Radiation of the Transient Electromagnetic Field above a plane, Non –Conducting, Earth

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Abstract: The purpose of this paper is to find out the electric and magnetic fields which exist in the medium by solving Maxwell's equations. The solution of this problem is facilitated by the introduction of the polarization potential. We shall now see that the whole electromagnetic field may be described by means of a single vector $\vec{\Pi}(r)$ or $\vec{\Pi}^*(r)$. .If the potential is known the electric and magnetic vectors are readily calculated for both sources .The Hertizian vedror corresponding to the reflected wave is expressed in terms of a single integral over a finite interval. The resulting electromagnetic field in the air is determined, it consist of a reflected wave which is superimposed upon the given incident wave. This integral is written in which a form of its numerical evaluation which can be easily performed.

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1. Introduction

Historically, the problem of electromagnetic radiation from a vertical magnetic dipole situated at a certain height h above a plane earth, all field quantities are usually assumed to a vary harmonically in time .One of the them well – known method for solving this steady – states problem is due to Sommerfeld [1], calculated the electromagnetic radiation from an electric vertical dipole, located above the plane interface of two media.

Many authors, Wait [2]; Moore [3] and Durrani [4] have considered this problem, the aim of the present work is to extend the study – state to transient excitation when no restrictions on the distance between receiving and transmitting ends are made. two integral transforms are applied to analyze the transient field of vertical electric dipole above a dielectric layer the distinction of different cases where the distance between the receiving and transmitting end are greater and lesser than the total reflection distance studied Abo Seliem [5] .

The problem has been studied by Arutaki and Chiba [6] and Abo Seliem [7] .This Integral is estimated by using the steepest descent method, along the count our Γ and around the branch –cuts, from the obtained results the Saddle point method show that the reflected waves and integrals Abo Seliem [8], the component of electric field strength is also arbitrary for the excitation function F (t) = t at some fixed.

The analyzes propagation of wideband electromagnetic (EM) wave in the ionosphere. The influence of total electron contents (TEC) of the ionosphere on the propagation of EM wave is investigated numerically. Our attention is paid to very high frequency (VHF) band, which is dominant frequency band of EM waves emitted from lightning discharges. A one-dimensional model of the ionosphere is considered for simplicity. The ionosphere is treated as an anisotropic and dispersive medium. Particularly, the altitude distribution of electron density is taken into consideration. Variation of pulse width and difference in the arrival time between each frequency component due to the dispersion in the ionosphere are revealed. For our numerical investigation, group delay of the EM wave is found to be dependent on the altitude distribution of electron density by T. Kahan, G .Eckart, [9] and D.S. Jones, [10].

2. Formulation of the problem

When the electromagnetic field is a prescribed distribution of oscillatory electric dipole moments per unit volume and activated by an external source as given by the vector field $\vec{P}(r)$ .We can write:-

$$\vec{D}(r) = \varepsilon\varepsilon_0 \vec{E}(r) + \vec{P}(r) \tag{1}$$

$$\vec{B}(r) = \mu\mu_0 \vec{H}(r) \tag{2}$$

where $\vec{E}(r)$ is the electric field intensity (in volts/meter) , $\vec{H}(r)$ is the magnetic field intensity (in amperes/meter) , $\vec{B}(r)$ is the magnetic induction, or the magnetic flux density (in webers /meter^2) and

$\vec{D}(r)$ is the electric flux density or electric displacement (in coulombs / meter^2) . The source distribution of the electric polarization $\vec{P}(r)$ is confined to a finite volume with in a finite distance from the arbitrary origin of coordinates. For our purpose, however, it is more convenient to introduce the electric current density $\vec{J}(r)$ in the medium (in amperes / meter ^{3}) as defined by :-

$$\vec{J}(r) = -i\omega\,\vec{P}(r) \qquad (3)$$

Where, ω is angular frequency, The Maxwell's equations are:-

$$\nabla \wedge \vec{E}(r) = i\omega\mu\mu_0\,\vec{H}(r)$$

$$\nabla \wedge \vec{H}(r) = (\sigma - i\omega\varepsilon\varepsilon_0)\,\vec{E}(r) + \vec{J}(r) \qquad (4)$$

$$\nabla \cdot \vec{H}(r) = 0$$

$$\nabla \cdot \vec{E}(r) = \frac{\nabla.\vec{J}(r)}{(\sigma + i\omega\varepsilon\varepsilon_0)}$$

