** The combination of these shortening components is unacceptable as a structural performance criterion. It is therefore necessary to accurately predict linear and non-linear components of differential axial shortening and control performance with design. Unacceptable cracking and deflection of floor plates, beams and secondary structural components, damage to facades, claddings, finishes, mechanical and plumbing installations and other non-structural walls can occur resulting from differential axial shortening. In addition, common effects on structural elements are sloping of floor plates, secondary bending moments and shear forces in framing **[Fintal and Fazlurl, 1987].**

Extensive research has been conducted to investigate differential column shortening phenomenon **[Fintel and Khan 1969 and 1971, Banavalkar and Wilkerson 1993, Gosh 1996, and Maru et al. 2001,** among others]. It has been shown that differential column shortening is affected by the relative axial stiffness and the tributary area of columns and walls. Moreover, the ratio between beam and column stiffness and the construction sequence also have significant influence on the axial force redistribution. **[Fintel and Khan (1969)]** originally introduced the method of quantifying axial shortening of reinforced concrete columns. They introduced a practical design methodology to estimate both the creep and shrinkage strains in vertical elements of multistory buildings, considering the effects of loading history, member size, and percentage of reinforcement. They have pointed out four main points as follows:

1-Although the magnitude of creep and shrinkage of plain concrete specimens may vary significantly, the final inelastic strains in reinforced concrete columns and walls have much less variation due to the restraining effect of the reinforcement.

2-Elements which receive a substantial loading at early stage, such as prestressed elements and columns in the upper stories of tall structures or columns of low-rise structures, are prone to higher shrinkage and creep strains.

3-Lower story columns of tall structures have significant smaller creep and shrinkage strains than commonly assumed as a result of: incremental loading over a longer period of time which reduces creep, a substantial volume-to-surface ratio which reduces shrinkage and a substantial percentage of reinforcement which reduces both shrinkage and creep in tall structures.

4- The differential shortening between columns and adjacent walls can cause Structural and non-structural distress unless proper design and details are provided.

A parametric study is conducted to investigate the effect of column shortening Fakher et.al. (2009) various Parameters were investigated when considering construction stages.These parameters include: a) Number of stories; thirty to sixty-floor building. b) Concrete compressive strength; values ranging between 45MPa and 60MPa. c) Number of days for a full construction of each storey; Values ranging between 11 and 63 days are used. d) Difference in stresses between the columns and the shear walls; values ranging from 1.0 to 0.5 are used to express the practical range of this variable. e) Variation in stiffness for the beams connecting the columns and the shear walls. They have pointed out four main points as follows:

1-Differences in the stresses between shear walls and columns lead to increasing the need for CS analysis. Almost, along whole building height, reducing the differences in the stresses between shear walls and columns lead to a reduction in the differences between CS analysis results and conventional analysis results.
2-Increasing of construction cycle resulted in an increase in the need for CS analysis. Almost along whole building height reducing of construction cycle resulted in a reduction in the differences between CS analysis results and conventional analysis results. Usually CS analysis results are greater than conventional analysis results within such lower portion of the building. Along the remaining upper portion of the building the increasing of construction cycle reduces the differences between CS analysis results and conventional analysis results.
3-Varying the concrete grade by + 15 MPa has relatively small effects on the differences between CS analysis results and conventional analysis results; it is pointed out that the effect of varying the concrete grade doesn't exceed 20%. However, there is no general trend for the effect of changing concrete grade.
4- Variation in stiffness for the beams connecting the columns and the shear walls resulted that the absolute straining actions values are increased for both construction stage analysis results and conventional analysis results, some straining actions changed their direction either totally along whole building height or partially along some stories. There is no clear trend for the effect of increasing beam depth on the differences between construction stage analysis results and conventional analysis results.

3. Numerical Model and Methodology

3.1Numerical Model

To obtain construction stage analysis for the investigation models well known finite element computer program called MIDAS/GEN **[11]**, was used in the analysis. The models consisted of two types of elements the first one is Beam element (2-node each node retains three translational and three rotational degrees of freedom) and the second one is plate element consisted of (4-node quadrilateral element with 6 degrees of freedom at each node). MIDAS/Gen provides the following coordinate systems: Global Coordinate System (GCS) and Element Coordinate System (ECS). The GCS is used for node data, the majority of data entries associated with nodes and all the results associated with nodes such as nodal displacements and reactions, The Element Coordinate System (ECS) uses lower case "x-y-z axes" in the conventional Cartesian coordinate system **[11].**see fig. 8; MIDAS/Gen separates the model into sub-models for each erection stage and assigns corresponding construction dead loads. The results for each stage are then superimposed to carry out the final erection sequence analysis. Analyses for all remaining loads other than the construction dead loads are carried out on the basis of the one step analysis; Fig 9 summarizes the analysis steps.

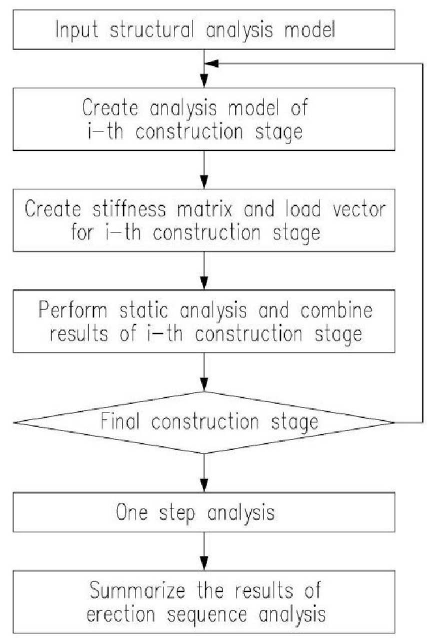


Figure 5: Flow chart for construction sequence analysis

Focus of this investigation is 20 to 60 floor residential buildings, which represents majority of medium- to high-rise building. A five bay structure as shown in Fig. 10 is adopted as a representative prototype of this type of building. The column spacing is 4mx4m, a typical value for residential building, which is also used herein as a lower bound for office usage. Various structural systems can be used for building construction depending on the floor numbers and load requirements. A survey of existing medium- to high-rise buildings **[10].** shows that shear wall structure is used extensively for buildings ranging from 20 to 60 stories, while shear wall with outrigger structure is used more common for buildings ranging from 40 to 80 stories. Therefore, these two structural systems are adopted in this study; Furthermore steel reinforcement has been considered in the analysis.

