Measuring Bullwhip Effect in a Three Stage Supply Chain with Exponential Smoothing Method

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Abstract: Today many of researchers study bullwhip effect, implies that demand variability increases as one moves up the supply chain. It is obvious that forecasting increases this phenomenon. In this paper bullwhip effect measured in a simple three stage supply chain consisting of a retailer, a supplier and a manufacturer. We analyze the model and quantify bullwhip effect when each stage uses exponential smoothing method to forecast future demand. At result we propose bullwhip effect related to lead time and smoothing factor.
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1. Introduction

It is becoming increasingly difficult to ignore the demand variations in supply chain management. Recently, researchers try to identify and reduce demand variations, stated as Bullwhip Effect.

The first recognition of bullwhip effect in supply chain management has been done by Forrester (1958,1961). He presented some evidences of bullwhip effect phenomenon and discusses the causes of this phenomenon.

Lee et al.(1997 a,b) identified five main causes of bullwhip effect, including demand forecasting, lead time, batch ordering, shortages and price fluctuations. They considered AR(1) demand process in a simple two stage supply chain. Chen et al.(2000a) quantified the bullwhip effect in a simple two stage supply chain with AR(1) demand process observed by the retailer. The retailer uses moving-average forecasting technique to predict future customer demands. They have derived a lower bound for the bullwhip effect. Chen et al.(2000b) extend their results to a case where the downstream retailer use exponential smoothing method. They measured the bullwhip effect and found that it depends on both the nature of customer demand process and the forecasting technique used by retailer. They concluded that if mean and variance of demand is known exactly, i.e. no forecasting method need to be used, there would no bullwhip effect. Furthermore bullwhip effect increases by increasing the lead time. Xiaolong Zhang(2004) considered a simple supply chain with order-up-to replenishment system and first-order autoregressive process describes the customer demand. He minimized the mean-squared forecasting error for the specified demand process and concluded that different forecasting techniques lead to different bullwhip effect in relation to lead time and demand process parameters. Rafiee and Akbar Jokar(2008) analyzed various inventory policies on the bullwhip effect. They examined moving average and exponential smoothing forecasting techniques in a two level supply chain.

However, there has been little discussion so far about bullwhip effect in three stage supply chain. This paper focuses on measuring bullwhip in a supply chain in which exponential smoothing is used for forecasting future demands.

2. A simple supply chain model

Consider a simple supply chain consisting of a retailer, a supplier and a manufacturer. In each period, the single retailer places an order, q_t, to the single supplier according to available inventory level. Then, the retailer observes and fills customer demand, D_t, for that period. Customer demands are i.i.d. with average μ and variance σ^2 The supplier observes retailer demands and places an order each n periods denoted by q_t^r, to the manufacturer. Any unfilled demands are backlogged. The retailer and supplier lead time, which are time spent during placing an order and receiving it, are fix and denoted consequently by L and L'. Retailer and supplier use order-up-to policy and forecast future demands by exponential smoothing method. For i.i.d. demands from a normal distribution, if there is no setup cost, order-up-to policy is optimal.(Nahmias, 1993) Order-up-to point in retailer and supplier are R_t and R'_t. Both of retailer and supplier use exponential method for predicting future demands.

The retailer observes customer demands ordered by customers and forecast t period demand by $\overline{D}_t$ as:

$$\overline{D}_t = \alpha D_{t-1} + (1-\alpha)\overline{D}_{t-1} \quad (1)$$

The order-up-to point is:

$$R_t = (L+T)D_t + SS \quad (2)$$

That SS is safety stock. According to Rafiee & Akbari Jokar(2009) order quantity is:

$$Q_i = R_i - R_{i-1} + D_{i-1} = (L+T)(\overline{D}_i - \overline{D}_{i-1}) + D_{i-1} \quad (3)$$

$$= (L+T)(\alpha)(D_{i-1} - \overline{D}_{i-1}) + D_{i-1} = [\alpha(L+T)+1]D_{i-1} - \alpha(L+T)\overline{D}_{i-1}$$