Where ε, μ, σ , respectively, are the dielectric permittivity (in farads / meter), the magnetic permeability (in henries /meter), and electric conductivity (in mhos /meter) of the medium, where ε_0, μ_0 are the dielectric permittivity and the magnetic permeability of vacuum, respectively are equal to

$\varepsilon_0 = 8.85 \times 10^{-12}$ F / m., $\mu_0 = 4\pi \times 10^{-7}$H / m .The product $\varepsilon_0\mu_0$ has the dimensions of (velocity)$^{-2}$, its value is $\varepsilon_0\mu_0 = \dfrac{1}{c^2}$

Where $c = 2.998 \times 10^8$ m / s

Thus the propagation constant k for plane homogenous wave as given by :-

$$k^2 = i\omega\mu\,(\sigma - i\omega\varepsilon) = i\omega\mu\sigma + \omega^2\varepsilon\mu \quad (5)$$

the quantity k is called the complex wave number or the propagation parameter. Equations (4) are readily verified to admit the solution: -

$$\vec{E}(r) = graddiv\ \vec{\Pi}(r) + k^2\,\vec{\Pi}(r) \quad (6)$$

$$\vec{H}(r) = \frac{k^2}{i\mu\mu_0\omega}\,curl\ \vec{\Pi}(r) \qquad (7)$$

In terms of the Hertz vector $\vec{\Pi}(r)$ or electric polarization that $\vec{\Pi}(r)$ satisfies the inhomogeneous Helmholtz vector equation.

$$(\nabla^2 + k^2)\,\vec{\Pi}(r) = \frac{i\mu\mu_0\omega}{k^2}\,\vec{J}(r) \qquad (8)$$

Where ∇^2 is the Laplace operator, Equation (8) exhibits the prescribed electric current density distribution $\vec{J}(r)$ as the sole source of the electromagnetic field. We can write the solution of (8) for an unbounded homogenous medium in the form

$$\vec{\Pi}(r) = \frac{i\mu\mu_0\omega}{4\pi k^2}\int_V \frac{e^{-iR}}{R}\,\vec{J}(r)\,dV \qquad (9)$$

Where $\vec{J}(r)$ denotes the impressed current density vector .Similarly, it can be shown that the fields associated with the vector $\vec{\Pi}^*(r)$ is set up by a distribution of the magnetic polarization and is known as the Fitzgerald vector. From the density $\vec{\Pi}(r)$ and $\vec{\Pi}^*(r)$ we can write Maxwell's equations in the form:-

$$\begin{aligned}
&\nabla \wedge \vec{E}(r) = i\mu\mu_0\omega\,\vec{H}(r) - \vec{M}(r)\\
&\nabla \wedge \vec{H}(r) = (\sigma - i\omega\mu\mu_0)\,\vec{E}(r)\\
&\nabla \cdot \vec{H}(r) = \frac{\nabla \cdot \vec{M}(r)}{i\omega\mu\mu_0}\\
&\nabla \cdot \vec{E}(r) = 0
\end{aligned} \qquad (10)$$

Which is readily verified to admit the solution

$$\vec{E}(r) = i\omega\mu\mu_0\nabla \wedge \vec{\Pi}^*(r) \qquad (11)$$

$$\vec{H}(r) = \nabla(\nabla \cdot \vec{\Pi}^*(r)) + k^2\,\vec{\Pi}^*(r) \qquad (12)$$

A previously mentioned, the Hertz vector satisfies equation (2.8), but in addition, we can assert Fitzgerald vector or magnetic polarization Helmholtz vector equation

$$(\nabla^2 + k^2)\,\vec{\Pi}^*(r) = \frac{-\vec{M}(r)}{\mu\mu_0} \qquad (13)$$

Equation (13) exhibits the prescribed magnetic current density distribution $\vec{M}(r)$ as the sole source of the electromagnetic field .We can write the solution of (13) for infinite homogenous medium in the form

$$\vec{\Pi}^{*}(r) = \frac{1}{4\pi\mu\mu_0\omega}\int_V \frac{e^{-iR}}{R}\vec{M}(r)dV \qquad (14)$$

Where $\vec{M}(r)$ is the distribution of the impressed magnetic current density enclosed by the volume V, In an isotropic and homogeneous medium , the physical properties in the neighborhood of an interior point are same in all directions and are constant from point to point , respectively . In this medium:-

$$\vec{D} = \varepsilon\varepsilon_0\vec{E}, \qquad (15)$$

$$\vec{B} = \mu\mu_0\vec{H} \qquad (16)$$

These are the basic equations for the propagation of electric and magnetic field vectors in an isotropic, homogeneous medium with physical properties $(\varepsilon, \mu, \sigma)$.