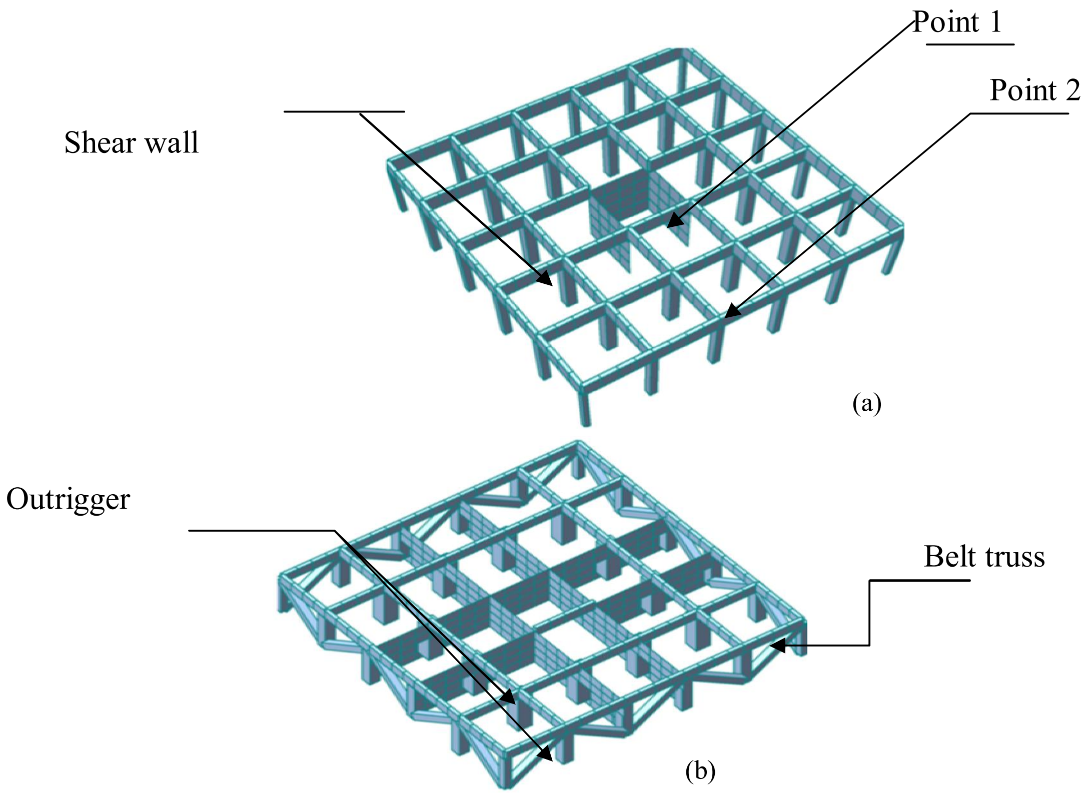


Figure 6: perspective for typical floor of the investigated models: (a) Shear wall structure system and (b) Shear wall with outrigger structure system

The default models are the basic structure models used for the study. Three default models have been created for mid- and high-rise residential buildings. A 20 and 40-floor models have been created for a mid-rise residential building and a 60-floor model has been created for a high-rise residential building Fig. 11 illustrates the investigated set of models. The bays are typically 4.0x4.0m, with typical beam of 25cm width and 70cm depth. The slab thickness is 14cm. Column sizes are determined by gravity loads. a utilization factor of 0.7 is used to account for possible additional load requirement due to wind or seismic loads, In addition the thickness of shear wall is not controlled by gravity loads but lateral loads (drift). For the purpose of column shortening investigation, an important characteristics for the shear wall is the Axial stress ratio between shear wall and columns. A survey on existing medium- to high-rise buildings reveals that this ratio varies between (0.2 to 0.5) **[10].** This ratio is used as a guideline to determine the RC wall thickness so we took stress ratio is equal to 0.5 the cross-sectional dimensions of vertical elements are variable with the aforementioned parameters to satisfy core/column stress ratio and the associated Compressive strength. Fig.12 shows a typical elevation for the 60-floor, 40-floor and 20-floor default models and sorts the columns and walls compressive strength within every twenty floor. Columns reinforcement ratio is fixed to 2%; accounting for the additional moment due to lateral loads would be by increasing the reinforcement from 2% to 4%. It is worth noting that columns dimensions and core thickness are reduced with height (every ten story) as per standard codes specifications. In the case of shear wall-outrigger structure, previous study has identified the optimal locations of outriggers for the purpose of increasing building lateral stiffness and controlling lateral drift **[12].** Based on the founding, in this research two levels of outriggers are placed at levels 20 and 40, respectively for a 60-story building; one level of outrigger is placed at level 20 for a 40-story building and one level of outrigger is placed on the top of a 20-story building. All outriggers are one story high, for all models the floor height is assumed to be 3.00 m and Construction cycle is 28 Days, Fig. 13 illustrates the construction schedule for the investigated models.

