# Optimal Synthesis of a 4-Bar Simple Toggle 

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#### Abstract

Optimal synthesis of mechanisms is a successful approach for mechanism design to satisfy all the desired characteristics of the designed mechanism. Toggles have wide industrial applications such as riveting, punching, pressing, and clamping. The toggle optimal design problem is a constrained multi-dimensional problem. Powell optimization technique is used to maximize the mechanical advantage of the toggle. 2 functional constraint functions are used to perform a successful optimization. The toggle force analysis in the static mode is performed in a dimensionless form. The results are tabulated for an easy reference to them without any calculations for any desired mechanical advantage of the toggle. Mechanical advantage in the range of 3.9 to 340 is selectable for specific constraints on the toggle input link length and operating conditions of the toggle. Using the optimal design table could satisfy the toggle objectives with errors less than $0.3 \%$. [Galal A. Hassaan, Mohammed A. Al-Gamil and Maha M. Lashin Optimal Synthesis of a 4-Bar Simple Toggle] Journal of American Science 2011;7(11):522-528]. (ISSN: 1545-1003). http://www.jofamericanscience.org. 65


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## 1. Introduction

Mechanical toggles represent the muscles of machinery required for some important processes such as riveting, punching, pressing, bonding and clamping. Successful synthesis of toggles leads to a successful machine design with minimum input effort either manual of mechanical. On the other hand, optimal toggle design leads to satisfying desired operation constraints such as coupler inclination and slider (tool) position from the fixed pivot of the toggle.

The presented approach in this work is essential for small industries especially in the developed countries.

Design of toggles has gained some interest among international researchers. Mostofi (1985) investigated the dynamic performance of a toggle mechanism. He showed that the toggle performance under Coulomb friction depends highly upon the input energy of the system [1]. Yossifon and Shivpuri (1993) optimized the design of a double knuckle toggle linkage for mechanical presses using kinematic simulation and an enumeration search procedure [2]. Howell and Midha (1995) studied the compliance of the work-piece on the input / output characteristics of a rigid-link toggle mechanism [3]. Fung et.al. (1997) used a constrained multi-body technique to calculate the position, velocity and acceleration of the toggle mechanism [4]. Fung and Yang (2001) studied the motion control of a toggle mechanism actuated by an electro-hydraulic system. The control system was a nonlinear timevarying one [5]. Fung et.al. (2001) investigated the dynamic problem of the control of a toggle mechanism
driven by a linear synchronous motor. They considered Coulomb friction in the toggle joints [6]. Tso and Liang (2002) proposed a 9-bar linkage for a toggle used in in mechanical forming presses. They optimized the toggle dimensions using linkage synthesis and the trial and error method [7]. Lin and Wai (2002) studied the dynamic response of a hybrid computed torque controlled toggle mechanism driven by a permanent magnet synchronous motor [8]. Lin and Chang (2002) proposes a force transmissivity index for planar linkage mechanisms. They concluded that the proposed index can be used as a better measure of force transmissivity for planar linkage mechanisms [9]. Wai (2003) proposed a robust control system using a Takagi Sugeno - Kang type fuzzy-neural network to control a nonlinear toggle mechanism driven by a permanent magnet synchronous servomotor [10]. Lin and Hsiao (2003) proposed an analytical formulation of the thrust of a 5-point double-toggle clamping mechanism during mold clamping. The effect of friction at in joints is investigated [11]. Englender (2007) studied reducing the cycle time of an injection molding machine by operating its clamping unit with minimum time. He proposed a nonlinear controller having a cascade structure [12]. Chuang et.al. (2008) studied numerically and experimentally the dynamic motion of an adaptive feedback-controlled punching machine. The machine was made of a toggle driven by a permanent magnet synchronous servomotor [13]. Huang et.al. (2009) derived the dynamic equations of a punching machine toggle mechanism using Hamilton's, Lagrange multiplier, geometric constraints
and partitioning method. They used the recursive least squares method to identify the motor-toggle unknown parameters [14]. Huang et.al.(2011) explored the effect of the key design parameters on the performance of a 5-point double-toggle clamping mechanism. They used the genetic algorithm to obtain the optimal solution of the clamping mechanism [15].