Maxwell's equations are coupled first – order differential equation which are difficult to apply when solving boundary- value problems . The difficulty is overcome by decoupling the first –order equation , thereby obtaining the wave equation ,a second-order differential equation which is useful for solving problems .To obtain the wave equation a linear , isotropic , homogeneous , source – free medium ($J = 0$, $\rho_v = 0$) .The time – varying electromagnetic fields are governed by physical laws expressed mathematically as :

$$\nabla\wedge\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad (17)$$

$$\nabla\wedge\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \qquad (18)$$

$$\nabla\cdot\vec{B} = 0 \qquad (19)$$

$$\nabla\cdot\vec{D} = \rho_v \qquad (20)$$

Where ρ_v is the volume charge density (in coulombs / meter ^{3}) .We take the curl of both sides of equation (17) this gives:-

$$\nabla\wedge\nabla\wedge\vec{E} = -\mu\frac{\partial}{\partial t}(\nabla\wedge\vec{H}). \qquad (21)$$

Where $\vec{B} = \mu\vec{H}$, from (18)

$$\nabla\wedge\vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t} \qquad (22)$$

Where $\vec{D} = \varepsilon\vec{E}$

From equations (21) and (22) we get:-

$$\nabla\wedge\nabla\wedge\vec{E} = -\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} \qquad (23)$$

Applying the vector identity in equation (23)

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} \qquad (24)$$

Since $\rho_v = 0$, $\nabla\cdot\vec{E} = 0$ we obtain:-

$$\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0 \qquad (25)$$

Which is the time - dependent vector Helmholtz equation or simply wave equation .If we had started the derivation with equation (18), we would obtain the wave equation for $\vec{H}$ as :-

$$\nabla^2\vec{H} - \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2} = 0 \qquad (26)$$

Equations (25) and (26) are the equations of propagation of electromagnetic waves in the medium under consideration .These wave are represented by the coupled $\vec{E}$ and $\vec{H}$ fields. The velocity of wave propagation is:-

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \qquad (27)$$

3 -Boundary Conditions

The material medium in which an electromagnetic field exists is usually characterized by its constitutive parameters σ, ε and μ . The medium is said to linear if σ, ε and μ are independent of $\vec{E}$ and $\vec{H}$ or nonlinear otherwise. It is homogeneous if σ, ε and μ are not function of space variables or inhomogeneous otherwise. It is isotropic if σ, ε and μ are independent of direction (scalar) or anisotropic otherwise. The boundary condition at the interface separating two different media with parameters $(\sigma_1, \varepsilon_1, \mu_1)$ and $(\sigma_2, \varepsilon_2, \mu_2)$ as shown in Fig. (1)

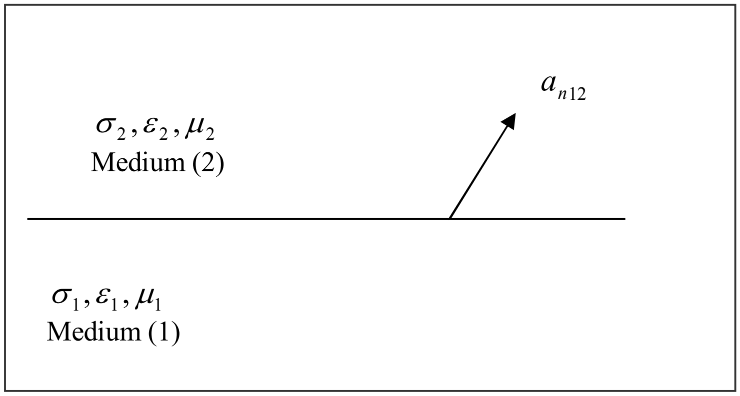

Fig. (1) Interface between two media.