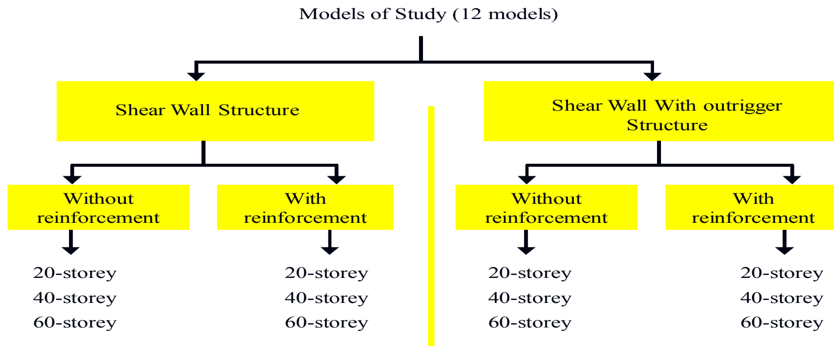
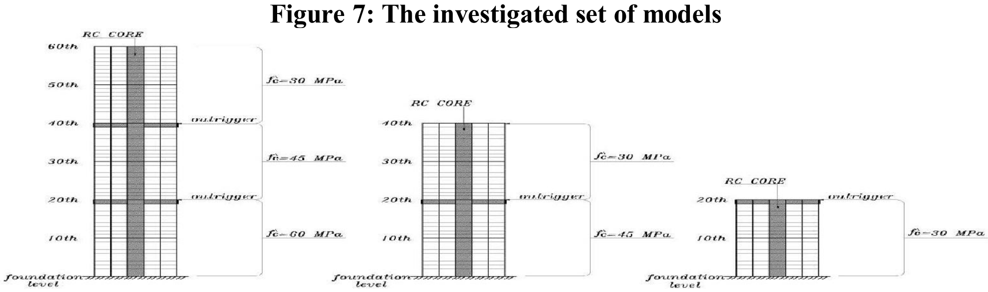


Figure 7: The investigated set of models



(a) (b) (c)

Figure 8: Typical elevation for: (a) 60-story Model, (b) 40-story Model, and (c) 20-story Model

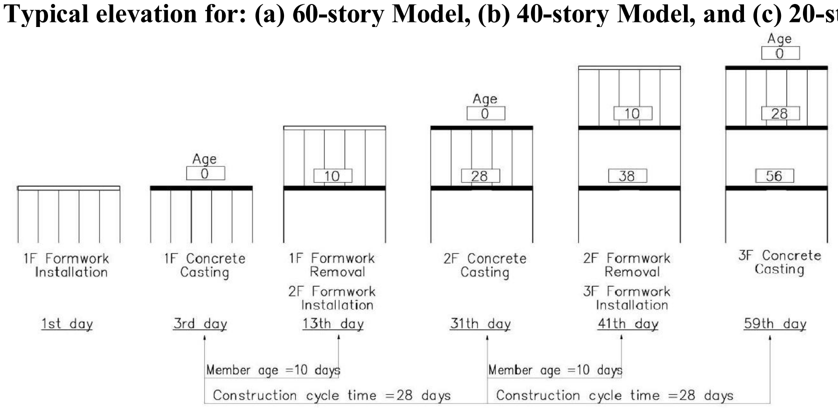


Figure 9: The construction schedule for the investigated models.

4 .Results and Discussion

Table 1 shows a summary for twelve analysis outputs. The nomenclature and the feature of each case are summarized in it, in the nomenclature, the first number represents number of story levels, the second character represents structure type (S: shear wall, O: outriggers), the third character represents inclusion of steel reinforcement or not in the analysis type (WOR: without reinforcement, WR: with reinforcement).

Table 1: Analysis Models Summary

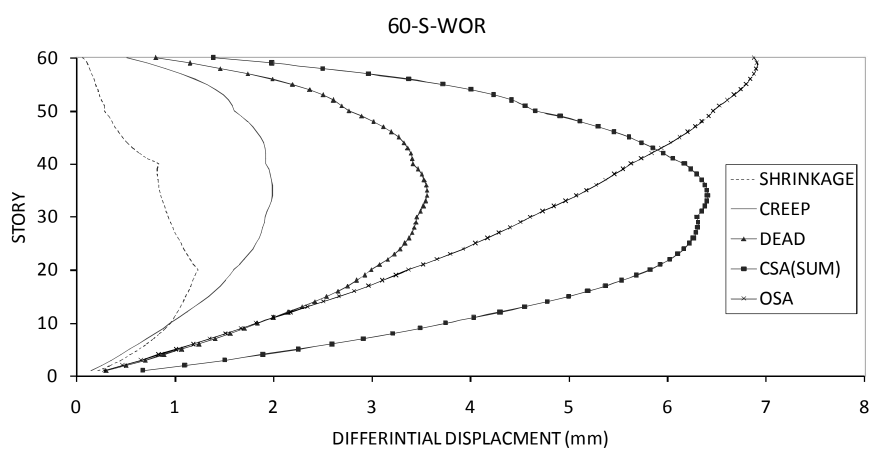
NO	MODEL	STORY	Structure Type	Inclusion of steel
1	60-S-WOR	60	S	WOR
2	60-S-WR	60	S	WR
3	60-O-WOR	60	O	WOR
4	60-O-WR	60	O	WR
5	40-S-WOR	40	S	WOR
6	40-S-WR	40	S	WR
7	40-O-WOR	40	O	WOR
8	40-O-WR	40	O	WR
9	20-S-WOR	20	S	WOR
10	20-S-WR	20	S	WR
11	20-O-WOR	20	O	WOR
12	20-O-WR	20	O	WR

By conducting level by level construction sequence analysis (CSA), the differential column shortening behavior in typical medium- to high-rise buildings is investigated. The investigation emphasizes the following issues:

1. Different results between CSA and one-step analysis (OSA) for the complete building model.
2. Differential vertical displacements between interior shear walls and exterior columns (Beams Differential Displacements).
3. Additional moments and shears in horizontal beams.
4. Axial forces in columns.

