The Leptonic e^-e^+ , $\mu^-\mu^+$, $\tau^-\tau^+$ Pair Production in the Photon Interaction with Electromagnetic Field of Light Nuclei

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Abstract: The analytical and numerical calculations for the problem of photoproduction of (lepton-anti lepton)- pairs in the electromagnetic field of light nuclei has been done. applying the obtained formulae for the energy distribution of the pair-production process to the cases of e^-e^+ , $\mu^-\mu^+$, -pairs in ultra relativistic regions of incident photon energy. and $\tau^-\tau^+$ .Comparing the results for the different cases of pair - production , we can show that the cross-sections for the $\tau^-\tau^+$ - pair due to the electric and magnetic fields of the target nucleus are larger than that for the e^-e^+ and $\mu^-\mu^+$ - pairs , and that the magnetic field of target nucleus is more effective than the electric field of the nucleus in the e^-e^+ , $\mu^-\mu^+$, and $\tau^-\tau^+$ - pair production processes. We can also show that the values cross-sections of the processes due to the electric quadrupole and magnetic octupole of the target nucleus are larger than the values of the cross-sections due to the electric charge distribution and the magnetic dipole moment of the target nucleus in the three cases of the pair-production processes.

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1. Introduction

The theoretical treatment of the process of the e^-e^+- pair photoproduction was firstly done in 1934 by **Nishina and others** [1] and by **Bethe and Heitler** [2]. In 1936, **Jaeget and Hulme** [3] shows that the calculations of the cross-section of the pair production process give a good results at high energies of incident photon. **Hubbell[4]**gives a historical review for the e^-e^+- pair production by photons from the prediction of the position $e^+$by Dirac in 1928 , to 2006. The cross-section data for e^-e^+- pair production by photons was given by **Hubbell and Seltzer**[5].The study of $\mu^-\mu^+$ and $\tau^-\tau^+$- pair production are given in many works ,e.g. [6-10].In this work we shall examine the process of the photoproduction of e^-e^+ , $\mu^-\mu^+$, and $\tau^-\tau^+$-pairs in the field of light nuclei , having electric charge distribution, magnetic dipole moment , electric quadrupole moment , and magnetic octupole moment , showing the effect of these electric and magnetic properties of target nucleus on the cross-section of the pair – production processes at high energy regions of incident photons.

2.Formulation of the Problem

The Feynman diagrams for the problem of photoproduction of leptonic pairs in the electromagnetic field of nuclei are shown in fig.(1)

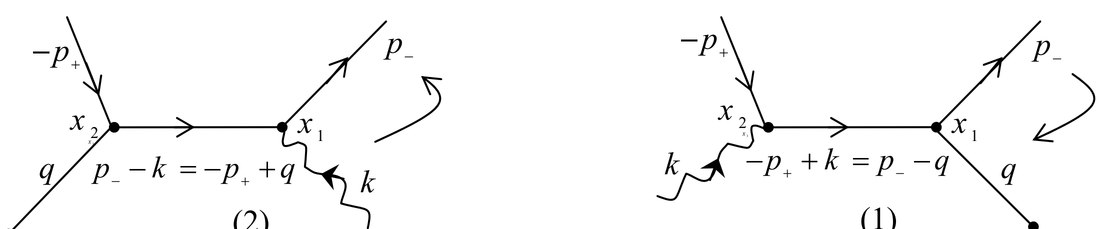

Fig.1

Where the momentum transferred to the nucleus is $q = p_- - k + p_+$. Ignoring the polarization of lepton and anti-lepton in the producing pair and also the polarization of the incident photon , we have the following formula for the differential cross-section of the process concerning the effects of the electric charge $d\sigma(BH)$, the magnetic dipole $d\sigma(2M)$, the electric quadrupole $d\sigma(4E)$, and the magnetic octupole moments $d\sigma(8M)$ of the target nucleus:

$$d\sigma_{tot}(\theta_+,\theta_-)=d\sigma_{BH}(\theta_+,\theta_-)+d\sigma_{2M}(\theta_+,\theta_-)+d\sigma_{4E}(\theta_+,\theta_-)+d\sigma_{8M}(\theta_+,\theta_-) \quad (1)$$