is easily derived from the integral form of Maxwell's equations. They are:-

$$\vec{E}_{t1} = \vec{E}_{t2} \text{ or } (\vec{E}_{t1} - \vec{E}_{t2}) \times \mathbf{a}_{n12} = 0 \quad (28)$$

$$\vec{H}_{t1} - \vec{H}_{t2} = \eta \text{ or } (\vec{H}_{t1} - \vec{H}_{t2}) \times \mathbf{a}_{n12} = \eta \quad (29)$$

$$\vec{D}_{n1} - \vec{D}_{n2} = \text{or} (\vec{D}_{n1} - \vec{D}_{n2}) . \mathbf{a}_{n12} = \rho_S \quad (30)$$

$$\vec{B}_{n1} - \vec{B}_{n2} = 0 \text{ or } (\vec{B}_{n1} - \vec{B}_{n2}) . \mathbf{a}_{n12} = 0 \quad (31)$$

where $\mathbf{a}_{n12}$ is a unite normal vector directed from medium 1 to medium 2 subscripts 1 and 2 denote field in region 1 and 2, and subscripts t and n respectively denote tangent and normal component of the fields .The physical meaning of Equations (.28) and (.31) is that the tangential components of $\vec{E}$ field as well as of normal components of $\vec{B}$ are continuous at the boundary. Equations (.29) states that the tangential component of $\vec{H}$ is discontinuous by the surface current density η on the boundary Equation (.30) states that the discontinuity in the normal component of $\vec{D}$ is the same as the surface charge density ρ_S on the boundary. In practice, only two of Maxwell's equations are used equations (.17) and (18) when a medium is source - free. Also, in practice, it is sufficient to make the tangential components of the fields satisfy the necessary boundary conditions since the normal components implicitly satisfy their corresponding boundary conditions, isotropic media in this text. L.M.Brekhovsikh, [11]

4-The scalar wave generated by an impulsive line source Inside the Ionosphere (in two dimensional).

Let x, y, z be Cartesian coordinates in three-dimensional space .A point in space will be located by either its Cartesian coordinates, its cylindrical coordinates r, ϕ, z defined through.

$$x = r\cos\phi, \qquad y = r\sin\phi, \qquad z = z \quad (32)$$

with

$$0 \prec r \prec \infty, \qquad 0 \prec \phi \prec 2\pi, \qquad -\infty \prec z \prec \infty,$$

or its spherical polar coordinates defined through

$$x = R\sin\theta\cos\phi, \qquad y = R\sin\theta\sin\phi, \qquad z = R\cos\theta \quad (33)$$

with

$$0 \le R \prec \infty \qquad 0 \le \theta \le \pi, \qquad 0 \le \phi \prec 2\pi$$

The two dimensional wave function $u = u(x, y; t)$ due to the presence of two–dimensional line source acting at $x = 0$, $y = 0$ satisfies of two– dimensional scalar wave equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = -\delta(x, y) f(t) \quad \mathbf{(34)}$$

Where $\delta(x, y)$ denotes the two – dimensional delta function and v is the wave velocity . f (t) is the function which determines the strength of line source as a function of time ; it is assumed that f (t) = 0 when $t \prec 0$. Further , it is assumed that the medium is at rest prior to the instant $t = 0$ and that everywhere outside the source $u = u(x, y; t)$ is continues and has continues partial derivatives of the first and second order .Follow Canard, all functions of time are subjected to a one-sided Laplace transform with respect to time .

Definition 1*.Let f (t) is a function on [0, ∞]. The Laplace transform of f is the function F defined by the integral*

$$F(s)=\int_0^\infty \exp(-st)f\ (t)dt \tag{35}$$

The domain of F (s) is all the values of s for which the integral in (34) exists. The Laplace transform of f is denoted by both F and L $\{f\}$.

We treat s as real valued out in certain applications s may be a complex variable.

$$u(x,y,s)=\int_0^\infty \exp(-st)u\ (x\ ,y\ ,t\ )dt \tag{36}$$

Where s is real ,positive, number large to ensure the convergence of the integral (34) and (35) it is assumed that the behavior of f (t) and $u(x,y,t)$ as $t\to\infty$ such that such a number s can be found since u and $\frac{\partial u}{\partial t}$ are continues, $u(x,y;s)$ satisfies the differential equation .

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{s^2}{v^2}u=-\delta(x,y)F(s) \tag{37}$$

In order to solve (36) we introduce the Fourier transform of $u(x,y;s)$ with respect to x let

$$\Pi\,(x,y;s)=\int_{-\infty}^{\infty}\exp(\ is\,\alpha x)u(x,\ y;\ s)dx \tag{38}$$

where $i=\sqrt{-1}$ and the factor s in the argument exponential function has been included for convenience with (38) the following equation for $\Pi(x,y;s)$ is obtained

$$\frac{d^2\Pi}{dy^2}-s^2\gamma^2\Pi=-\delta(y)F(s) \tag{39}$$

$$\gamma=\gamma(\alpha)=\sqrt{(\alpha^2+\frac{1}{v^2})}\qquad (\mathrm{Re}\,\gamma\ge 0) \tag{40}$$

As indicated in (40), γ is defined as that branch of the square root at the right-hand side of (40) for which Re γ ≥ 0. The solution of (39) that is bounded as $|\ y\ |\to\infty$ is given by

$$\Pi(x,y;s)=\frac{F(s)}{2s\gamma}\exp(-s\gamma|y|) \tag{41}$$

with the aid of Fourier's inversion theorem we then obtain for $u(x,y,s)$ the expression

$$u(x,y;s)=\frac{F(s)}{2\pi}\int_{-\infty}^{\infty}\exp(-is\alpha x-s\gamma|y|)\frac{1}{2\gamma}d\alpha \tag{42}$$

In the right – hand side of (12) we write $\alpha=-ip$ and consider p as a complex variable in the p -plan. This leads to:-

$$u(x,y;s)=\frac{F(s)}{2\pi i}\int_{-i\infty}^{i\infty}\exp[-s(px+\gamma|y|)]\frac{1}{2\gamma}dp \tag{43}$$

In which

$$\gamma=\sqrt{(\frac{1}{v^2}-p^2)}\qquad \mathrm{Re}\,\gamma\ge 0 \tag{44}$$

The only singularities of the integrand in (43) are branch points at $p=\frac{1}{v}$, and $p=-\frac{1}{v}$. In view of subsequent deformations of the path of integration we take Re γ ≥ 0, everywhere in the p -plane . This implies that branch cuts are introduced along Im p = 0, $\frac{1}{v}<|\mathrm{Re}\ p\ |<\infty$.

The next step towards the solution of the transient problem is to perform the integration in the p -plane along such a path that the right - hand side of (43) can be recognized as the Laplace transform of a certain function of time. The analysis which follows will show that the path has to be selected such that

$$px+\gamma\ |\mathrm{y}|=\tau \tag{45}$$

Where τ is real and positive? If $\frac{r}{v}<\ \tau<\infty$, equation (45) represents the branch Γ of a hyperbola, where Γ is given through.

$$p=\frac{x}{r^2}\tau\pm i\frac{|y|}{r^2}\sqrt{(\tau^2-r^2/v^2)}\ (\frac{r}{v}<\tau<\infty) \tag{46}$$

In which the square root is taken positive. It is easily verified that by virtue of Cauchy's theorem and Jordan's lemma [54] ,the integral along the imaginary p -axis is equal to the integral along Γ . Along Γ we have :-

$$\gamma=\frac{|y|}{r^2}\tau\mp i\frac{x}{r^2}\sqrt{(\tau^2-r^2/v^2)} \tag{47}$$

and

$$\frac{\partial p}{\partial \tau} = \pm \frac{i\gamma}{(\tau^2 - r^2/v^2)^{\frac{1}{2}}} \tag{48}$$

In (46) , (47) and (48) the upper and lower signs belong together . Taken into account the symmetry of the path of integration with respect to the real axis and introducing τ as variable of integration we obtain

$$u(x,y;s) = \frac{F(s)}{2\pi} \int_{r/v}^{\infty} \exp(-s\tau)(\tau^2 - r^2/v^2)^{-\frac{1}{2}} d\tau \tag{49}$$

This expression is of the general form:-

$$u(x,y;s) = F(s) \int_0^{\infty} \exp(-s\tau) g(x,y;\tau) d\tau \tag{50}$$

where , in our case ,

$$g(x,y,\tau) = \begin{cases} 0 & (0 \prec \tau \prec \frac{r}{v}) \\ \frac{1}{2\pi}(\tau^2 - \frac{r^2}{v^2})^{-\frac{1}{2}} & (\frac{r}{v} \prec \tau \prec \infty) \end{cases} \tag{51}$$

Application of the shift rule for Laplace transform to the function $F(s)\exp(-s\tau)$ directly yields the function $u(x,y;t)$ We obtain

$$u(x,y,t) = \int_0^t f(t-\tau) g(x,y,\tau) d\tau \quad (t>0) \tag{52}$$

While, from our assumptions, $u = u(x,y;t) = 0$ when $t < 0$.In our case we have

$$u(x,y,t) = \begin{cases} 0 & (0 \prec t \prec \frac{r}{v}) \\ \frac{1}{2\pi}\int_{\frac{r}{v}}^{t} f(t-\tau)(\tau^2 - \frac{r^2}{v^2})^{-\frac{1}{2}} d\tau & (\frac{r}{v} \prec t \prec \infty) \end{cases} \tag{53}$$

Form the final result (53) it is clear that $g(x,y,t)$ can be regarded as the wave function corresponding to a delta function time dependence of the source.

5-The scalar wave generated by an impulsive point source Inside The Ionosphere. (in three-dimensional)

The three-dimensional wave function $u = u(x,y,z;t)$ due to the presence of a point source acting at $x = 0, y = 0, z = 0$ satisfies the three-dimensional wave function

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = -\delta(x,y,z) f(t) \tag{54}$$

Where $\delta(x,y,z)$ denotes the three – dimensional delta function. Again, we assume that, outside the source, u are continuous and have continuous partial derivatives of the first and second order. Further , $f(t) = 0$ when $t \prec 0$ and $u = 0$ when $t \prec 0$ .The following one – sided Laplace transforms with respect to time are introduced

$$F(s) = \int_0^{\infty} \exp(-st) f(t) dt \tag{55}$$

and

$$u(x,y,z;s) = \int_0^{\infty} \exp(-st) u(x,y,z,t) dt \tag{56}$$

Since u and $\frac{\partial u}{\partial t}$ are continues , $u(x,y,z;s)$ satisfies the differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{s^2}{v^2} u = -\delta(x,y,z) F(s) \tag{57}$$

In order to solve (57) we introduce the two-dimensional Fourier transform of $u(x,y,z;s)$ with respect to x and y . Let

$$\Pi(\alpha,\beta,z;s) = \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \exp[is(\alpha x + \beta y)] u(x,y,z;s) dx \tag{58}$$

Then $\Pi(\alpha,\beta,z;s)$ satisfies the differential equation

$$\frac{d^2\Pi}{dz^2} - s^2\gamma^2\Pi = -\delta(z) F(s) \tag{59}$$

where

$$\gamma = \gamma(\alpha,\beta) = \sqrt{(\alpha^2 + \beta^2 + \frac{1}{v^2})}, (\mathrm{Re}\,\gamma \geq 0) \tag{60}$$

the solution of (3.28) that is bounded as $|z| \to \infty$ is given by

$$\Pi(\alpha,\beta,z;s) = \frac{F(s)}{2s\gamma} \exp(-s\gamma|z|). \tag{61}$$

with the aid of Fourier's inversion theorem we obtain the following expression for $u(x,y,z;s)$

$$u(x,y,z;s) = \frac{sF(s)}{4\pi^2} \int_{-\infty}^{\infty} d\beta \int_{-\infty}^{\infty} \exp[-is(\alpha x + \beta y) - s\gamma|z|] \frac{1}{2\gamma} d\alpha \tag{62}$$

Again , we shall try to cast the integral on the right-hand side of (62) in such a form that $u = u(x, y, z; t)$ can be found more or less by inspection .It will be advantageous to transform the exponential function into a form which resembles the one occurring in the two dimensional problem .