4.1 Floor differential displacement

The CSA results for the typical floor differential displacement between exterior column and interior shear wall (between Points 1 and 2 in Fig. 10(a)) for both a shear wall structure and an outrigger structure are shown in Figs. 14a&14bThe results obtained from OSA are also shown in the same figures for the sake of comparison. It can be observed that for a shear wall structure, the maximum floor differential displacement occurs at a level around 2/3 of the building height. For an outrigger structure, since the outriggers separate the vertical structure into several segments, a local maximum floor differential displacement occurs at about 1/2 to 4/5 height of each segment. The absolute floor differential displacement occurs at the lowest segment for the 40-story building and the highest segment for the 60-story. From the analyses it can be concluded that for shear wall structure, as a general rule, when story Number increases, or column steel ratio increases, or beam stiffness decreases, the floor differential displacement increases. The maximum floor differential displacement of the building varies from 0.80 to 2.50 mm for 20-story building, to 0.9 to 5.0 mm for 40-story building, and to 1.0-6.50 mm for 60-story building. These numbers indicates that for shear wall structure less than 20 stories, the column shortening effect can be ignored. On the other hand, when the building reaches 60 stories, the floor differential displacement due to column shortening increases. The behavior of outrigger structures is quite different. Although the increasing of story numbers have small or negligible effects. This again is due to the fact that the rigid outriggers separate the building into several relatively isolated segments. Because of this, the floor differential displacement of the outrigger structures generally falls into the range of 0.4 to 3.0 mm. It is interesting to note that this value is close to that of a 20-story shear wall structure, as of the sub-structure separated by the outriggers. Again, from design practice of view, this floor differential displacement can be ignored, for both structural systems; the analyses show that OSA reports the maximum floor differential displacement at top of the building Contrary to reality. In addition, this method overestimates the maximum floor differential displacement by twofold to threefold. On the other hand inclusion of steel Reinforcement in Analysis reduces the differential displacements with maximum reduction of 7% as shown in Figs. 14c, 14f and 14g. Another problem associated with using OSA for outrigger structures is the column differential displacements, Taking Case 20-O-WOR for example (see fig. 14h) in which case the outriggers are placed on the roof of the building, due to large stiffness of the outriggers, the exterior column behaves as a hanger column hanging from the outriggers under OSA analysis. As a result, differential displacement in the upper floors due to OSA is less than CSA in this case. Apparently, this differential displacement is not realistic, since the outriggers are not in place until all lower column settlement has occurred.



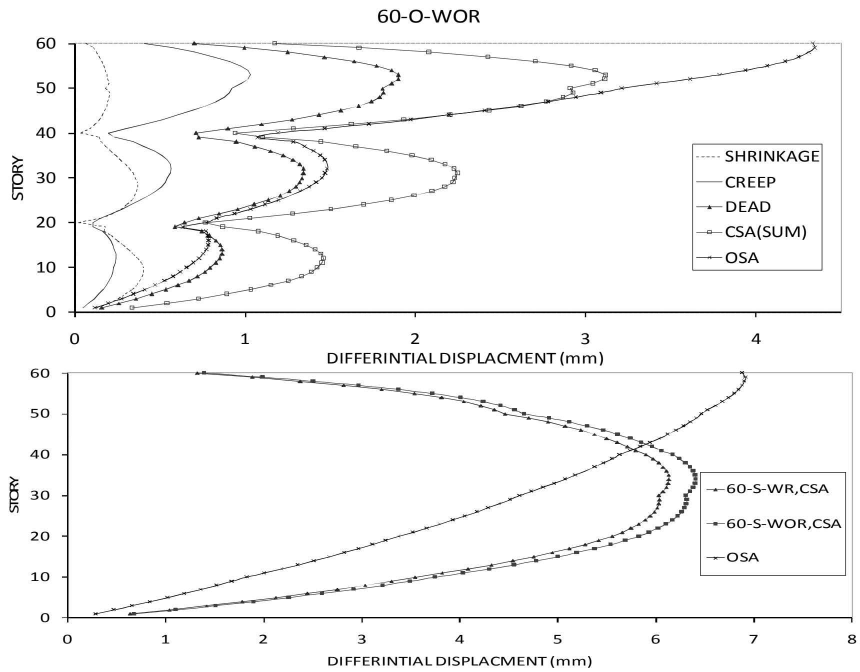


Figure 10: Floor differential displacement of 60-story model: (a) shear wall structure, (b) outrigger structure, and (c) reinforcement effect for both shear wall and outrigger

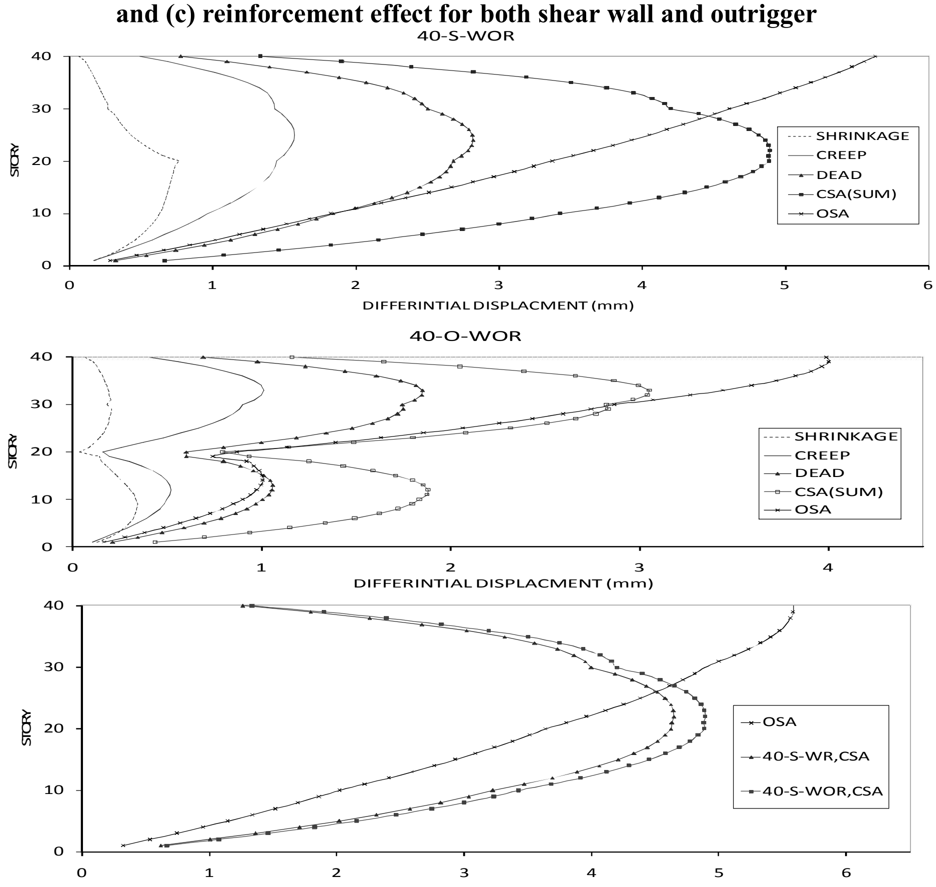


Figure 11: Floor differential displacement of 40-story: (a) shear wall structure, (b) outrigger structure, and (c) reinforcement effect for both shear wall and outrigger structure

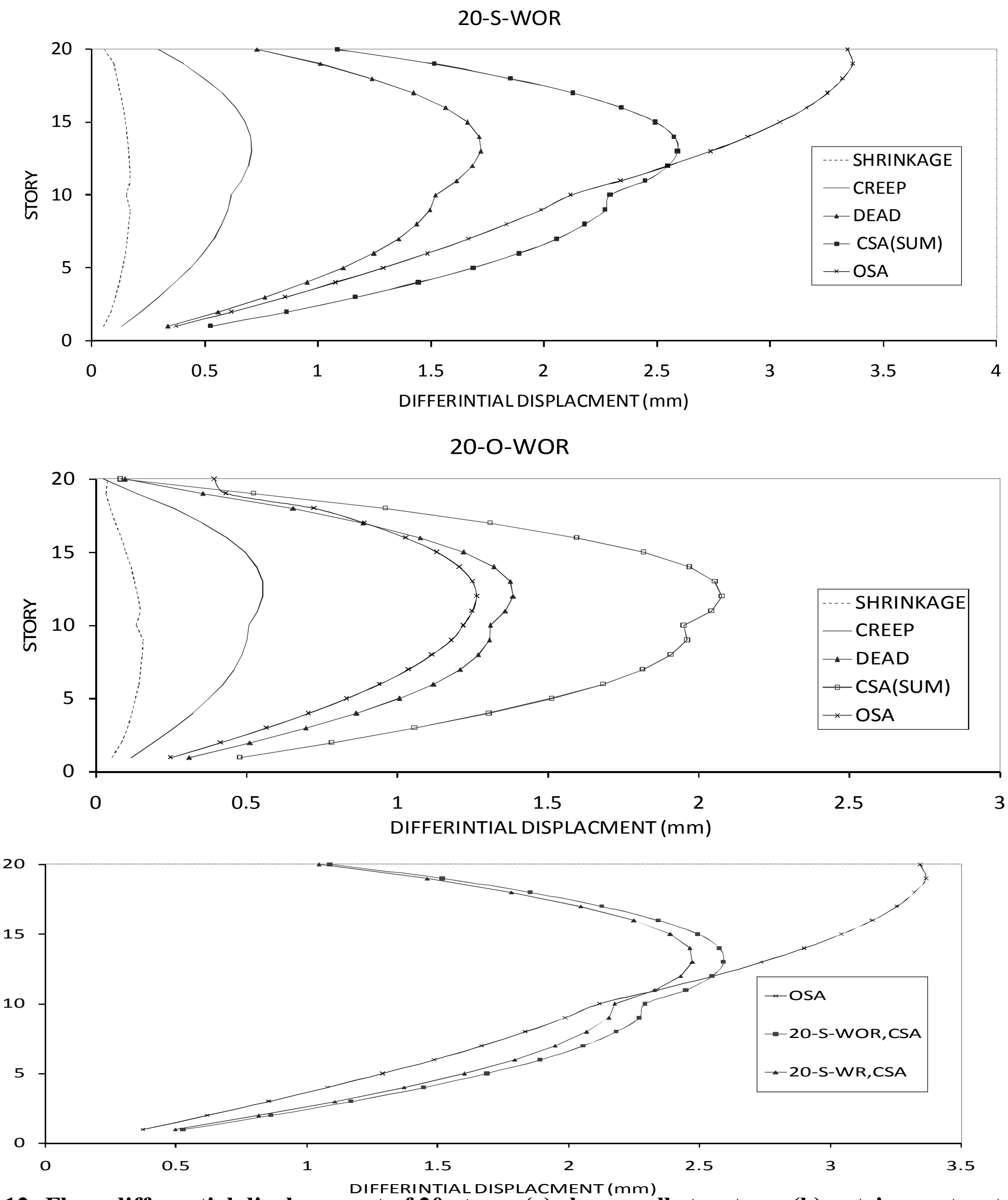


Figure 12: Floor differential displacement of 20-story: (a) shear wall structure, (b) outrigger structure, and (c) reinforcement effect for both shear wall and outrigger structure

4.2 Beams Straining Actions

When transferring axial forces from columns to shear walls, additional forces are introduced in horizontal beams such as bending moments and shear forces at Shear Wall. Figures no. 15&16 illustrates the absolute bending moment values and shear forces for both analysis methods. It is shown that at the lower 2/3 of building height, in CSA analysis results are more than OSA analysis results by (25%for20-story,40% for 40-Storey & 60% for 60-Storey), to (0.0%), at lowest storey and 2/3 of building height, respectively. These results vary almost linearly; hence the effect of CSA analysis can be estimated at any storey within the lower 2/3 of building height. At the higher 1/3 of building height, CSA analysis results are less than OSA analysis results by (0.0%) to (50% for 20-Storey, 60% for 40-Storey & 70% for 60- Storey); at 2/3 of building height and highest storey, respectively. These results can be considered varying according to 2nd degree polynomial equation.

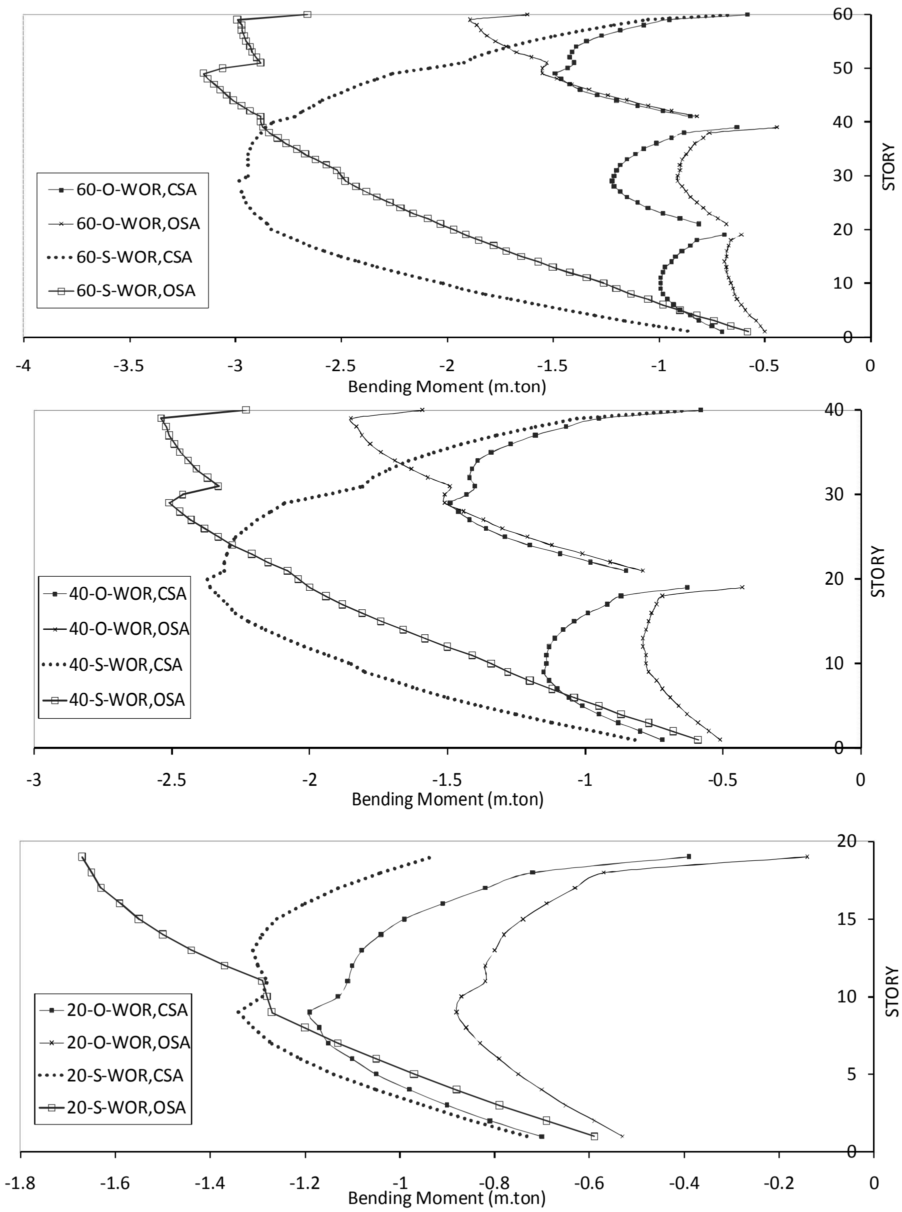
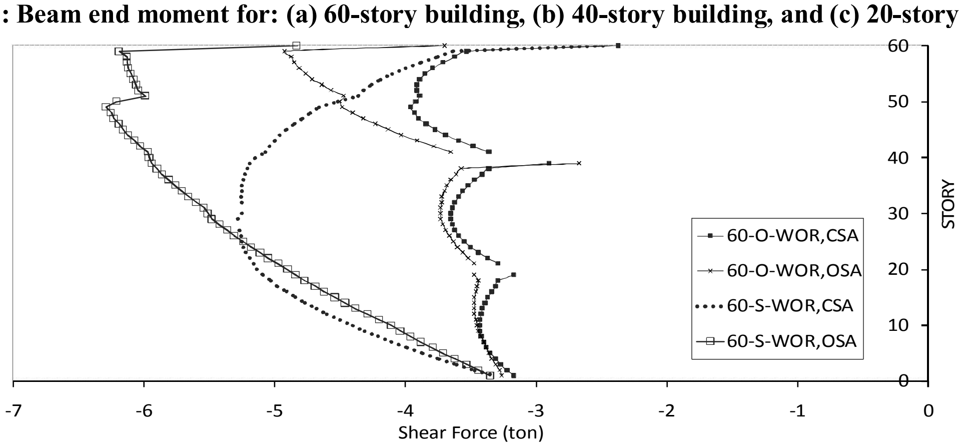


Figure 13: Beam end moment for: (a) 60-story building, (b) 40-story building, and (c) 20-story building



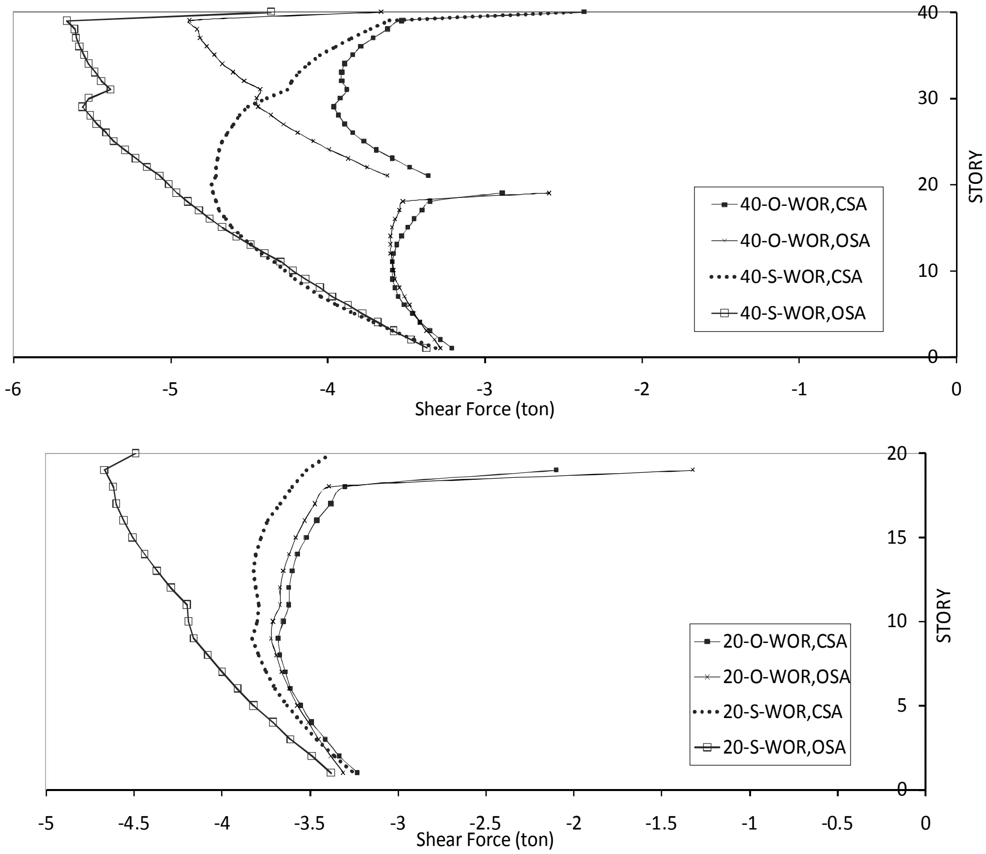


Figure 14: Beam end shear for: (a) 60-story building, (b) 40-story building, and (c) 20-story building

4.3 Columns axial force

The vertical deformation of columns are greater than that of shear walls, partial of the column axial forces are released through horizontal beams to the shear walls. Fig. 17 shows a typical column force distribution along height of the building for both a shear wall structure and an outrigger structure. However, analyses of all the cases show that the axial force redistribution between columns and shear wall is not significantly affected by the variation of floor levels. For shear wall structures, the difference between CSA & OSA in the axial forces of columns is less than 2.7% for 20-story buildings, 4.6% for 40-story buildings and goes up to 5.8% for 60-story buildings. In the case of outrigger structures, the existence of outriggers greatly change the forces in the columns, with each outrigger shifts significant amount of column forces to the shear walls (see Fig. 17), because of outriggers have large stiffness and tend to attract more loads, OSA underestimates the columns forces for an amount about 15% to 20%. Another problem associated with using OSA for outrigger structures is the column forces immediately below the outriggers. Taking Case 20-O-WOR for example in which case the outriggers are placed on the roof of the building, due to large stiffness of the outriggers, the exterior column behaves as a hanger column hanging from the outriggers under OSA analysis. As a result, tension forces are introduced in the exterior columns (in this case 7.50 ton tension force is observed). Apparently, this tension force is not realistic, since the outriggers are not in place until all lower column settlement has occurred (see Fig. 18).

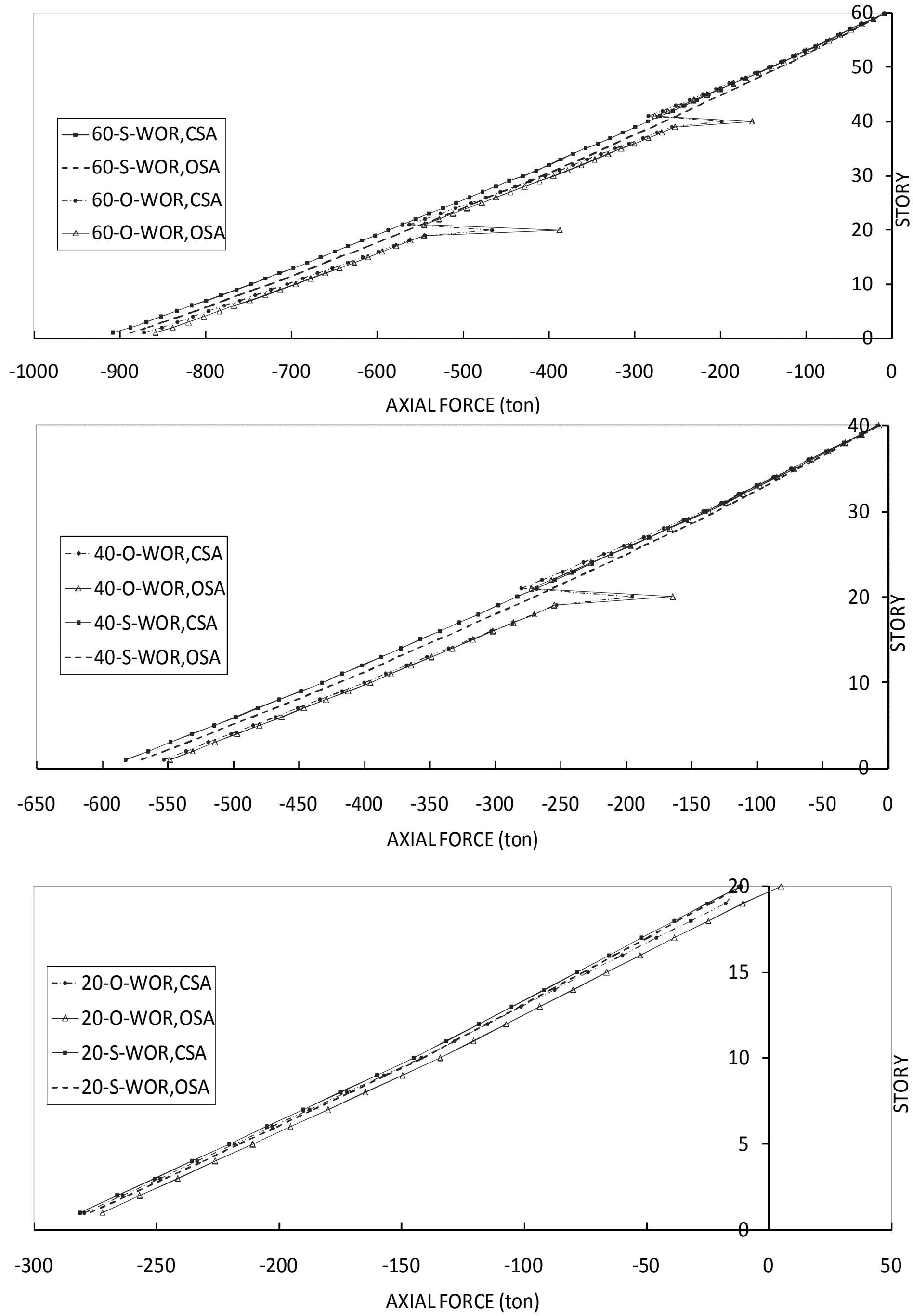


Figure 15: Column axial force for: (a) 60-story building, (b) 40-story building, and (c) 20-story building

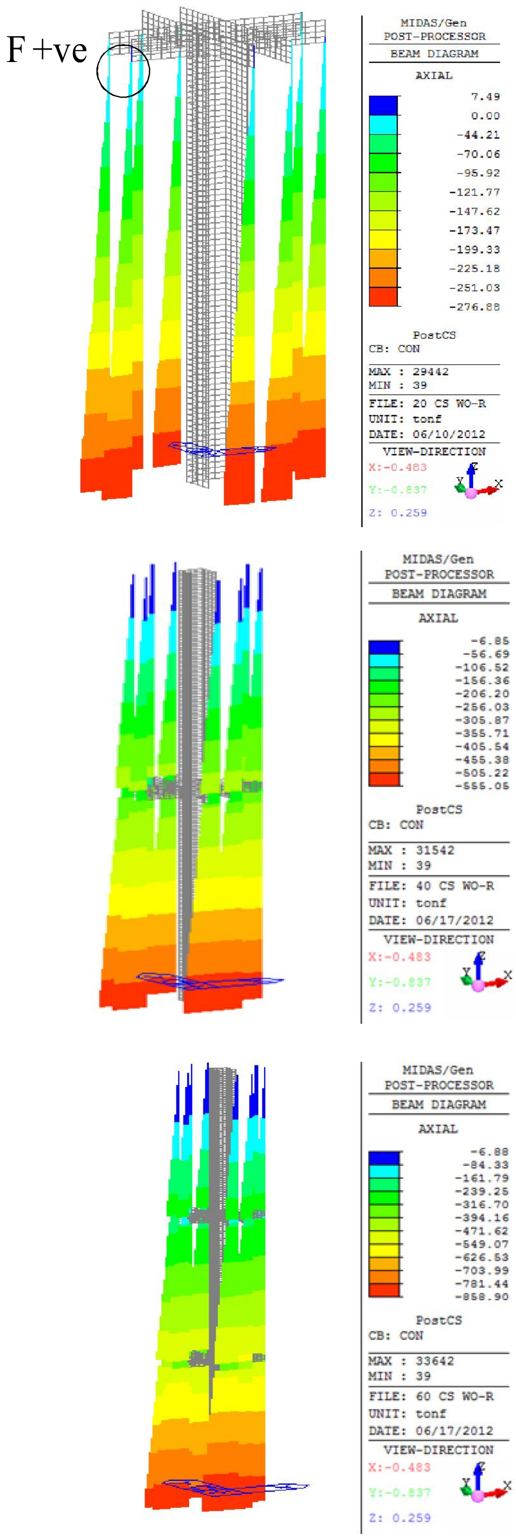


Figure 16: Exterior column axial force for: (a) 20-O-WOR, it behaves as a hanger column hanging from the outrigger, (b) 40-O-WOR, and (c) 60-O-WOR

1. Conclusions

Differential column shortening behavior in medium- to high- rise shear wall and shear wall plus outrigger structures is investigated, the main conclusions of the present study have been summarized in the following points:

1) For shear wall structures, differential column shortening between exterior columns and interior shear wall causes noticeable floor differential displacement, especially when the building exceeds 40 stories. In addition, it introduces considerable additional moments in horizontal beams, but has small effects on the variation of column forces.
2) In the case of outrigger structures, the effects of column shortening are close to those in the sub-structures isolated by the rigid outriggers. When the outriggers are placed at 20 story spacing, the floor differential displacement can be ignored. Similar to shear wall structures, column shortening in outrigger structures significantly increases the moments in horizontal beams, but has small effects on the force redistribution between Columns and shear walls.
3) Inclusion of steel reinforcement in analysis reduces the absolute differential displacements with maximum limit of 7%. Meanwhile, it doesn't reduce the differences between CSA & OSA along building height. Hence, it is not true that CSA can be avoided by having steel reinforcement.
4) Although, the construction stage analysis is inclusive for only the construction loads; its effect is comparable to that of service loads; This is because construction loads represent about 70% to 80% of the total service loads.
5) For high rise buildings, engineer should investigate carefully each individual floor within a complete model of the building. This should be whether there will be CSA analysis or not. Sometimes CSA analysis changes the direction of the straining action. Therefore, engineer should pay more attention.
6) Percentages of CSA analysis individual components: erection sequence, creep and shrinkage are totally different from straining actions to another Generally, erection sequence values (own weight) represent the maximum share.
7) In order to capture the effects of differential column shortening, it is recommended to use construction sequence analysis for concrete, since a one-step analysis for the complete building always erroneously estimates the behavior of column shortening.

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