Where:

$$d\sigma_{BH}(\theta_+,\theta_-)=\eta\left(\frac{1}{q^4}\right)^2 F_{0e}(\theta_+,\theta_-)d\Omega_+d\Omega_- \quad (2)$$

$$d\sigma_{2M}(\theta_+,\theta_-)=\eta\left(\frac{\mu_I}{Ze}\right)^2 a_\mu(I)\frac{1}{q^2}F_{0m}(\theta_+,\theta_-)d\Omega_+d\Omega_- \quad (3)$$

$$d\sigma_{4E}(\theta_+,\theta_-)=\eta\left(\frac{Q}{Ze}\right)^2 a_q(I)F_{0e}(\theta_+,\theta_-)d\Omega_+d\Omega_- \quad (4)$$

$$d\sigma_{8M}(\theta_+,\theta_-)=\eta\left(\frac{\Omega}{Ze}\right)^2 a_\Omega(I)F_{0m}(\theta_+,\theta_-)d\Omega_+d\Omega_- \quad (5)$$

$$\eta=\left(\frac{Z^2\alpha^3}{4\pi^2}\right)\frac{P_+P_-dE_+}{\omega^3} \quad , \quad \omega=E_++E_- \quad \text{is the energy incident photon}$$

Ze, μ_I, Q, Ω are the charge, the magnetic dipole moment, the electric quadrupole moment, and the magnetic octupole moment of the target nucleus. $d\Omega_\mp=\sin\theta_\mp\, d\theta_\mp\, d\phi$ are the solid angles of the emitted lepton (anti-lepton) respectively.

$$a_\mu(I)=\frac{I+1}{3I} \quad , \quad a_q(I)=\frac{1}{180}\frac{(I+1)(2I+3)}{I(2I-1)} \quad , \quad a_\Omega(I)=\frac{2}{4725}\frac{(I+1)(I+2)(2I+3)}{I(I-1)(2I-1)}$$

are the magnetic dipole, electric quadrupole, and magnetic octupole coefficients of the nucleus with spin I. The functions $F_{0e}(\theta_+,\theta_-)$ and $F_{0m}(\theta_+,\theta_-)$ are given by the relations:

$$F_{0e}(\theta_+,\theta_-)=\frac{p_-^2\sin^2\theta_-}{\Delta_-^2}(q^2-4E_+^2)+\frac{p_+^2\sin^2\theta_+}{\Delta_+^2}(q^2-4E_-^2)+\frac{2\omega^2}{\Delta_-\Delta_+}(p_+^2\sin^2\theta_++p_-^2\sin^2\theta_-)-\frac{2F(\theta_+,\theta_-)}{\Delta_-\Delta_+}(q^2+4E_+E_--2\omega^2) \quad (6)$$

$$F_{0m}(\theta_+,\theta_-)=2\omega^2\left(\frac{\Delta_+}{\Delta_-}+\frac{\Delta_-}{\Delta_+}+\frac{q^2}{\Delta_+\Delta_-}-2\right)-\frac{2F(\theta_+,\theta_-)}{\Delta_-\Delta_+}\left[4(E_+E_-+m_0)-q^2\right]$$

$$-\frac{p_+^2\sin^2\theta_+}{\Delta_+^2}(4p_-^2+q^2)-\frac{p_-^2\sin^2\theta_-}{\Delta_-^2}(4p_+^2+q^2) \quad (7)$$

$$F(\theta_+,\theta_-)=2p_+p_-\sin\theta_+\sin\theta_-\cos\phi \quad , \quad \Delta_+=E_+-p_+\cos\theta_+ \quad , \quad \Delta_-=E_--p_-\cos\theta_-$$