Now , if (α, β) - plane is rotated by angle ϕ ,this introduces new variables of integration ω and q through

$$\alpha = \omega\cos\phi - q\sin\phi, \beta = \omega\sin\phi + q\cos\phi \quad (63)$$

since $d\alpha d\beta = d\omega dq$ we obtain :-

$$u(x, y, z; s) = \frac{sF(s)}{4\pi^2}\int_{-\infty}^{\infty} dq \int_{-\infty}^{\infty} \exp[-i\omega r - s\gamma|z|]\frac{1}{2\gamma}d\omega \quad (64)$$

Where $\alpha x + \beta y = \omega r$ (65)

In which ,as

$$\therefore \alpha^2 + \beta^2 = \omega^2 + q^2 \quad (66)$$

where :-

$$\gamma = (\omega^2 + q^2 + \frac{1}{v^2})^{1/2}, \operatorname{Re}\gamma \geq 0 \quad (67)$$

In order to bring the right-hand side of (63) in a form which is analogous to the two dimensional case , we introduce the variable $p = i\omega$ and regard p as a complex variable in the $p-$ plane , while q is kept real .the result is

$$u(x, y, z; s) = \frac{sF(s)}{4\pi^2 i}\int_{-\infty}^{\infty} dq \int_{-i\infty}^{i\infty} \exp[-s(pr - \gamma|z|)]\frac{1}{2\gamma}dp \quad (68)$$

In which

$$\gamma = (q^2 + \frac{1}{v^2} - p^2)^{\frac{1}{2}}, \operatorname{Re}\gamma \geq 0 \quad (69)$$

From now on , the procedure is similar to the one outlined in III.1 by virtue of Cauchy's theorem and Jordan's lemma the integration along the imaginary p -axis can be replaced by an integration along the branch Γ of a hyperbola, where Γ is given through

$$p = \frac{r}{R^2}\tau \pm i\frac{|z|}{R^2}[\tau^2 - R^2(q^2 + \frac{1}{v^2})]^{1/2},$$

$$(R(q^2 + 1/v^2)^{1/2} \prec \tau \prec \infty). \quad (70)$$

Along Γ we have

$$\gamma = \frac{|z|}{R^2}\tau \mp i\frac{r}{R^2}[\tau^2 - R^2(q^2 + \frac{1}{v^2})]^{\frac{1}{2}} \quad (71)$$

and

$$\frac{\partial p}{\partial \tau} = \pm\frac{i\gamma}{[\tau^2 - R^2(q^2 + \frac{1}{v^2})]^{\frac{1}{2}}} \quad (72)$$

In (70), (71) and (72) the upper and lower signs belong together. Taken into account the symmetry of the path of integration with respect to the real axis and introducing τ as variable of integration we obtain that:-

$$u(x, y, z; s) = \frac{sF(s)}{4\pi^2}\int_{-\infty}^{\infty} dq \int_{R^2(q^2+\frac{1}{v^2})^{\frac{1}{2}}}^{\infty} \exp(-s\tau)[\tau^2 - R^2(q^2 + \frac{1}{v^2})]^{-\frac{1}{2}}d\tau \quad (73)$$

Now we interchange the order of integration , which leads to

$$u(x, y, z; s) = \frac{sF(s)}{4\pi^2}\int_{R/v}^{\infty} \exp(-s\tau)d\tau \int_{-(\tau^2/R^2 - 1/v^2)^{\frac{1}{2}}}^{(\tau^2/R^2 - 1/v^2)^{\frac{1}{2}}} [\tau^2 - R^2(q^2 + 1/v^2)]^{-\frac{1}{2}}dq$$

$$= \frac{sF(s)}{4\pi R}\int_{R/v}^{\infty} \exp(-s\tau)d\tau \quad (74)$$

or

$$u(x, y, z; s) = F(s)\frac{\exp(-sR/v)}{4\pi R} \quad (75)$$

Where $t \prec \tau = R/v$, R denotes the spherical distance between the source and the point of observation.

6- Numerical results

We define a normalized time τ , which means that the beginning of the $\tau-$axis coincides with the arrival time of the spherical wave originating directly from the source; clearly this normalization depends upon the point of observation. Includes the potential theory, the Hertz vector, the Fitzgerald vector, wave equations, boundary conditions, classification of EM problem, classification of solution region and classification of EM methods one result is given in the form of a definite integral over a finite integral can easily be computed numerically. In Fig.[2-5]

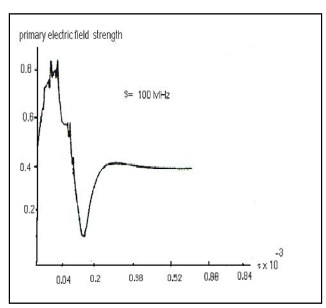

Fig. (2) Z-component of primary electric field strength as a function of normalize time τ , R= 5 km , s = 100 MHz