Then: (7)

$$Var(Q_i) = [\alpha(L+T)+1]^2 Var(D) + \alpha^2 (L+T)^2 Var(\overline{D}_{i-1})$$

$$-2\alpha(L+T)[\alpha(L+T)+1]Cov(D_{i-1}, \overline{D}_{i-1}) \quad (4)$$

According to this fact that:

$$Cov(D_{i-1}, \overline{D}_{i-1}) = 0 \quad (5)$$

And :

$$Var(Q_i) = [\alpha(L+T)+1]^2 Var(D) + \frac{\alpha^3 (L+T)^2}{2-\alpha} Var(D)$$

$$= \{[\alpha(L+T)+1]^2 + \frac{\alpha^3 (L+T)^2}{2-\alpha}\} Var(D) \quad (6)$$

Bullwhip Effect can be written as:

$$\text{B.E.} = \left\{[\alpha(L+T)+1]^2 + \frac{\alpha^3 (L+T)^2}{2-\alpha}\right\} \quad (7)$$

They concluded that α and $L+T$ are affecting on bullwhip effect. Increasing lead time, ordering period and smoothing factor causes increasing bullwhip effect.

3. Supplier Replenishment System

We develop the research to 3 echelon supply chain. Suppose the supplier orders to a manufacturer every n period. Like last stage, order-up-to point is:

$$R'_i = (L' + T')\overline{D'_i} + SS' \quad (8)$$

The demands that the supplier is observing, are the retailer orders, in other words $\overline{D'_i} = Q_i$. Using this reality we can write:

$$Q'_i = R'_i - R'_{i-n} + Q_{i-n} + Q_{i-n+1} + Q_{i-n+2} + \ldots + Q_{i-1} \quad (9)$$

$$= (L'+T')(Q_i - Q_{i-n}) + Q_{i-n+1} + Q_{i-n+2} + \ldots + Q_{i-1}$$

And:

$$Q_i - Q_{i-n} = [\alpha(L+T)+1]D_{i-1} - \alpha(L+T)\overline{D}_{i-1}$$

$$-[\alpha(L+T)+1]D_{i-n-1} + \alpha(L+T)\overline{D}_{i-n-1}$$

$$= [\alpha(L+T)+1](D_{i-1} - D_{i-n-1}) \quad (10)$$

$$-\alpha(L+T)\sum_{j=1}^{\infty}\alpha(1-\alpha)^{j-1}[D_{i-j-1} - D_{i-n-j-1}]$$

And the second segment can be written as:

$$Q_{i-n} + Q_{i-n+1} + Q_{i-n+2} + \ldots + Q_{i-1} =$$

$$[\alpha(L+T)+1][D_{i-n-1} + D_{i-n} + D_{i-n+1} + \ldots + D_{i-2}] \quad (11)$$

$$-\alpha(L+T)[\overline{D}_{i-n-1} + \overline{D}_{i-n} + \overline{D}_{i-n+1} + \ldots + \overline{D}_{i-2}]$$

$$= [\alpha(L+T)+1][D_{i-n-1} + D_{i-n} + D_{i-n+1} + \ldots + D_{i-2}]$$

$$-\alpha(L+T)\sum_{j=1}^{\infty}\alpha(1-\alpha)^{j-1}[D_{i-j-n-1} + D_{i-j-n} + \ldots + D_{i-j-2}] \quad (5)$$

Using two latest relations we can write:

$$Q'_i = (L'+T')[\alpha(L+T)+1](D_{i-1} - D_{i-n-1}) \quad (12)$$

$$-\alpha(L+T)(L'+T')\sum_{j=1}^{\infty}\alpha(1-\alpha)^{j-1}[D_{i-j-1} - D_{i-n-j-1}]$$

$$+[\alpha(L+T)+1][D_{i-n-1} + D_{i-n} + D_{i-n+1} + \ldots + D_{i-2}]$$

$$-\alpha(L+T)\sum_{j=1}^{\infty}\alpha(1-\alpha)^{j-1}[D_{i-j-n-1} + D_{i-j-n} + \ldots + D_{i-j-2}]$$

Now for calculating the variance, we should calculate covariance between above terms as below:

$$Cov(D_i - D_{i-n-1}, \overline{D}_{i-n} - \overline{D}_{i-n-1}) =$$

$$Cov(D_i, \overline{D}_{i-n} - \overline{D}_{i-n-1})$$

$$-Cov(D_{i-n-1}, \overline{D}_{i-n} - \overline{D}_{i-n-1}) \quad (13)$$

$$= 0 - \alpha Var(D) = -\alpha Var(D)$$

$$Cov(D_i - D_{i-n-1}, D_{i-n-1} + ... + D_{i-2}) =$$
$$Cov(D_i, D_{i-n-1} + D_{i-n} + D_{i-n+1} + ... + D_{i-2}) \quad (14)$$
$$- Cov(D_{i-n-1}, D_{i-n-1} + D_{i-n} + D_{i-n+1} + ... + D_{i-2})$$
$$= 0 - (1)Var(D) = -Var(D)$$

$$Cov(D_i - D_{i-n-1}, \overline{D}_{i-n-1} + ... + \overline{D}_{i-2}) \quad (14)$$
$$= Cov(D_i, \overline{D}_{i-n-1} + \overline{D}_{i-n} + \overline{D}_{i-n+1} + ... + D_{i-2})$$
$$- Cov(D_{i-n-1}, \overline{D}_{i-n-1} + \overline{D}_{i-n} + \overline{D}_{i-n+1} + ... + \overline{D}_{i-2})$$
$$= 0 - [\alpha + \alpha(1-\alpha) + ... + \alpha(1-\alpha)^{n-2}]Var(D)$$
$$= -[1 - (1-\alpha)^{n-1}]Var(D)$$

$$Cov(\overline{D}_{i-n} - \overline{D}_{i-n-1}, D_{i-n-1} + ... + D_{i-2}) =$$
$$Cov(\overline{D}_{i-n}, D_{i-n-1} + D_{i-n} + ... + D_{i-2}) \quad (15)$$
$$- Cov(\overline{D}_{i-n-1} - \overline{D}_{i-n-1}, D_{i-n-1} + ... + D_{i-2})$$
$$= Cov(\sum_{j=1}^{\infty} \alpha(1-\alpha)^{j-1} D_{i-n-j}, D_{i-n-1} + ... + D_{i-2})$$
$$- Cov(\sum_{j=1}^{\infty} \alpha(1-\alpha)^{j-1} D_{i-n-j-1}, D_{i-n-1} + ... + D_{i-2})$$
$$= \alpha Var(D) - 0 = \alpha Var(D)$$

$$Cov(\overline{D}_{i-n} - \overline{D}_{i-n-1}, \overline{D}_{i-n-1} + \overline{D}_{i-n} + ... + \overline{D}_{i-2})$$
$$= Cov(\overline{D}_{i-n}, \overline{D}_{i-n-1} + \overline{D}_{i-n} + ... + \overline{D}_{i-2}) \quad (16)$$
$$- Cov(\overline{D}_{i-n-1}, \overline{D}_{i-n-1} + \overline{D}_{i-n} + ... + \overline{D}_{i-2})$$
$$= \frac{\alpha(1-\alpha) + (1-\alpha)^n - (1-\alpha)^{n-1}}{2-\alpha} Var(D)$$

$$Cov(D_{i-n-1} + D_{i-n} + D_{i-n+1} + ... + D_{i-2},$$
$$\overline{D}_{i-n-1} + \overline{D}_{i-n} + \overline{D}_{i-n+1} + ... + \overline{D}_{i-2}) =$$
$$Cov(D_{i-n-1}, \overline{D}_{i-n-1} + \overline{D}_{i-n} + ... + \overline{D}_{i-2})$$
$$+ Cov(D_{i-n}, \overline{D}_{i-n-1} + \overline{D}_{i-n} + \overline{D}_{i-n+1} + ... - \quad (18)$$
$$+ ... + \mathrm{cov}(D_{i-2}, \overline{D}_{i-n-1} + \overline{D}_{i-n} + ... + \overline{D}_{i-2}) =$$
$$\{[0 + \alpha + \alpha(1-\alpha) + ... + \alpha(1-\alpha)^{n-2}] +$$
$$[0 + 0 + \alpha + \alpha(1-\alpha) + ... + \alpha(1-\alpha)^{n-3}] +$$
$$... + [\alpha] + [0]\}Var(D)$$
$$= \{(n-1) - \frac{(1-\alpha) - (1-\alpha)^n}{\alpha}\}Var(D)$$

Regarding to these relations:

$$Var(Q') = 2(L' + T')^2[\alpha(L+T)+1]^2 Var(D) +$$
$$\alpha^2(L+T)^2(\frac{2\alpha}{2-\alpha})Var(D) + n(\alpha(L+T)+1)^2 Var(D)$$
$$+ \alpha^2(L+T)^2(\frac{n\alpha}{2-\alpha})Var(D)$$
$$-2(L' + T')[\alpha(L+T)+1]\alpha(L+T)\{-\alpha Var(l \quad (17)$$
$$+2(L' + T')[\alpha(L+T)+1]^2[-Var(D)]$$
$$-2(L' + T')[\alpha(L+T)+1]\alpha(L+T)\{-[1-(1-\alpha)^{n-1}]\}Var(D)$$
$$-2\alpha(L+T)[\alpha(L+T)+1]\{\alpha Var(D)\}$$
$$+2\alpha^2(L+T)^2[\frac{\alpha(1-\alpha)+(1-\alpha)^n-(1-\alpha)^{n-1}}{2-\alpha}]$$
$$-2[\alpha(L+T)+1]\alpha(L+T)\{\alpha Var(D)\} \quad (18)$$

And bullwhip effect calculated as:

$$\frac{Var(Q')}{Var(D)} = 2(L' + T')^2[\alpha(L+T)+1]^2$$
$$+(\frac{2\alpha^3}{2-\alpha})(L+T)^2 + n(\alpha(L+T)+1)^2$$
$$+(\frac{n\alpha^2}{2-\alpha})(L+T)^2 + 2\alpha^2(L' + T')[\alpha(L+T)+1](L+T)$$
$$-2(L' + T')[\alpha(L+T)+1]^2$$
$$+2\alpha(L' + T')(L+T)[\alpha(L+T)+1][1-(1-\alpha)^{n-1}]$$
$$-2\alpha^2(L+T)[\alpha(L+T)+1]$$
$$+2\alpha^2(L+T)^2[\frac{\alpha(1-\alpha)+(1-\alpha)^n-(1-\alpha)^{n-1}}{2-\alpha}]$$
$$-2\alpha^2[\alpha(L+T)+1](L+T) \quad (19)$$

It can be concluded that all of parameters including lead time, reviewing period, smoothing coefficient have second order effect on bullwhip effect.

4. Conclusions and Discussions

The purpose of the current study is determination of factors affecting bullwhip effect in a supply chain where it is developed to the third stage. The most obvious finding to emerge from the study is that each stage forecasting increases the bullwhip effect and lead time, ordering period and smoothing factor are the main factors that lead to this phenomenon. It is supposed that each stage uses exponential smoothing method for forecasting future demand.

Further future more studies on the issue are therefore recommended. To develop this study, forecasting technique can be changed and different stages may use different forecasting techniques. Of course trying to get public bullwhip effect formula among the different stages of the supply chain is great.

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