## 2. Analysis

Assumptions:

- Neglecting element weight.
- The design problem is static (neglecting inertia forces and moments).
Links orientation and positions: See Fig.1:


Notations:

| $r_{2}:$ | length of input link OA. |
| :--- | :--- |
| $r_{3}:$ | length of AC. |
| $\mathrm{r}_{4}:$ | length of coupler link AB. |
| $\mathrm{X}:$ | slider position relative to origin O. |
| $\theta:$ | orientation of coupler. |
| $\varphi:$ | orientation of input link. |

Known parameters:

- Coupler angle, $\theta$.
- Slider position, X.


Fig.2: Free body diagram of Link 2

- Moment about O gives:

$$
\mathrm{F}_{\mathrm{i}}\left[\left(\mathrm{r}_{2}+\mathrm{r}_{3}\right) \cos \varphi\right]=\mathrm{F}_{42} \mathrm{r}_{2} \cos (90-\theta+\varphi)
$$

- Giving:

$$
\begin{equation*}
F_{42}=-----------\cdots------r_{i} \tag{2}
\end{equation*}
$$

- Eq. 2 becomes:

$$
F_{42 n}=\frac{(1+-----\cdots)}{\cos (90-\theta+\varphi)}
$$

free body diagram for the toggle slider show in Fig. 3

- Normalizing the dimensions and force through:

$$
r_{3 n}=r_{3} / r_{2}, \quad r_{4 n}=r_{4} / r_{2}, \quad F_{42 n}=F_{42} / F_{i}
$$



Fig.3: Free body Diagram of 4-bar Mechanism's slider

Using $\sum \mathrm{F}_{\mathrm{y}}=0$ :
$\mathrm{F}_{14}=\mathrm{F}_{24} \sin \theta$
Using $\sum \mathrm{F}_{\mathrm{x}}=0$ :
$\mathrm{F}_{\mathrm{o}}=\mathrm{F}_{24} \cos \theta-\mu \mathrm{F}_{14}$
Combining Eqs. 3, 4 and 5 with $\mathrm{F}_{24}=\mathrm{F}_{42}$ gives:

$$
\begin{equation*}
\mathrm{F}_{\mathrm{o}}=\frac{(\cos \theta-\mu \sin \theta)\left(\mathrm{r}_{2}+\mathrm{r}_{3}\right) \cos \varphi}{\mathrm{r}_{2} \cos \left(90-\theta+---------------\mathrm{F}_{\mathrm{i}}\right)} \tag{6}
\end{equation*}
$$

Mechanical advantage, MA:
Definition: $\quad \mathrm{MA}=\mathrm{F}_{\mathrm{o}} / \mathrm{F}_{\mathrm{i}}$
Equation: Using Eq. 6 and the normalized parameters:

$$
\begin{aligned}
& (\cos \theta-\mu \sin \theta)\left(1+r_{3 n}\right) \cos \varphi \\
& \text { MA = ----------------------------------------- } F_{i} \\
& \cos (90-\theta+\varphi)
\end{aligned}
$$

## 3. Optimal toggle design

Objective function, F: The objective function of the toggle design problem is selected as the mechanical advantage (MA) which has to be maximized.
Mechanism design variables: $\quad r_{3 n}$ and $r_{4 n}$.
Design inputs:

- Normalized slider position: $X_{n}\left(X / r_{2}\right)$
- Coupler angle: $\theta$
- Coefficient of friction: $\mu$

Coefficient of friction, $\mu$ : The coefficient of friction depends on the surface material, sliding speed, normal force and lubrication oil temperature [16]. Depending on the above parameters the coefficient of friction for lubricated sliders lie in the range: $0.04 \leq \mu \leq 1.20$ [17].

