A ga algorithm for a two–echelon inventory system with space constraint & compare this with simulated annealing

Seyyed Jamal Hosseini [1], L.Hojagani [2]

[1]Department of Management, Economics, Accounting ,payame noor unvierstiy,I.R.of IRAN.poBox19395-3697tehran,IRAN.PHD candidat of.industrial Management in AMEA
[2.]Azad university of Qazvin, Iran
pnu_zonoz @yahoo.com

Abstract: One of the key areas of operations and supply chain management is inventory control. Inventory control determines which quantity of a product should be ordered when to achieve some objective, such as minimizing cost. This paper presents a two-echelon non-repairable spare parts inventory system that consists of one warehouse with space constraint and m identical retailers and implements the reorder point, order quantity (R, Q) inventory policy . We formulate the policy decision problem in order to minimize the total annual inventory investment subject to average annual ordering frequency and expected number of backorder constraints.
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1. Introduction

Research on inventory control can be traced back to Harris, who developed the well-known economic-order-quantity (EOQ) model in 1915. Since then, hundreds of papers on inventory control have been published. Most of these papers essentially follow the same approach.

First, the inventory-control problem is translated into a mathematical model. Second, an inventory-control policy that optimizes the mathematical model is derived. Third, an algorithm for finding the optimal values of the decision variables of the inventory-control policy is developed. The modeling, the optimization, and the development of the algorithm are performed by highly skilled experts and can be quite time consuming. Since skilled experts are expensive and sought after for a variety of projects in any institution, it would be beneficial to have an alternative approach that requires less expert involvement than the traditional approach.

2. Literature review

One of the most important multi-echelon, multi- item inventory models for spare parts management is METRIC. METRIC is the Multi-Echelon Tech- nique for Recoverable Items Control, developed by Sherbrooke (1968)[10] and it is used for setting repairable items inventory control policies using the base stock model. The base stock model is a special case of the reorder point, order quantity inventory policy, where the reorder quantity Q = 1 and it is usually used with expensive, slow moving items, and when the holding and back order costs dominate. The objective function in METRIC is minimizing the expected number of backorders at the base level, subject to budget constraints while setting optimal inventory policy parameters. In the case of low or medium cost items with medium to high demand rates, the (R, Q) policy may be more appropriate. Many inventory models have been developed for expensive, low demand, and repairable spare parts (e.g. Sherbroo ke, 1968 ; Grave s, 1985 ; Diaz an d Fu, 1997; Caglar et al., 2004[10],[5],[4],[1]), where the base stock model is implemented at least at one echelon of the supply network. In other research,rosetti in 2007 [14]research about two-echelon non-repairable spare parts inventory system that consists of one warehouse and m identical retailers and implements the reorder point, order quantity (R, Q) inventory policy.

3. Problem definition and model formulation

We have a two-echelon inventory system that consists of an external supplier that can supply any item with a given lead time and a single warehouse that supplies any number of independent identical retailers. Under this system, the retailers are faced with demands that are generated by random failures of the spare parts at the customer's sites according to a Poisson process. Since the demand process at each retailer for each item is a Poisson process, the demand process at any warehouse is a superposition of the retailer's ordering processes. Specifically, it is a superposition of renewal processes each with an Erlang inter renewal processes time with Qri stages and rate per state λ_{ri} (Svoronos and Zipkin, 1988[11]).

The above two-echelon (R, Q) inventory system operates as follows. When a retailer is faced with a demand, the demand is satisfied from shelves if the amount demanded is less or equal to the number of units available. Otherwise, the demand is backordered. Under an (R, Q) policy, item i's inventory position at retailer r is checked continuously, if it drops to or below its reorder point R is placed at the warehouse. The inventory position is defined as the on-hand inventory plus the on-order inventory minus the number of outstanding backorders. After receiving the replenishment order, the outstanding backorders at the retailer are immediately satisfied according to a first-in-first-out (FIFO) policy. Since the same policy is followed at the warehouse. Before proceeding in developing the model, we state our assumptions as follows. We model a two echelon inventory system, where each retailer is replenished by only one warehouse. The demand process at each retailer occurs according to a Poisson process. All orders that are not satisfied from on hand inventory are backordered (i.e. lost sales are not considered). The warehouse's supplier has infinite capacity with a fixed lead time, the warehouse with space constraint has limited supply, delay time in warehouse (because of shortage) is considered zero and no lateral shipments are permitted between the retailers.

We do not model the delivery process from the retailer to the end customer. The following is a list of the notation that we will use throughout the paper:

w warehouse index
r retailer index
i item index
m number of retailers
N number of items
F_r target order frequency at retailer r (orders per year)
F_w target order frequency at the ware house (orders per year)
B_r target number of backorders at retailer r
B_w target number of backorders ast the ware house
λ_{ri} Item i demand rate at retailer r (unit/ year)
λ_{wi} Item i demand rate at the ware house (in units of item i batch size at retailer per year)
L_{ri} item i lead time (ordering and transportation)at retailer r (year)
L_{wi} item i lead time (ordering and transportation)at the warehouse (years)
ℓ_{ri} Item i effective lead time at retailer r (years)
C_i total inventory investment at both echelons($)
C superscript that represents the current value
P superscript that represents the previous value
Qri item i replenishment batch size at retailer r(units)
Rri item i reorder point at retailer r(units)
Qwi item i replenishment batch size at the warehouse(in units of Qri)
Rwi item i reorder point at the ware house (in units of Qri)
Ī ri(Rri,Qri) item i expected on-hand inventory at retailer r(units)
Īwi(Rwi, Qwi) item i expected on-hand inventory at the ware house (in units of Qri)
Item i expected number of backorders at retailer r(units).Also,Bri $\overline{B}_{ri}(\overline{R}_{ri}Q_{ri})$
$\overline{B}_{Wi}(R_{wi}Q_{wi})$ Item i expected number of backorders at warehouse(in units of Qri)).Also Bwi
$\phi(X)$ the pdf of the standard normal distribution function
the cdf of the standard normal distribution function $\Phi(X)$
the inverse of the standard normal distribution function $\Phi^{-1}(X)$
Fri item i average order frequency at retailer r
Fwi item i average order frequency at the ware house
x_i item i space
X warehouse space

We assumed identical retailers and formulate the two-echelon (R,Q)policy problem in order to minimize the total annual inventory investment at both echelons subject to the following average annual order frequency and average number of backorder constraint:

Average annual order frequency at each retailer ≤ Fri, (1)
Average annual order frequency at the ware house ≤Fw, (2)
Total expected number of back orders at each retailer ≤ Br, (3)
Total expected number of backorders at the ware house ≤Bw. (4)

We represent the above model mathematically as follows:

$$Minimize C = m\sum_{i=1}^{N} c_i \bar{I}_{ri}(R_{ri},Q_{ri}) + \sum_{i=1}^{N} c_i Q_{ri} \bar{I}_{wi}(R_{wi},Q_{wi}) \tag{5}$$

Subject to

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\lambda_{wi}}{Qwi} \le F_w, \tag{6}$$

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\lambda_{ri}}{Q_{ri}} \le F_r, \tag{7}$$

$$\sum_{i=1}^{N} \overline{B}_{ri}(R_{ri}, Q_{ri}) \le B_r \tag{8}$$

$$\sum_{i=1}^{N} \overline{B}_{wi}(R_{wi}, Q_w) \le B_w \tag{9}$$

$$\sum_{i=1}^{\mathrm{N}} \mathrm{Xi\,Qwi} = \mathrm{X} \tag{10}$$

$$R_{ri} \ge Q_{ri} \qquad i = 1.2...., N. \tag{11}$$

$$R_{wi} \ge Q_{wi} \qquad i = 1.2...., N. \tag{12}$$

$$Q_{ri} \ge 1 \qquad i = 1.2...., N. \tag{13}$$

$$Q_{wi} \ge 1 \qquad i = 1.2...., N. \tag{14}$$

$Q_{ri}, R_{ri}, Q_{wi}, \& R_{wi}$:Integers , i=1,2,…,N (15)

Constraint (11) and(12) are used to make sure that the outstanding backorders are satisfied when a replenishment order is received.. Constraints (13) and (14) are used to make sure that the mini mum allow able replenishment order size is one . Constraint (15) is necessary, since in real life no partial parts are ordered. Later on, in order to simplify the problem, constraint (15) will be relaxed to allow for continuous values . Under an(R, Q) policy the expected on-hand inventory for item i at any location when the demand during lead time is modeled using a discrete distribution (under which the inventory level declines in discrete steps) is defined as follows (Hadley and Whitin, 1963 [6]):

$$\bar{I}_i = \overline{B}_i(R_i Q_i) + R_i + \frac{Q_i + 1}{2} - E[D_i] \tag{16}$$

Where E[Di] is item i expected lead time demand and $B_i(R_i, Q_{ri})$ is item i expected number of backorders at any time. Since almost all real- world systems involve discrete inventory, it generally makes sense to use the discrete inventory formula (Eq. (16)) even when a continuous model is used to compute the policy parameters(hopp and spearman,2001[8]).

Hence, we evaluate the inventory level using Eq.(16). Since the demand process for item i at retailer r is a simple poisson process with an annual rate is:

$$E[D r_i] = \lambda_{ri} \times \ell_{ri} \tag{17}$$

$$\ell_{ri} = L_{ri} + d_{ri} \tag{18}$$

The first part of Eq.(18), specifically Lri, represents item i's transportation time from the warehouse to retailer r. Since non-repairable spare parts are considered, no parts are shipped back to the warehouse. Hence, no explicit assumption is made on the transportation time from any retailer to the warehouse. Also, ordering times are assumed to be negligible and transportation times are assumed to be deterministic.

Since the demand process at each retailer is a poisson process and an (R,Q) policy is implemented at each retailer, the demand process at the warehouse is a super position of independent renewal processes each with an erlang inter-renewal time with Qri stages and rate per state λri (svoronos and zipkin, 1988[11]). Item i's order frequency at retailer r is:

$$Fri = \frac{\lambda ri}{Qri} \tag{19}$$

Under the assumption of identical retailers item i's demand rate at the warehouse (λwi) is:

$$\lambda_{wi} = mf_{ri} = \frac{m\lambda_{ri}}{Q_{ri}} \tag{20}$$

Svoronos and zipkin (1988)[11],derived the following expressions for the mean and variance of the warehouse lead time demand under the assumption of identical independent retailers :

$$E[D_{wi}] = \frac{m\lambda_{ri}L_{wi}}{Q_{ri}} \tag{21}$$

$$V[D_{wi}] = \frac{\lambda_{ri} L_{wi} m}{Q_{ri}^2} + \frac{m}{Q_{ri}^2} \sum_{k=1}^{Q_{ri}-1} \left(\frac{[1 - \exp(-\alpha k \lambda_{ri} L_{wi}) \cos(\beta_k \lambda_{ri} L_{wi})]}{\alpha k} \right) \tag{22}$$

Where

$$\alpha_k = 1 - \cos(2\pi k / Q_{ri}), \tag{23}$$

$$\beta_k = \sin(2\pi k / Q_{ri}) \tag{24}$$

We use the normal approximation to the poisson distribution to approximate the distribution of the retailer's lead time demand .in addition, We approximate the distribution of the warehouse leadtime demand using a normal distribution with mean and variance as given by eqs.(21)and (22).

Underan (R, Q) policy, item i's expected number of backorders is (see Hopp and Spearman, 2001,[8])

$$\overline{B}_i(R_i, Q_i) = \frac{1}{Q_i}\,[B(R_i) - B(R_i + Q_i)] \tag{25}$$

$$\beta(x) = \frac{\sigma^2}{2}\{(z^2+1)[1 - I(z)] - z\phi(z)\}, \tag{26}$$

$$z = \frac{(x-\theta)}{\sigma} \tag{27}$$

Where θ and δ are the mean and standard deviation of the demand during replenishment lead time, respectively. Eq. (26) is the continuous analog of the second-order loss function $\beta(x)$ (Hopp and Spearman, 2001[8]). The second-order loss function represents the time -weighted backorders arising from lead time demand in excess of x (Hopp et al., 1997).

4. Solution Procedure

The above two-echelon (R, Q) optimization model is a large-scale, non- linear, integer optimization problem (M.H.Al-rafai,M.D.Rossetti, 2007[14]) .

Under the above assumptions, modeling each echelon independent of the other echelons is not attainable due to the dependency between them. In order to model the warehouse, the retailer's order batch size must be known a priori. To solve the above two-echelon inventory system, we assumed identical retailers and decomposed the problem into two levels; the retailer and the warehouse.

Decomposition has been used widely in many areas such as inventory management and queuing systems (e.g .Cohen et al.1990 [2]).

To solve the problem, we have used two algorithms: genetic and simulated annealing.

Finally we have compared these two algorithms, to introduce a proper solution algorithm.

5. Experimentation and Analysis

In order to asses the quality of the solutions obtained via the above heuristc optimization algorithm we compared the solutions obtained using algorithm genetic with the solutions obtained using algorithm simulated annealing .

For the sake of experimentation, we set the following target values of the order frequency and the expected number of back order constraints at the retailer and the warehouse (F_r =10,F_w = 15,B_r = 15, B_w = 10).

also, we set the number of retailers equals to three.

The data of tens sample has been shown in table1 :

Table 1

λ_1	λ_2	x_1	x_2	C_1	C_2
200	250	10	20	120	140

In table2, we have presented ten time repetition results of above sample in both algorithms and their runtimes :

Table 2: ten time repetition results

	First response				Improvement in GA					Improvement in SA				
Row	Q1	Q2	K	c	Q1	Q2	k	c	Runtime(s)	Q1	Q2	K	c	Runtime(s)
1	37	30	2	3.03e5	17	42	7	302520	61.33	49	224	15	1002200	61.59
2	28	24	2	270.84e5	33	28	1	270870	68.67	32	494	3	138400	93.28
3	499	23	3	0.482e5	499	23	3	482000	58.85	29	31	4	481960	52.32
4	40	34	2	366e5	20	39	1	365720	54.89	90	20	3	145300	123.16
5	31	30	2	255e5	32	29	7	408310	60.36	107	21	59	856100	48.98
6	27	30	2	203e5	17	42	7	302520	58.03	49	224	15	100220	58.61
7	28	34	2	271e5	33	28	1	270870	66.55	32	494	3	138400	91.71
8	29	31	4	482e5	29	31	4	481960	55.55	499	23	3	482000	41.05
9	40	34	2	366e5	20	39	1	365720	53.22	90	20	3	145300	123.9
10	37	30	2	255e5	17	42	7	302520	58.97	49	224	15	1002200	57.6

5.1 .comparison of two solution algorithms

In this paper, we compare GA algorithm with SA algorithm according to reply quality and problem solving's time.

At first, following supposition tests is used to compare reply quality and run time :

1)

$\mu(fitnessGA) = \mu(fitnessSA)$ H0 :

H1 : $\mu(fitnessGA) \neq \mu(fitnessSA)$

2)

H0: $\mu(RuntimeGA) = \mu(RuntimeSA)$

H1: $\mu(RuntimeGA) \neq \mu(RuntimeSA)$

By t-student test in SPSS software, we analyze our data . in two following tables, analysis results of the first supposition test and the second supposition test have been presented separately:

1)

Table 3: Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	cost ga	2.4000	10	2.36643	.74833
	cost sa	5.1000	10	3.60401	1.13969

TABLE 4: Paired Samples Correlations

			Correlation	Sig.
Paoir 1	Cost ga & Cost Sa	10	-.435	.209

TABLE 5: Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	cost ga – cost sa	-2.70000	5.10011	1.61280	-6.34840	.94840	-1.674	9	.128

TABLE :Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 2	time ga	63.6000	10	35.14478	11.11376
	time sa	63.9000	10	17.89755	5.65970

Table 7: Paired Samples Correlations

		N	Correlation	Sig.
Pair 2	time ga & time sa	**10**	**.441**	**.202**

Table 8 :Paired Samples Test

		Paired Differences					t	Df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 2	time ga- time sa	-.30000	31.62647	10.00117	-22.92421	22.32421	-.030	9	.977

According to the results, in both tests zero supposition is rejected . now we should examine two following tests :

3) H0: $\mu(fitnessGA) \succ \mu(fitnessSA)$

H1: $\mu(fitnessGA \prec \mu(fitnessSA)$

4) H0: $\mu(timeGA) \succ \mu(timeSA)$

H1: $\mu(timeGA) \prec \mu(timeSA)$

To doing these tests, we use Minitab software.software output for both tests has been presented :

3) Two-sample T for FITNESS(COST) GA vs FITNESS(COST)SA

	N	Mean	StDev	SE Mean
COST GA	10	355301	80095	25328
COST SA	10	449208	376980	119212

Difference = mu (COST GA) - mu (COST SA)
Estimate for difference: -93907
95% lower bound for difference: -317313

T-Test of difference = 0 (vs >): T-Value = -0,77 P-Value = 0,770 DF = 9

4) Two-sample T for RUN TIME GA vs RUNTIME SA

	N	Mean	StDev	SE Mean
RUNTIME GA	10	59,70	5,01	1,6
RUNTIME SA	10	75,3	30,5	9,7

Difference = mu (RUN TIME GA) - mu (RUNTIME SA)
Estimate for difference: -15,60
95% lower bound for difference: -33,54
T-Test of difference = 0 (vs >): T-Value = -1,59 P-Value = 0,927 DF = 9

According to the analysis results, zero supposition of the third test is acceoted . so reply quality of SA algorithm is better than that of GA algorithm . And zero supposition of the forth test is accepted, there fore Run time of SA algorithm is less than Runtime of GA algorithm .

6.Conclusion and future work

We modeled a two – echelon inventory system that implements (R,Q) policies at each facility. In order to solve the two-echelon inventory system we decomposed it by echelon . by GA and SA algorithms, we have solved the model . the results shows that reply quality of SA algorithm is better than that of GA algorithm and SA algorithm reach to reply in short of time.

Corresponding Author:
Dr. Seyyed Jamal Hosseini
Department of Management, Economics, Accounting ,payame noor unvierstiy,I.R.of IRAN.poBox19395-3697tehran,IRAN.PHD candidat of.industrial Management in AMEA
E-mail: pnu_zonoz @yahoo.com

References

1. Caglar, D., Li, C.-L., Simchi-Levi, D., 2004. Two-echelon spare parts inventory system subject to a service constraint. IIETransactions 36, 655–666.
2. Cohen, M.A., Kamesam, P.V., Kleindorfer, P., Lee, H., Tekerian, A., 1990. Optimizer: IBM's multi-echelon inventory system for managing service logistics. