

Multi Depot Multiple Traveling Salesman Problem Using Genetic Algorithm

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Abstract: The multi Depot multiple traveling salesman (MmTSP) is the normal status of multiple traveling salesman problem (mTSP) whereas there is more than one depot and multiple salesmen in each depot. The functional purpose of this problem includes minimizing all travels for each salesman as each salesman starts its own travel from one specific depot and returns to the same depot. Since this problem is related to NP-Hard, it is impossible to solve it in this real world. Thus; we used some Meta heuristic methods in order to achieve some approximate efficient results. We used Meta Heuristic standard genetic algorithm to solve this problem. Despite comparing the final results of limited problem with efficient result, the efficiency of parameters as well as the strategy of choosing are compared in large scaled in this Meta Heuristic method. In addition, the suggestive parameters (New-MX cross-over parameter) in genetic algorithm in an extremely large sizes (100 cities), large (150 cities), and extra large scale (200 cities) are compared separately, the results show the better combinations of UX#2 cross-over parameter and the selection strategy of elite parents in extra large scale, the combination of suggestive New-MX cross-over parameter, and the competitive binary strategy in large scale as well as the combination of PMX cross-over parameter with the binary selection strategy in an extremely large scale.

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1. Introduction

Today by the development of manufacturing industry, strong global competition among companies and corporations, the transient life cycle of goods, the necessary time for marketing as well as various needs of customers in different places with different distances from manufacturing place made some corporations in goods and raw material delivery or transiting companies promote their efficiency. These transiting and delivery systems must be able to deliver goods with the least cost in the least possible time to submit them to the customers on time. One more general form of the famous traveling salesman problem (TSP) is the multiple traveling salesman problems (mTSP) that consists of finding a collection of travels for m salesmen who start from one specific depot and return to it. In addition, the multi depots multiple traveling salesman problem (MmTSP) is the general form of single depot multiple traveling salesman problem whereas despite multi depots, there should be multiple salesmen in each depot. The required problem of this research is determining some specific numbers of travels for each traveling salesman as that salesman is obliged to return to the depot where they stated their travel. This problem is known as fixed depot traveling salesman problem.

Of the first heuristic methods for solving m travel in TSP was introduced by Russel [5] with some limitations. Although the method of solving was

based on converting the problem to simple TSP and developing it on a graph, its algorithm was the expanded model of heuristic method which was given earlier by Lin and Kernighan [10] for TSP.

The other heuristic method based on the exchange procedure for mTSP was introduced by Potvin et al [11]. The Parallelism Processing Approach for solving mTSP by the use of evolutionary programming was first framed by Fogel [12]. There are two salesmen with one purpose traveling this approach that minimizes the variance between the undertaken travels for each salesman. Some problems by the limitation of 25 or 50 cities were solved by this improving method which showed some suitable responses approximate to optimum. Wachodler et al [13] expanded Hopfield-Tank ANN model for mTSP. But this model is considered complex due to the disability to guaranty the feasible solutions [14]. Hsu et al [15] offered one Neural networks Approach for solving mTSP based on solving m problems of standard TSP. These writers pointed out that they achieved better results comparing with what Wachodler et al mentioned. One self-organized approach to neural networks for mTSP is related to Wakhotinsky and Golden [16] which was offered based on Elastic Net approach for TSP problem. Recently, Modares et al [17] was well as Somhom et al [14] introduced one self-organized NN approach for mTSP with one minimizing purpose travel that

minimizes the cost of the most expensive travel among salesmen.

Using genetic algorithm (GA) for solving mTSP was first offered by Zhang et al [1]. One recent uses of this is related to Tang et al [18] who used genetic Algorithm for solving the expanded model of mTSP to program hot rolling Scheduling. The solving is as follow: first, this problem is modeled in the form of mTSP. Then; this problem is converted to a simple TSP. Finally, one refined genetic algorithm is applied on it to find the respond. Yu et al [2] also used genetic algorithm for solving mTSP in designing the travels. Taboo search algorithm (TS) for mTSP was used by Ryan et al [3] by an open timing. These writers offered one Integer numerical programming formulation which was solved by TS algorithm in the format of discreet event stimulation.