The integration of (1) with respect to the solid angle of the emitted lepton $d\Omega_-=\sin\theta_-d\theta_-d\phi$, gives a general formula for the angular distribution of the process into the form:

$$d\sigma_{tot}(\theta_+)=d\sigma_{BH}(\theta_+)+d\sigma_{2M}(\theta_+)+d\sigma_{4E}(\theta_+)+d\sigma_{8M}(\theta_+) \quad (8)$$

The integration of (8) with respect to the solid angle of emitted anti-lepton $d\Omega_+=\sin\theta_+d\theta_+d\phi$, gives the following formulas for the energy distributions in the ultra relativistic case, where ε_γ, E_-, $E_+ >> m_0c^2$.

$$d\sigma_{tot}(E_+,E_-)=d\sigma_{BH}(E_+,E_-)+d\sigma_{2M}(E_+,E_-)+d\sigma_{4E}(E_+,E_-)+d\sigma_{8M}(E_+,E_-) \quad (9)$$

Where

$$d\sigma_{BH}(E_+,E_-)=d\sigma(BH)=\eta_E\left(E_+^2+E_-^2+\frac{2}{3}E_+E_-\right)\left(2\left(\ln\frac{2E_+E_-}{\omega m_0c^2}\right)-1\right) \quad (10)$$

$$d\sigma_{2M}(E_+,E_-)=d\sigma(2M)=4\eta_E\left(\frac{\mu_I}{Ze}\right)^2 a_\mu(I)\left(\frac{m_0c}{\hbar}\right)^2\left[\omega^2\left(\ln\frac{2E_+E_-}{\omega m_0c^2}\right)+\left(E_+^2+E_-^2\right)\left(\ln\frac{2E_+}{m_0c^2}\right)\left(\ln\frac{2E_-}{m_0c^2}\right)\right] \quad (11)$$

$$d\sigma_{4E}(E_+,E_-)=d\sigma(4E)=8\eta_E\left(\frac{Q}{Ze}\right)^2 a_q(I)\left(\frac{m_0c}{\hbar}\right)^2(E_+E_-)\left[6\omega^2-10E_+E_-+2(E_+^2+E_-^2).\right.$$

$$\left.\cdot\left(\frac{E_-}{E_+}\left(ln\frac{2E_+}{m_0c^2}\right)+\frac{E_+}{E_-}\left(ln\frac{2E_-}{m_0c^2}\right)\right)\right] \quad (12)$$

$$d\sigma_{8M}(E_+,E_-)=d\sigma(8M)=8\eta_E\left(\frac{\Omega}{Ze}\right)^2 a_\Omega(I)\left(\frac{m_0c}{\hbar}\right)^2(E_+E_-)\frac{112}{3}(E_+^4+E_-^4)+$$

$$\frac{4}{9}(E_+E_-).(81E_+^2+81E_-^2+80E_+E_-)+$$

$$+\frac{40}{3}(E_+^2+E_-^2)\left(\frac{E_+^3}{E_-}\left(ln\frac{2E_-}{m_0c^2}\right)+\frac{E_-^3}{E_+}\left(ln\frac{2E_+}{m_0c^2}\right)\right)\Bigg] \quad (13)$$

$$\eta_E=2\alpha\left(\frac{Ze^2}{m_0c^2}\right)^2\frac{dE_+}{\omega^3} \quad , \quad \alpha=\frac{e^2}{\hbar c}=\frac{1}{137}$$

The formulas (8 -13) represents the energy distribution cross-sections of the photon production of the (lepton-anti lepton)-pair in the field of nuclei having electric charge distribution, (dB), magnetic dipole moment (2M), electric quadrupole moment(4E), and magnetic octupole moment (8M).

3.Results and Discussion

The electric charge $d\sigma(BH)$, magnetic dipole $d\sigma(2M)$, electric quadrupole $d\sigma(4E)$, magnetic pctupole $d\sigma(8M)$, total electric $d\sigma_{tot}(E)$, and total magnetic $d\sigma_{tot}(M)$ cross sections for the different leptonic - pairs e^-e^+, $\mu^-\mu^+$, $\tau^-\tau^+$ production using formulas for the energy distribution are obtained for the nucleus B_5^{11} and for different values of energies $\varepsilon_\gamma = (1\,Gev, 5\,Gevl, 10Gev)$.