### 3.1. Functional constraints:

The functional constraints of the optimization problems are set to obtain a successful optimization process. In the toggle optimization problem, the functional constraints are:

- Functional constraint $1: r_{4}$ has to be greater than the projection of $r_{2}$ on the vertical. Thus:
$\mathrm{r}_{4 \mathrm{n}}>\mathrm{r}_{2} \sin \varphi$
Using the normalized variables, the first functional constraint is:

$$
\begin{equation*}
\mathrm{FF}(1)=\mathrm{r}_{4 \mathrm{n}}-\sin \varphi>0 \tag{8}
\end{equation*}
$$

- Functional constraint 2: It is about the slider position X from O (see Fig.1). X is related to the toggle parameters through:
$X=r_{2} \cos \varphi-r_{4} \cos \theta$
Using the normalized dimensions:
$X_{\mathrm{n}}=\cos \varphi-\mathrm{r}_{4 \mathrm{n}} \cos \theta$

Eq. 9 represents the second functional constraint:
$\mathrm{FF}(2)=\cos \varphi-\mathrm{r}_{4 \mathrm{n}} \cos \theta$
(10)

It has an upper limit defined by the desired position of the slider during processing (it is set as a fraction of $r_{2}$ ).

## 4. Optimization results

- The objective function given by Eq. 7 is maximized considering the constraints given by Eqs. 9 and 10 using any suitable optimization technique.
- Optimization technique: The Powell optimization technique of unconstrained problems is applied after problem transformation to cope with constrained multi-variables problems without functions differentiation [18-20].
- The optimization results using Powell's technique for an 0.08 coefficient of friction and a variable values of the maximum $r_{3 n}$ are given in:
- Tables $1-8$ for $X_{n}=0.1$.
- Tables $9-16$ for $X_{n}=0.2$.
- Tables $17-24$ for $X_{n}=0.3$.

Table 1: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.1$ and $\theta=5^{\circ}$ :

| $\mathrm{r}_{\text {3nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.1 | 0.1 | 0.9 | 135.753 |
| 0.2 | 0.2 | 0.9 | 135.761 |
| 0.4 | 0.4 | 0.9 | 158.387 |
| 0.6 | 0.6 | 0.9 | 181.006 |
| 0.8 | 0.8 | 0.9 | 203.633 |
| 1.0 | 1.0 | 0.9 | 226.268 |
| 1.2 | 1.2 | 0.9 | 248.882 |
| 1.4 | 1.4 | 0.9 | 271.512 |
| 1.6 | 1.6 | 0.9 | 294.116 |
| 1.8 | 1.8 | 0.9 | 316.776 |
| 2.0 | 2.0 | 0.9 | 339.364 |

Table 2: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.1$ and $\theta=10^{\circ}$ :

| $\mathrm{r}_{\text {3nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.901 | 66.264 |
| 0.4 | 0.4 | 0.901 | 77.307 |
| 0.6 | 0.6 | 0.901 | 88.351 |
| 0.8 | 0.8 | 0.901 | 99.392 |
| 1.0 | 1.0 | 0.901 | 110.436 |
| 1.2 | 1.2 | 0.901 | 121.479 |
| 1.4 | 1.4 | 0.901 | 132.527 |
| 1.6 | 1.6 | 0.901 | 143.559 |
| 1.8 | 1.8 | 0.901 | 154.609 |
| 2.0 | 2.0 | 0.901 | 165.653 |

Table 3: Optimal toggle design for $X_{n}=0.1$ and $\theta=15^{\circ}$ :

| $\mathrm{r}_{\text {3nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.3 | 0.3 | 0.903 | 46.159 |
| 0.4 | 0.4 | 0.903 | 49.710 |
| 0.6 | 0.6 | 0.903 | 56.810 |
| 0.8 | 0.8 | 0.903 | 63.912 |
| 1.0 | 1.0 | 0.903 | 71.013 |
| 1.2 | 1.2 | 0.903 | 78.116 |
| 1.4 | 1.4 | 0.903 | 85.215 |
| 1.6 | 1.6 | 0.903 | 92.315 |
| 1.8 | 1.8 | 0.903 | 99.418 |
| 2.0 | 2.0 | 0.903 | 106.519 |

Table 4: Optimal toggle design for $X_{n}=0.1$ and $\theta=20^{\circ}$ :

| $\mathrm{r}_{3 \mathrm{nmax}}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.3 | 0.3 | 0.905 | 32.971 |
| 0.4 | 0.4 | 0.905 | 35.508 |
| 0.6 | 0.6 | 0.905 | 40.580 |
| 0.8 | 0.8 | 0.905 | 45.905 |
| 1.0 | 1.0 | 0.905 | 50.725 |
| 1.2 | 1.2 | 0.905 | 55.798 |
| 1.4 | 1.4 | 0.905 | 60.870 |
| 1.6 | 1.6 | 0.905 | 65.941 |
| 1.8 | 1.8 | 0.905 | 71.014 |
| 2.0 | 2.0 | 0.905 | 76.085 |

Table 5: Optimal toggle design for $X_{n}=0.1$ and $\theta=25^{\circ}$ :

| $\mathrm{r}_{3 \text { max }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.908 | 22.875 |
| 0.3 | 0.3 | 0.905 | 24.779 |
| 0.4 | 0.4 | 0.905 | 26.687 |
| 0.6 | 0.6 | 0.905 | 30.499 |
| 0.8 | 0.8 | 0.905 | 34.311 |
| 1.0 | 1.0 | 0.905 | 38.124 |
| 1.2 | 1.2 | 0.905 | 41.936 |
| 1.4 | 1.4 | 0.905 | 45.749 |
| 1.6 | 1.6 | 0.905 | 49.561 |
| 1.8 | 1.8 | 0.905 | 53.374 |
| 2.0 | 2.0 | 0.905 | 57.185 |

Table 6: Optimal toggle design for $X_{n}=0.1$ and $\theta=30^{\circ}$ :

| $\mathrm{r}_{3 \mathrm{nmax}}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.3 | 0.3 | 0.912 | 19.110 |
| 0.4 | 0.4 | 0.912 | 20.581 |
| 0.6 | 0.6 | 0.912 | 23.523 |
| 0.8 | 0.8 | 0.912 | 26.461 |
| 1.0 | 1.0 | 0.912 | 29.404 |
| 1.2 | 1.2 | 0.912 | 32.344 |
| 1.4 | 1.4 | 0.912 | 35.285 |
| 1.6 | 1.6 | 0.912 | 38.224 |
| 1.8 | 1.8 | 0.912 | 41.165 |
| 2.0 | 2.0 | 0.912 | 44.105 |

Table 7: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.1$ and $\theta=35^{\circ}$ :

| $\mathrm{r}_{3 \text { nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.916 | 13.760 |
| 0.3 | 0.3 | 0.916 | 14.909 |
| 0.4 | 0.4 | 0.916 | 16.054 |
| 0.6 | 0.6 | 0.916 | 18.347 |
| 0.8 | 0.8 | 0.916 | 20.641 |
| 1.0 | 1.0 | 0.916 | 22.935 |
| 1.2 | 1.2 | 0.916 | 25.231 |
| 1.4 | 1.4 | 0.916 | 27.521 |
| 1.6 | 1.6 | 0.916 | 29.818 |
| 1.8 | 1.8 | 0.916 | 32.109 |
| 2.0 | 2.0 | 0.916 | 34.402 |

Table 8: Optimal toggle design for $X_{n}=0.1$ and $\theta=40^{\circ}$ :

| $\mathrm{r}_{3 \text { nax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.4 | 0.4 | 0.921 | 12.540 |
| 0.6 | 0.6 | 0.921 | 14.331 |
| 0.8 | 0.8 | 0.921 | 16.125 |
| 1.0 | 1.0 | 0.921 | 17.916 |
| 1.2 | 1.2 | 0.921 | 19.705 |
| 1.4 | 1.4 | 0.921 | 21.499 |
| 1.6 | 1.6 | 0.921 | 23.289 |
| 1.8 | 1.8 | 0.921 | 25.083 |
| 2.0 | 2.0 | 0.921 | 26.874 |

Table 9: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.2$ and $\theta=5^{\circ}$ :

| $\mathrm{r}_{3 \mathrm{nmax}}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.3 | 0.3 | 0.801 | 73.590 |
| 0.4 | 0.4 | 0.801 | 79.246 |
| 0.6 | 0.6 | 0.801 | 90.572 |
| 0.8 | 0.8 | 0.801 | 101.893 |
| 1.0 | 1.0 | 0.801 | 113.216 |
| 1.2 | 1.2 | 0.801 | 124.537 |
| 1.4 | 1.4 | 0.801 | 135.859 |
| 1.6 | 1.6 | 0.801 | 147.180 |
| 1.8 | 1.8 | 0.801 | 158.500 |
| 2.0 | 2.0 | 0.801 | 168.823 |

Table 10: Optimal toggle design for $X_{n}=0.2$ and $\theta=10^{\circ}$ :

| $\mathrm{r}_{3 \mathrm{nmax}}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.3 | 0.3 | 0.802 | 35.985 |
| 0.4 | 0.4 | 0.802 | 38.753 |
| 0.6 | 0.6 | 0.802 | 44.289 |
| 0.8 | 0.8 | 0.802 | 49.825 |
| 1.0 | 1.0 | 0.802 | 55.361 |
| 1.2 | 1.2 | 0.802 | 60.898 |
| 1.4 | 1.4 | 0.802 | 66.430 |
| 1.6 | 1.6 | 0.802 | 71.975 |
| 1.8 | 1.8 | 0.802 | 77.506 |
| 2.0 | 2.0 | 0.802 | 83.047 |

Table 11: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.2$ and $\theta=15^{\circ}$ :

| $\mathrm{r}_{3 \text { nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.805 | 21.430 |
| 0.3 | 0.3 | 0.805 | 23.215 |
| 0.4 | 0.4 | 0.805 | 25.002 |
| 0.6 | 0.6 | 0.805 | 28.572 |
| 0.8 | 0.8 | 0.805 | 32.141 |
| 1.0 | 1.0 | 0.805 | 35.716 |
| 1.2 | 1.2 | 0.805 | 39.286 |
| 1.4 | 1.4 | 0.805 | 42.861 |
| 1.6 | 1.6 | 0.805 | 46.429 |
| 1.8 | 1.8 | 0.805 | 50.000 |
| 2.0 | 2.0 | 0.805 | 53.571 |

Table 12: Optimal toggle design for $X_{n}=0.2$ and $\theta=20^{\circ}$ :

| $\mathrm{r}_{3 \text { nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.810 | 15.378 |
| 0.3 | 0.3 | 0.810 | 16.659 |
| 0.4 | 0.4 | 0.810 | 17.941 |
| 0.6 | 0.6 | 0.810 | 20.504 |
| 0.8 | 0.8 | 0.810 | 23.066 |
| 1.0 | 1.0 | 0.810 | 25.630 |
| 1.2 | 1.2 | 0.810 | 28.192 |
| 1.4 | 1.4 | 0.810 | 30.755 |
| 1.6 | 1.6 | 0.810 | 33.317 |
| 1.8 | 1.8 | 0.810 | 35.883 |
| 2.0 | 2.0 | 0.810 | 38.444 |

Table 13: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.2$ and $\theta=25^{\circ}$ :

| $\mathrm{r}_{3 \text { max }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.815 | 13.328 |
| 0.3 | 0.3 | 0.815 | 14.438 |
| 0.4 | 0.4 | 0.815 | 15.549 |
| 0.6 | 0.6 | 0.815 | 17.770 |
| 0.8 | 0.8 | 0.815 | 19.992 |
| 1.0 | 1.0 | 0.815 | 22.212 |
| 1.2 | 1.2 | 0.815 | 24.434 |
| 1.4 | 1.4 | 0.815 | 26.655 |
| 1.6 | 1.6 | 0.815 | 28.875 |
| 1.8 | 1.8 | 0.815 | 31.097 |
| 2.0 | 2.0 | 0.815 | 33.315 |

Table 14: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.2$ and $\theta=30^{\circ}$ :

| $\mathrm{r}_{3 \text { max }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.822 | 10.940 |
| 0.3 | 0.3 | 0.822 | 11.851 |
| 0.4 | 0.4 | 0.822 | 12.763 |
| 0.6 | 0.6 | 0.822 | 14.586 |
| 0.8 | 0.8 | 0.822 | 16.409 |
| 1.0 | 1.0 | 0.822 | 18.233 |
| 1.2 | 1.2 | 0.822 | 20.055 |
| 1.4 | 1.4 | 0.822 | 21.879 |
| 1.6 | 1.6 | 0.822 | 23.703 |
| 1.8 | 1.8 | 0.822 | 25.525 |
| 2.0 | 2.0 | 0.822 | 27.350 |

Table 15: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.2$ and $\theta=35^{\circ}$ :

| $\mathrm{r}_{3 \text { nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.830 | 9.201 |
| 0.3 | 0.3 | 0.830 | 9.967 |
| 0.4 | 0.4 | 0.830 | 10.734 |
| 0.6 | 0.6 | 0.830 | 12.267 |
| 0.8 | 0.8 | 0.830 | 13.800 |
| 1.0 | 1.0 | 0.830 | 15.334 |
| 1.2 | 1.2 | 0.830 | 16.867 |
| 1.4 | 1.4 | 0.830 | 18.401 |
| 1.6 | 1.6 | 0.830 | 19.933 |
| 1.8 | 1.8 | 0.830 | 21.468 |
| 2.0 | 2.0 | 0.830 | 23.000 |

Table 16: Optimal toggle design for $X_{n}=0.2$ and $\theta=40^{\circ}$ :

| $\mathrm{r}_{3 \text { nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.838 | 5.619 |
| 0.3 | 0.3 | 0.838 | 6.087 |
| 0.4 | 0.4 | 0.838 | 6.555 |
| 0.6 | 0.6 | 0.838 | 7.491 |
| 0.8 | 0.8 | 0.838 | 8.428 |
| 1.0 | 1.0 | 0.838 | 9.363 |
| 1.2 | 1.2 | 0.838 | 10.301 |
| 1.4 | 1.4 | 0.838 | 11.237 |
| 1.6 | 1.6 | 0.838 | 12.173 |
| 1.8 | 1.8 | 0.838 | 13.110 |
| 2.0 | 2.0 | 0.838 | 14.047 |

Table 17: Optimal toggle design for $X_{n}=0.3$ and $\theta=5^{\circ}$ :

| $\mathrm{r}_{3 \mathrm{mmax}}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.701 | 45.312 |
| 0.3 | 0.3 | 0.701 | 49.089 |
| 0.4 | 0.4 | 0.701 | 52.866 |
| 0.6 | 0.6 | 0.701 | 60.414 |
| 0.8 | 0.8 | 0.701 | 67.970 |
| 1.0 | 1.0 | 0.701 | 75.522 |
| 1.2 | 1.2 | 0.701 | 83.074 |
| 1.4 | 1.4 | 0.701 | 90.623 |
| 1.6 | 1.6 | 0.701 | 98.173 |
| 1.8 | 1.8 | 0.701 | 105.725 |
| 2.0 | 2.0 | 0.701 | 113.272 |

Table 18: Optimal toggle design for $X_{n}=0.3$ and $\theta=10^{\circ}$ :

| $\mathrm{r}_{3 \text { max }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.703 | 22.197 |
| 0.3 | 0.3 | 0.703 | 24.047 |
| 0.4 | 0.4 | 0.703 | 25.897 |
| 0.6 | 0.6 | 0.703 | 29.596 |
| 0.8 | 0.8 | 0.703 | 33.296 |
| 1.0 | 1.0 | 0.703 | 36.994 |
| 1.2 | 1.2 | 0.703 | 40.695 |
| 1.4 | 1.4 | 0.703 | 44.392 |
| 1.6 | 1.6 | 0.703 | 48.092 |
| 1.8 | 1.8 | 0.703 | 51.792 |
| 2.0 | 2.0 | 0.703 | 55.490 |

Table 19: Optimal toggle design for $X_{n}=0.3$ and $\theta=15^{\circ}$ :

| $\mathrm{r}_{3 \text { max }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.707 | 14.361 |
| 0.3 | 0.3 | 0.707 | 15.558 |
| 0.4 | 0.4 | 0.707 | 16.755 |
| 0.6 | 0.6 | 0.707 | 19.148 |
| 0.8 | 0.8 | 0.707 | 21.542 |
| 1.0 | 1.0 | 0.707 | 23.935 |
| 1.2 | 1.2 | 0.707 | 26.328 |
| 1.4 | 1.4 | 0.707 | 28.722 |
| 1.6 | 1.6 | 0.707 | 31.115 |
| 1.8 | 1.8 | 0.707 | 33.509 |
| 2.0 | 2.0 | 0.707 | 35.903 |

Table 20: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.3$ and $\theta=20^{\circ}$ :

| $\mathrm{r}_{3 \text { nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.713 | 10.348 |
| 0.3 | 0.3 | 0.713 | 11.210 |
| 0.4 | 0.4 | 0.713 | 12.072 |
| 0.6 | 0.6 | 0.713 | 13.796 |
| 0.8 | 0.8 | 0.713 | 15.521 |
| 1.0 | 1.0 | 0.713 | 17.246 |
| 1.2 | 1.2 | 0.713 | 18.970 |
| 1.4 | 1.4 | 0.713 | 20.696 |
| 1.6 | 1.6 | 0.713 | 22.420 |
| 1.8 | 1.8 | 0.713 | 24.145 |
| 2.0 | 2.0 | 0.713 | 25.869 |

Table 21: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.3$ and $\theta=25^{\circ}$ :

| $\mathrm{r}_{3 \text { nax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.720 | 7.866 |
| 0.3 | 0.3 | 0.720 | 8.522 |
| 0.4 | 0.4 | 0.720 | 9.177 |
| 0.6 | 0.6 | 0.720 | 10.488 |
| 0.8 | 0.8 | 0.720 | 11.799 |
| 1.0 | 1.0 | 0.720 | 13.111 |
| 1.2 | 1.2 | 0.720 | 14.422 |
| 1.4 | 1.4 | 0.720 | 15.733 |
| 1.6 | 1.6 | 0.720 | 17.044 |
| 1.8 | 1.8 | 0.720 | 18.354 |
| 2.0 | 2.0 | 0.720 | 19.666 |

Table 22 Optimal toggle design for $\mathbf{X}_{\mathrm{n}}=0.3$ and $\theta=30^{\circ}$ :

| $\mathrm{r}_{3 \text { nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.729 | 6.154 |
| 0.3 | 0.3 | 0.729 | 6.667 |
| 0.4 | 0.4 | 0.729 | 7.179 |
| 0.6 | 0.6 | 0.729 | 8.205 |
| 0.8 | 0.8 | 0.729 | 9.230 |
| 1.0 | 1.0 | 0.729 | 10.256 |
| 1.2 | 1.2 | 0.729 | 11.282 |
| 1.4 | 1.4 | 0.729 | 12.307 |
| 1.6 | 1.6 | 0.729 | 13.333 |
| 1.8 | 1.8 | 0.729 | 14.359 |
| 2.0 | 2.0 | 0.729 | 15.384 |

Table 23: Optimal toggle design for $\mathrm{X}_{\mathrm{n}}=0.3$ and $\theta=35^{\circ}$ :

| $\mathrm{r}_{3 \text { max }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.739 | 4.884 |
| 0.3 | 0.3 | 0.739 | 5.291 |
| 0.4 | 0.4 | 0.739 | 5.698 |
| 0.6 | 0.6 | 0.739 | 6.512 |
| 0.8 | 0.8 | 0.739 | 7.325 |
| 1.0 | 1.0 | 0.739 | 8.139 |
| 1.2 | 1.2 | 0.739 | 8.953 |
| 1.4 | 1.4 | 0.739 | 9.767 |
| 1.6 | 1.6 | 0.739 | 10.581 |
| 1.8 | 1.8 | 0.739 | 11.395 |
| 2.0 | 2.0 | 0.739 | 12.209 |

Table 24: Optimal toggle design for $X_{n}=0.3$ and $\theta=40^{\circ}$ :

| $\mathrm{r}_{3 \text { nmax }}$ | $\mathrm{r}_{3 \mathrm{n}}$ | $\mathrm{r}_{4 \mathrm{n}}$ | MA |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.751 | 3.894 |
| 0.3 | 0.3 | 0.751 | 4.218 |
| 0.4 | 0.4 | 0.751 | 4.543 |
| 0.6 | 0.6 | 0.751 | 5.192 |
| 0.8 | 0.8 | 0.751 | 5.841 |
| 1.0 | 1.0 | 0.751 | 6.490 |
| 1.2 | 1.2 | 0.751 | 7.139 |
| 1.4 | 1.4 | 0.751 | 7.788 |
| 1.6 | 1.6 | 0.751 | 8.437 |
| 1.8 | 1.8 | 0.751 | 9.086 |
| 2.0 | 2.0 | 0.751 | 9.735 |

## 5. Tables application

The 24 tables of the toggle optimal design covers:

- The slider normalized positions:
$0.1,0.2$ and 0.3 .
- The coupler orientation: $5,10,15,20,25,30$, 35 and 40 degrees
- Maximum normalized input link part: 0.1-2

The tables also cover:

- A toggle mechanical advantage from 3.894 up to 339.364 with small increments.
- Small mechanical advantage suits weak materials such as plastics and thin metallic sheets.
- This allows easy location of a desired mechanical advantage without need to curve fitting or interpolation.
- Presenting the results in a normalized form gives the mechanical engineer a good chance to assign the toggle dimensions to suit his application.
- The next section illustrates the use of the optimal design tables in designing a toggle for a specific application.


## 6. Case Study

A mechanical process requires a force of 5 KN . The operator can exert a hand force of 100 N . We want to find the optimal dimensions and other parameters of the simple toggle studied in this paper.
The desired mechanical advantage is:
$M A=5000 / 100=50$.
Using Table 11, the toggle parameters are:

- Normalized slider position:

$$
\begin{aligned}
& \mathrm{X}_{\mathrm{n}}=0.2 . \\
& \theta=15^{\circ} . \\
& \mathrm{r}_{3 \mathrm{n}}=1.8 \\
& \mathrm{r}_{4 \mathrm{n}}=0.805
\end{aligned}
$$

- Coupler orientation:
- Normalized input force part:
- Normalized coupler:

Let $\mathrm{r}_{2}=100 \mathrm{~mm}$.
This gives the toggle dimensions:

| - | Slider position from $\mathrm{O}:$ | $\mathrm{X}=20 \mathrm{~mm}$. |
| :--- | :--- | :--- |
| - | Input link part: | $\mathrm{r}_{3}=180 \mathrm{~mm}$. |
| - | Coupler: | $\mathrm{r}_{4}=80.5 \mathrm{~mm}$. |

Calculations using the simple toggle analysis:

- Input link orientation: $\quad \varphi=12.026^{\circ}$
- Mechanism transmission angle: TA $=105^{\circ}$.
- Coupler force: $\quad \mathrm{F}_{42}=527.77 \mathrm{~N}$
- Mechanical advantage: $\quad$ MA $=49.886$
- Slider position: $\quad X=20.048 \mathrm{~mm}$


## 6. Discussions

- Optimization is a powerful technique which leads to successful kinematic design of machinery.
- It tries to satisfy all the kinematic constraints assigned by the designer.
- The optimal design process of the simple 4links toggle is reduced to the assignment of the input link length, coupler length for a desired mechanical advantage.
- Mechanical advantage ranged from 3.894 to 339.364 .
- The optimization results is tabulated in 24 table to simplify using them without need to curve fitting or interpolation.
- A range $0.1-0.3$ is covered for the normalized toggle slider position.
- A range $5-40$ degrees is covered for the coupler orientation.
- A case study about using the toggle optimal design tables showed that:
- The error in the desired mechanical advantage is only $0.228 \%$.
- The error in the slider position is only 0.242 \%.
- This error is due to rounding the toggle dimensions in the tables.


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