Recently, Song et al [19] suggested simulated Annealing Approach for mTSP with fixed costs for salesmen. This approach was applied for mTSP for 400 cities and 3 salesmen that responded in a suitable time table. Gomes and Von Zuben [20] offered one method based on Neuro-Fuzzy system for solving mTSP used in capacitated VRP which is a network approach based on Fuzzy limitations. Sofge et al [21] performed and compared some developed algorithm for solving mTSP that consisted of neighborhood attractor schema, Shrink-Wrap algorithm for Local neighborhood Optimization, particle Swarm optimization, Monte-Carlo Optimization, Genetic Algorithm, and other developing strategies. Table 1 illustrated various solving methods of mTSP:

Table 1- mTSP Solutions

Problem approach	Solving methods
Exact solution methods	Linear programming formulas [4], [5] Cutting planes [6] Branch and bound [8], [28] Lagrange's method with branch and bound [7]
Heuristic methods	Simple Heuristic methods [9], [11] Evolutional algorithms [12] Simulated Annealing (SA) [19] Taboo search (TA) [3] * Genetic algorithms (GA) [1], [2], [18] Neural networks (NN) [13], [14], [15], [16], [17]
Converts	Asymmetric mTSP to asymmetric TSP [22] Symmetric mTSP to symmetric TSP [23], [24], [25] MmTSP to TSP [26], [27]

2. Material and Methods

Input parameters:

cij: Distance between i and j cities

- N: the number of total locations
- M: The number of salesmen
- d: the number of depots
- Mk: the number of salesmen in k depot
- V: The total locations (cities)
- V': The total intermediate locations (intermediated cities)
- D: Total depots
- L: the maximum number if intermediate cities for each salesman to visit
- K: the minimum number of intermediate cities for each salesman to visit
- ui: The total visited cities for each salesman before arriving to i city (including depot)

Statement of the problem

Consider the complete directed graph $G = (V, A)$ in which V is the total n points (vertices), A is the total arcs, and $C = (c_{ij})$ is the cost matrix (distances) for each arc $(i, j) \in A$. The cost matrix C could be symmetrical, asymmetrical or Euclidean. The total set of points is divided as $V=V'UD$ as d is the first point from V, makes the set of depots (D). At first, there are mk salesmen in k depot while the total set of salesmen is m. the total intermediated points (cities) includes:

$$V' = \{d+1, d+2 \dots n\}$$

indicated the total points which exist along the travel from depot to i point for each salesman (concluding depot), and L is the highest number if points where a salesman must visit. Thus, for each $i \geq 2$; we have: $1 \leq u_i \leq L$

k is also the least numbers of points where one salesman must visit. It means that if $x_{ijk} = 1$, $K \leq u_i \leq L$ must be sustained. In figure 1, one mTSP problem with 5 cities and 2 salesmen is shown. Also in figure 2, one mTSP problem with 10 cities and 4 salesmen, 2 depots, and 2 salesmen in each depot is illustrated which indicated one response with 4 travels (one travel for each salesman):

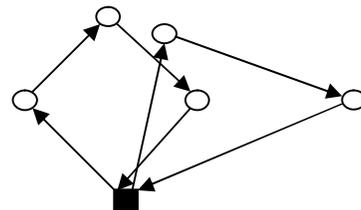


Figure 1: mTSP with 5 cities and 2 salesmen

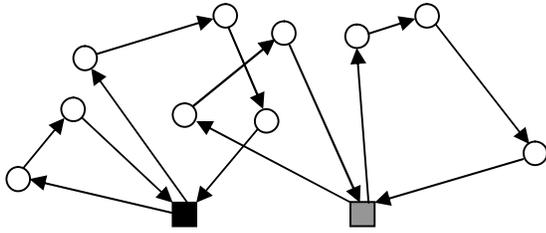


Figure 2: mTSP with 10 cities and 4 salesmen, 2 depots,

Formulation

This formulation for some traveling salesmen with fixed depots and destinations are as follow:

This formulation is valid in the case that $2 \leq K \leq \lfloor n/m \rfloor$ and $K \leq L \leq n - (m-1)K$ is

$$\text{Minimize } \sum_{k \in D} \sum_{j \in V'} (c_{kj} x_{kjk} + c_{jk} x_{jkk}) + \sum_{k \in D} \sum_{i \in V'} \sum_{j \in V'} c_{ij} x_{ijk}$$

S.t.

$$\sum_{j \in V'} x_{kjk} = m_k, \quad k \in D,$$

$$\sum x_{kjk} + \sum \sum x_{ijk} = 1, \quad j \in V',$$

$$x_{kjk} + \sum_{i \in V'} x_{ijk} - x_{jkk} - \sum_{i \in V'} x_{jik} = 0, \quad k \in D, \quad j \in V',$$

$$\sum_{j \in V'} x_{kjk} - \sum_{j \in V'} x_{jkk} = 0, \quad k \in D,$$

$$u_i + (L-2) \sum_{k \in D} x_{kik} - \sum_{k \in D} x_{ikk} \leq L-1, \quad i \in V'$$

$$u_i + \sum_{k \in D} x_{kik} + (2-K) \sum_{k \in D} x_{ikk} \geq 2, \quad i \in V'$$

$$\sum_{k \in D} x_{kik} + \sum_{k \in D} x_{ikk} \leq 1, \quad i \in V',$$

$$u_i - u_j + L \sum_{k \in D} x_{ijk} + (L-2) \sum_{k \in D} x_{jik} \leq L-1, \quad i \neq j,$$

$$x_{ijk} \in \{0,1\}, \quad i, j \in V' \quad k \in D.$$

$m \geq 1$ and integer

feasible. Thus, one salesman can not leave one depot, visit only one city, and returns to the same depot. As the result, the number of cities one salesman must visit without considering the depot, must be at least 2 cities. The first limitation guarantees that m_k salesmen exactly exist each $K \in D$ depot. The second limitation indicated that each point (city) is exactly visited once.

The unity of travels for intermediate points and depots are illustrated by the third and fourth limitation respectfully. Limitations (5) and (6) apply the upper and lower limits of visited points to the travels. In general, if i is the first point in each travel, these limitations set u_i equals to 1. The travels with only one intermediated point are prohibited by (7) limitation. Finally, (8) limitation is subtour elimination constraint (SEC) that avoids the existence of any sub-travels (under-travel) among intermediated points. These sub-travels are some blocked travels which are made without any depot as start or end point by intermediate points. These could appear in the case that there would be no suitable SEC in responses. It obviously appears that this problem is related to NP-Hard. There is some elimination in terms of the number of $O(dn^2)$

binary variable as well as $O(n^2)$ in this formulation. One Genetic Algorithm for solving the Mathematical Model of MmTSP with Stable Depots and Destination

The purpose is to formulate one mathematical model of MmTSP with stable depots and destinations based on specific Genetic Algorithm. Hence, there will be one almost efficient response for medium average problems and one suitable response in one logical time for larger problems since we are not able to find an optimum response. In addition, we studied and compared various response parameters of genetic algorithm. Six basic parameters must be identified for each genetic algorithm at first. Each formulated requires its own parameters genetically. The major parameters for the genetic algorithm specified for MmTSP model with stable depots and destinations are considered as follow:

The partially matched crosses over Parameter (PMX):

In this parameter, two Childs are provided by two selected parents as one random place is chosen from two parents. Then, the genes related to the parents are substituted to be replaced in two Childs. The related number of this substituted gene in 1 parent as well as the related number of gene in the 2 parent is substituted with each other to be planted in saves. This practice is performed for almost half of the parents.

As an example in figure 4, the permutation number 4 was selected in the parent's chromosome and the related numbers of genes (7 and 9) are substituted. Then, number 9 in the first parent found to be substituted by number 7 in Second parent.

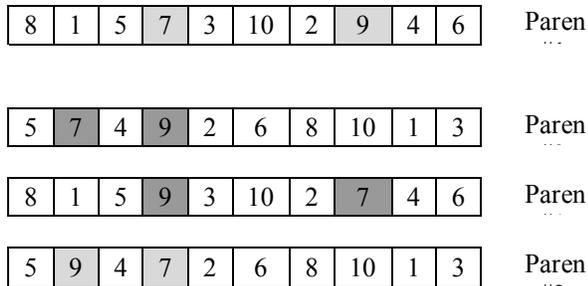


Figure 3- The partial matched crosses over Parameter (PMX)

Union Cross-over #2 parameter (UX#2):

In this parameter, one salve is produced by two parents as one sub-branch of the available genes in parent 2 is planted under the sub-branch S1 first. Then, the rest the parent genes are planted in sub-branch S2 like the 1 parent. One of the sub-branches is chosen randomly. Next; the first available gene in the selected sub-branch is planted in the stable to delete it from sun-branch. This process is continued till all the available genes under these two sub-branches would be planted in two Childs. As an example, we have the same process in figure 5:

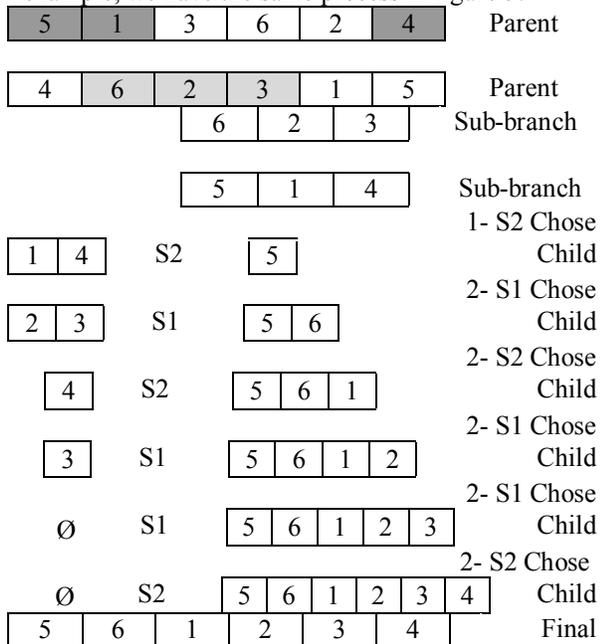


Figure 4- Union Cross-over #2 parameter (UX#2)

Mutation Parameter:

Parameters 1 bit and 2-opt are used as mutation parameters for the production of generation randomly along with all cross-over parameters. The illustration of 1-bit and 2-opt parameters along the travels are provided in figure7. As you have noticed, in 1-bit method, two travels from two salesmen are chased

randomly from one parent. Then, one point is deleted from the travel to be added to the other travel. In 2-opt method, two travels from two salesmen are chosen from one parent randomly, and two points (cities) of two travels are chosen randomly to be substituted.

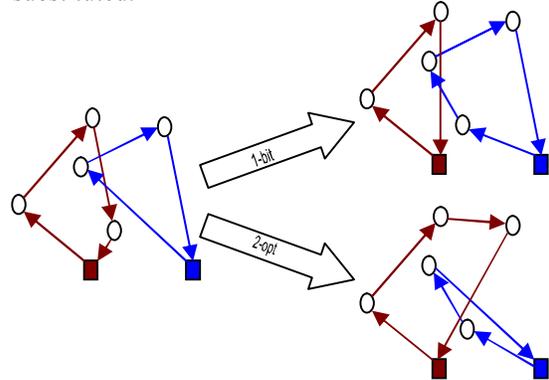


Figure 5- 1 bit and 2-opt Mutation Parameter

3. Results

Comparing the results of optimum solution by branch and bound methodology with Lingo v.8 software and the genetic algorithm result for the submission purpose of total distances is illustrated in table 3. The final column of the table shows the variance percentage between optimum solution and GA responses which is measured by equation (11).

$$Gap\% = \frac{GA(fitness) - Lingo(fitness)}{Lingo(fitness)} \times 100$$

Table 3- Comparing the results of optimum solution and the genetic algorithm

ID	Real results		GA results		Gap%
	Quantity	Time (s)	Quantity	Time (s)	
S1	542	223	546	124	0.74
S2	639	70	648	127	1.25
S3	749	317	772	161	3.07
S4	774	451	797	165	2.97
S5	861	137	896	227	4.06
S6	891	242	930	241	4.37

Considering the results of table 3, it is clarified that we can achieve to some results quite approximate to optimum results in medium scale problems by the help of genetic algorithm in a shorter time though this contiguity of responses will eliminate by increasing the problem scale (figure 8). Moreover, the comparing of the time of solving problems by genetic algorithm and optimum solution by branch and bound is illustrated in figure 9.

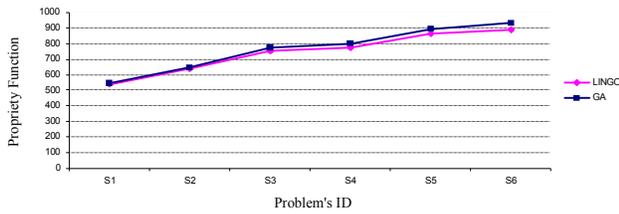


Figure 8- Comparing the GA's propriety function vs. optimum solution in average scale problems

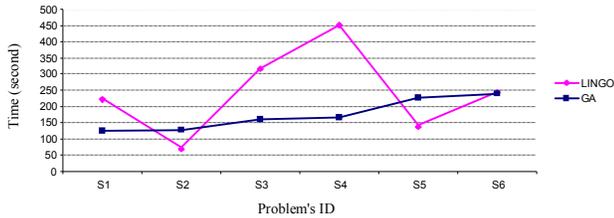


Figure 9- Comparing the GA's time vs. optimum solution in average scale problems

In this section, we were able to compare the results achieved by GA and optimum solution by the capability of Lingo in solving the problems in this scale. For larger sale problems, the most important effective parameters in the explained genetic algorithm such as cross-over parameters as well as selections of parents are compared. According to the results of genetic algorithm, these two parameters (cross-over parameter and selection strategy of parent selection), were more effective comparing with other parameters in GA. Any changes in them resulted some more changes in the algorithm responses.

Comparing Cross-over parameter and Parent selection strategy in GA

In this section, the results of the performing GA by cross-over parameters and selection strategy of different parents are offered. The considered cross-over parameters include:

New-MX, OX, PMX and UX#2.

The parent selection strategy also includes:

Elitism strategy, Roulette wheel and Tournament Selection strategy

Comparing the results generally with all cross-over parameters and selection strategies for the problems with one similar scale, the mean propriety for the problems in largish scale, large scale, and extra large scale are shown in 10,11 and 12 graphs based on the kind of cross-over parameter and parent selection strategy.

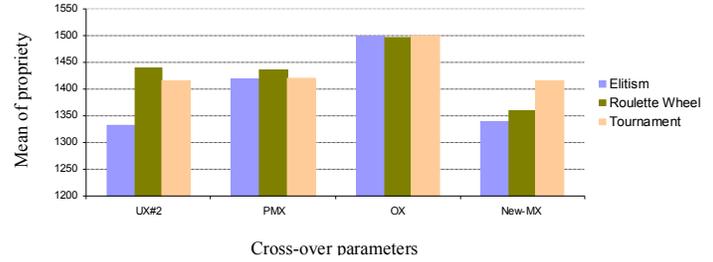


Figure 10- Comparing the propriety function of parameters with Various selection strategies in largish scale problems (n=100)

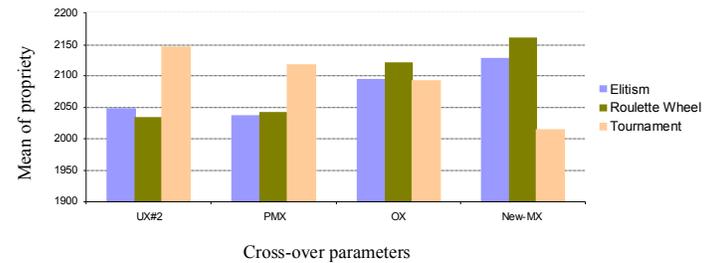


Figure 11- Comparing the propriety function of parameters with Various selection strategies in large scale problems (n=150)

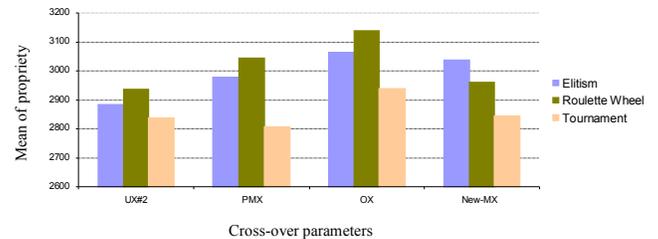


Figure 12- Comparing the propriety function of parameters with Various selection strategies in extra large scale problems (n=200)

Considering figure 10, it is shown that cross-over parameter UX#2 with elitism parent selection strategy has the best mean propriety submission in largish scale problems (problems with M1, M2 and M3 indicators). As the result, the best function of GA explained in largish scale problems is UX#2 cross-over parameter and elitism parent selection strategy along with other shared parameters described in the sixth part. Similarly, figure 11 can depot the New-MX cross-over parameter with tournament parent selection strategy has the best mean propriety in large scale problems (problems with L1, L2 and L3 indicators). Thus, The best performance of GA in terms of large scale problem is New-MX cross-over parameter and tournament parent selection strategy

along with other shared parameters described in part six. Figure 12 also illustrates that PMX cross-over parameter with tournament selection strategy has the best mean propriety in extra large scale problem (problems with E1, E2 and E3 indicators). As the final result, the best performance of GA in terms of extra large scale problems is PMX cross-over parameter and tournament parent selection strategy along with other shared parameters described in part six.

conclusion

In this chapter, Genetic algorithm was first introduced as one of randomly search methods. These algorithms search the sample problems by the help of performing its parameters on group of possible solutions for finding approximate optimum solutions. The important parameters in creating GA are the mode of chromosome illustrations based on the given model, the method of designing the parameters as well as parent selection strategy. Then, the models related to MmTSP with fixed depots and destinations are offered. The purpose of this model is finding the possible travels for each salesman as to minimize the total undertaken distances by each salesman. Then, one heuristic genetic algorithm was offered for solving this problem. We noticed that evaluating the achieved results, Performing GA comparing with optimum results by branch and bound methodology had a small difference in small and medium scale problem. The time was also better than the achieved time of optimum solution. Although this exact method is not able to achieve to optimum result gently by increasing the scale of the problem, the presented genetic algorithm can achieved some acceptable results for problems with the scale of 350 cities.

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