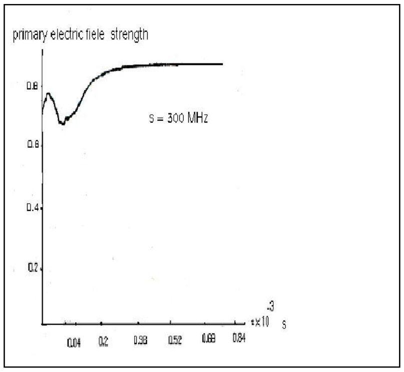

Fig. (3) Z-component of primary electric field strength as a function of normalize time τ , R= 5 km, s = 300 MHz.

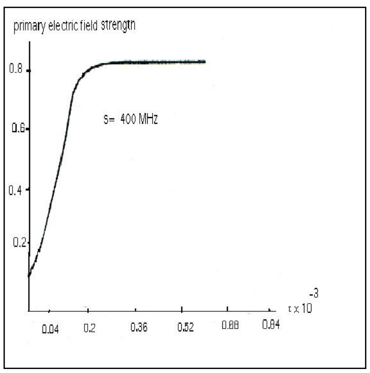

Fig. (4) Z-component of primary electric field strength as a function of normalize time τ , R= 5 km , s = 400 MHz .

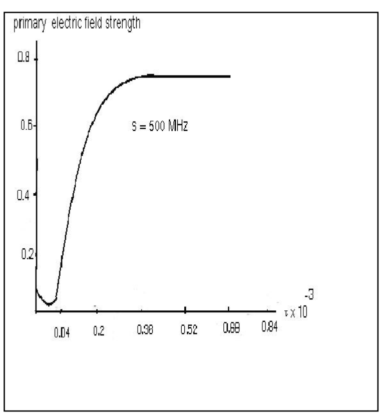

Fig.(5) Z-component of primary electric field strength as a function of normalize time τ , R= 5 km , s = 500 MHz

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.we have studied Cagniard's method , the scalar wave generated by an impulsive line source.(in two-dimensional and in three-dimensional) .The application of Cagniard's method in obtaining exact solution of the three – dimensional pulse problems leads to complicated expression for the components of the displacement vector in the ionosphere.

8. Conclusion

The author try to give the exact solution of the electric field strength above a two layer medium .The integral represent of the physical point of view; also, the integral is evaluated by two mathematical methods: Residue and Saddle point method. A disadvantage of the method is not it cannot be used to calculate the potential in the dielectric half – space outside the layer in similar manner .

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9. References

[1] A.Sommerfeld ,Uber die Ausbreitung der wellen in der drahlosen telegraphic, Ann.d.Phys.Lpz **28**,pp665-736(1909).
[2] J.R. Wait , Electromagnetic waves in stratified Media ,Pergumon Press, Oxford , pp.608(1970).
[3] R.K.Moore,W.E.Blair ,Dipole radiation in a conducting half-space ,J Res Nat.Bar..standards **65D** PP.547-583(1961)
[4] S.H.Durrani. Air to under sea communication with magnetic dipole ,trans.IEEE. **Ap-12**(1964)
[5] A.A. Abo Seliem, The transient reponse above an evaporation duct, J.Phys. D.Appl.Phys **31**, pp3046-3050(1998) .
[6] A.Arutaki,J.Chiba,Commincation in a three – layered conducting media with a vertical magnetic dipole , Trans.IEEE,Antennas Propagat .**28** ,pp.551-556(1980).
[7] A.A. Abo Seliem Propagation of the transient electromagnetic field above atmospheric surface duct Applied Mathematics and Computation, 145, 25 December),vol.9 ,no. 2 ,pp. 631-639(2003).
[8] A.A. Abo Seliem, Evaluation of the transient electromagnetic field in evaporation duct, Applied Mathematics and Computation ,vol.151, pp.411-421(2004) .
[9] T. Kahan, G .Eckart , Theorie de la propagation des ondes electromagne-tiques le guide d'ondes atmospheriques, Ann.Phys.vol.5,pp.641-705(1950).
[10] D.S. Jones, Theory of electromagnetism, Pergamon Press, London, England, (1964).
[11]L.M.Brekhovsikh,Wave in Layered Media.Academic .Press.New York.(1960) .