The results are tabulated in the following table:

Table (1) Energy

energy	pair	d(BH)	d(2M)	d(4E)	d(8M)	d(E)	d(M)
1000	e e	$3.65217\ 10^{51}$	$2.00318\ 10^{46}$	$2.97399\ 10^{49}$	$1.10247\ 10^{43}$	$3.01051\ 10^{49}$	$1.10447\ 10^{43}$
5000	e e	$7.98674\ 10^{52}$	$4.37008\ 10^{47}$	$1.487000\ 10^{48}$	$1.43995\ 10^{41}$	$1.4878\ 10^{48}$	$1.43995\ 10^{41}$
10000	e e	$4.14031\ 10^{52}$	$2.26336\ 10^{47}$	$2.97399\ 10^{48}$	$1.17327\ 10^{40}$	$2.97441\ 10^{48}$	$1.17327\ 10^{40}$
1000		$7.61882\ 10^{49}$	$4.17884\ 10^{44}$	$6.20407\ 10^{47}$	$8.51781\ 10^{37}$	$6.28025\ 10^{47}$	$8.51781\ 10^{37}$
5000		$1.66612\ 10^{49}$	$9.11645\ 10^{45}$	$3.10203\ 10^{46}$	$1.12089\ 10^{34}$	$3.1037\ 10^{46}$	$1.12089\ 10^{34}$
10000		$8.63713\ 10^{50}$	$4.72162\ 10^{45}$	$6.20407\ 10^{46}$	$9.1606\ 10^{34}$	$6.20493\ 10^{46}$	$9.1606\ 10^{34}$
1000		$1.27004\ 10^{47}$	$6.96698\ 10^{43}$	$1.03427\ 10^{45}$	$3.58182\ 10^{33}$	$1.04697\ 10^{45}$	$3.58182\ 10^{33}$
5000		$2.77738\ 10^{48}$	$1.51991\ 10^{43}$	$5.17137\ 10^{45}$	$4.73742\ 10^{31}$	$5.17414\ 10^{45}$	$4.73742\ 10^{31}$
10000		$1.43979\ 10^{48}$	$7.87196\ 10^{44}$	$1.03428\ 10^{44}$	$3.87957\ 10^{30}$	$1.03442\ 10^{44}$	$3.87957\ 10^{30}$

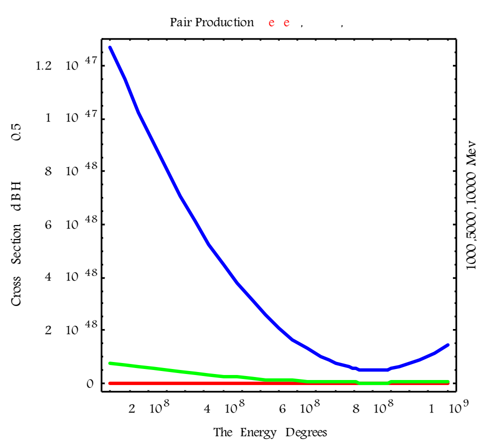

Fig.2

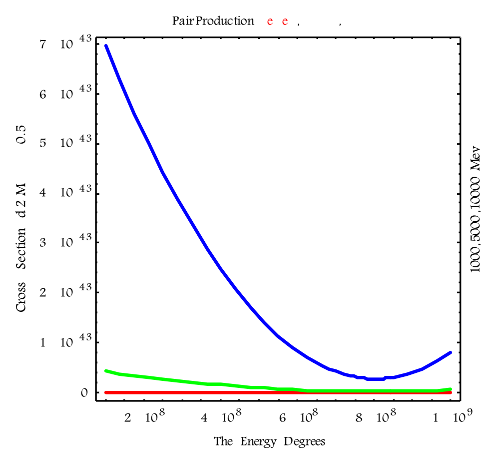

Fig.3

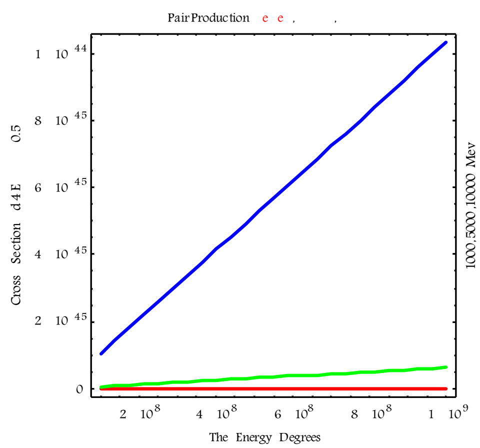

Fig.4

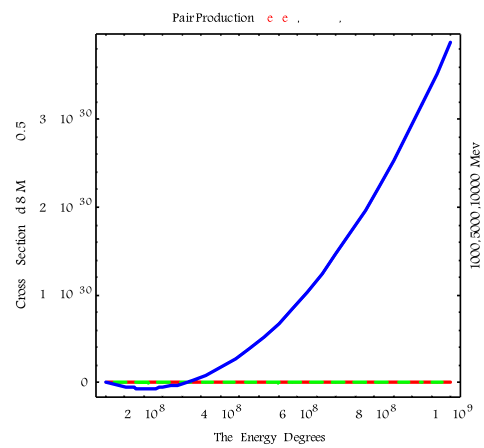

Fig.5

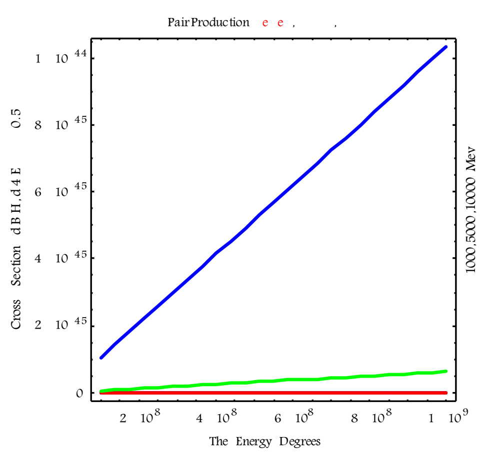

Fig.6

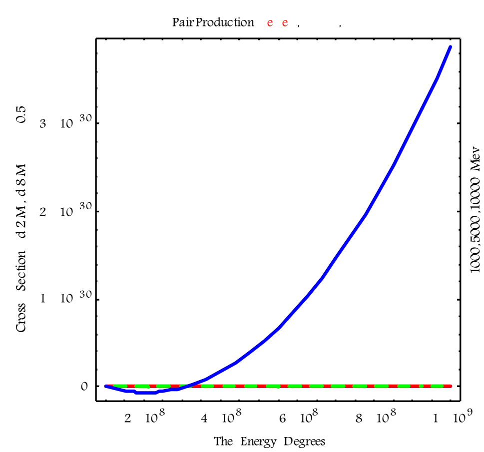

Fig.7

The energy distribution curves for the different leptonic pairs e^-e^+ , $\mu^-\mu^+$, $\tau^-\tau^+$ and for different values of energies $\varepsilon = \varepsilon_\gamma = (1000\ ,5000\ ,10000\ Mev)$, are shown in figures (2-7). From the table and figures we have the following results:

1. For e^-e^+-pair:

Knowing that the rest mass of electron is: $m_e = 9.10941 \times 10^{-28}\ gm = 0.51100 Mev/c^2$

i) The cross-section $d\sigma(BH)$ decreases with increasing energies: e.g.

at energy $\varepsilon_\gamma = 1Gev : d\sigma(BH) = 3.65217 \times 10^{-51}$, at $\varepsilon_\gamma = 10Gev : d\sigma(BH) = 4.14031 \times 10^{-52}$

ii) The cross-section $d\sigma(2M)$ decreases with increasing energies: e.g.

at energy $\varepsilon_\gamma = 1Gev : d\sigma(2M) = 2.00318 \times 10^{-46}$, at $\varepsilon_\gamma = 10Gev : d\sigma(2M) = 2.26336 \times 10^{-47}$

iii) The cross-section $d\sigma(4E)$ increasing with increasing energies: e.g.

at energy $\varepsilon_\gamma = 1Gev : d\sigma(4E) = 2.97399 \times 10^{-49}$,

at $\varepsilon_\gamma = 10Gev : d\sigma(4E) = 2.97399 \times 10^{-48}$

iv) The cross-section $d\sigma(8M)$ increasing with increasing energies: e.g.

at energy $\varepsilon_\gamma = 1Gev : d\sigma(8M) = 1.10247 \times 10^{-43}$,

at $\varepsilon_\gamma = 10Gev : d\sigma(8M) = 1.17327 \times 10^{-40}$

2. for $\mu^-\mu^+$-pair:

Knowing that the rest mass of electron is: $m_\mu = 1.90032 \times 10^{-25}\ gm = 105.65837 Mev/c^2$

i) The cross-section $d\sigma(BH)$ decreases with increasing energies: e.g.

at $\varepsilon_\gamma = 1Gev : d\sigma(BH) = 7.61882\times10^{-49}$, at $\varepsilon_\gamma = 10Gev : d\sigma(BH) = 8.63713\times10^{-50}$

ii) The cross-section $d\sigma(2M)$ decreases with increasing energies: e.g.
at
energy $\varepsilon_\gamma = 1Gev : d\sigma(2M) = 4.17884\times10^{-44}$,
at $\varepsilon_\gamma = 10Gev : d\sigma(2M) = 4.72162\times10^{-45}$

iii) The cross-section $d\sigma(4E)$ increasing with increasing energies: e.g.
at energy $\varepsilon_\gamma = 1Gev : d\sigma(4E) = 6.20407\times10^{-47}$,
at $\varepsilon_\gamma = 10Gev : d\sigma(4E) = 6.20407\times10^{-46}$

iv) The cross-section $d\sigma(8M)$ increasing with increasing energies: e.g.
at $\varepsilon_\gamma = 1Gev : d\sigma(8M) = 8.51781\times10^{-37}$, at $\varepsilon_\gamma = 10Gev : d\sigma(8M) = 9.1606\times10^{-34}$

3. For $\tau^-\tau^+$-pair:
Knowing that the rest mass of electron is: $m_\tau = 3.16779\times10^{-24}\ gm = 1776.99990 Mev/c^2$

i) The cross-section $d\sigma(BH)$ decreases with increasing energies: e.g.
at $\varepsilon_\gamma = 1Gev : d\sigma(BH) = 1.27004\times10^{-47}$, at $\varepsilon_\gamma = 10Gev : d\sigma(BH) = 1.43979\times10^{-48}$

ii) The cross-section $d\sigma(2M)$ decreases with increasing energies: e.g.
at
energy $\varepsilon_\gamma = 1Gev : d\sigma(2M) = 6.96698\times10^{-43}$,
at $\varepsilon_\gamma = 10Gev : d\sigma(2M) = 7.87196\times10^{-44}$

iii) The cross-section $d\sigma(4E)$ increasing with increasing energies: e.g.
at energy $\varepsilon_\gamma = 1Gev : d\sigma(4E) = 1.03427\times10^{-45}$,
at $\varepsilon_\gamma = 10Gev : d\sigma(4E) = 1.03428\times10^{-44}$

iv) The cross-section $d\sigma(8M)$ increasing with increasing energies: e.g.
at $\varepsilon_\gamma = 1Gev : d\sigma(8M) = 3.58182\times10^{-33}$, at $\varepsilon_\gamma = 10Gev : d\sigma(8M) = 3.87957\times10^{-30}$

From the above analysis , and from diagrams (2-7)we conclude that:

(i) The cross-sections $d\sigma(BH)$ and $d\sigma(2M)$ are decreases with increasing energies for the e^-e^+ , $\mu^-\mu^+$, $\tau^-\tau^+$-pairs , while the cross-section $d\sigma(4E)$ and $d\sigma(8M)$ are increases with increasing energies for the e^-e^+ , $\mu^-\mu^+$, $\tau^-\tau^+$-pairs.

(ii) The values of the electric and magnetic cross-sections $d\sigma(BH), d\sigma(2M), d\sigma(4E)$ and $d\sigma(8M)$ for the $(\tau^-\tau^+)$-pair production are larger than that for the (e^-e^+) -and $(\mu^-\mu^+)$-pairs production , and the same values for $(\mu^-\mu^+)$ are larger than that for the (e^-e^+) -pair production.

(iii) The cross- sections of the pair-production processes in the three cases e^-e^+, $\mu^-\mu^+$, $\tau^-\tau^+$, due to the electric quadrupole $d\sigma(4E)$ and the magnetic octupole $d\sigma(8M)$ are more larger than the cross-sections due to the electric charge distribution $d\sigma(BH)$ and the magnetic dipole moment $d\sigma(2M)$ of the target nucleus.

For the total electric $d\sigma_{tot}(E) = d\sigma(BH) + d\sigma(4E)$ and the total magnetic $d\sigma_{tot}(M) = d\sigma(2M) + d\sigma(8M)$ cross-sections we have also the following results [see the table]:

i- For the cross-section $d\sigma_{tot}(E)$, the cross-section $d\sigma(4E)$ is more effective than the cross-section $d\sigma(BH)$ i.e. $d\sigma_{tot}(E) \approx d\sigma(4E)$ or $d\sigma(4E) \rangle\rangle\ d\sigma(BH)$.[Fig.(4),(6)].

ii- For the cross-section $d\sigma_{tot}(M)$, the cross-section $d\sigma(8M)$ is more effective than the cross-section $d\sigma(2M)$ i.e. $d\sigma_{tot}(M) \approx d\sigma(8M)$ or $d\sigma(8M) \rangle\rangle\ d\sigma(2M)$. [Fig.(5),(7)].

iii- The total magnetic cross-section $d\sigma_{tot}(M)$ for the 3-types of pair e^-e^+ , $\mu^-\mu^+$, $\tau^-\tau^+$ is more effective than the total electric cross-section $d\sigma_{tot}(E)$ .i.e. $d\sigma_{tot}(M) \rangle\rangle\ d\sigma_{tot}(E)$.

e.g. for the $\tau^-\tau^+$-pair production: at energy
$\varepsilon_\gamma = 1Gev: d\sigma_{tot}(E) = 1.04697\times10^{-45}$, $d\sigma_{tot}(M) = 3.58182\times10^{-33}$
at energy
$\varepsilon_\gamma = 10Gev: d\sigma_{tot}(E) = 1.03442\times10^{-44}$, $d\sigma_{tot}(M) = 3.87957\times10^{-30}$

iv- The values of $d\sigma_{tot}(E)$ and $d\sigma_{tot}(M)$ for the $\tau^-\tau^+$-pair are larger than that for the e^-e^+ and -$\mu^-\mu^+$ pair.

e.g at energy $\varepsilon_\gamma = 5Gev$:

$$d\sigma_E(\tau^-\tau^+) = 5.17414\times10^{-45}\ ,\ d\sigma_E(\mu^-\mu^+) = 3.1037\times10^{-46}\ ,\ d\sigma_E(e^-e^+) = 1.4878\times10^{-48}$$
$$d\sigma_M(\tau^-\tau^+) = 4.73742\times10^{-31}\ ,\ d\sigma_M(\mu^-\mu^+) = 1.12089\times10^{-34}\ ,\ d\sigma_M(e^-e^+) = 1.43995\times10^{-41}$$

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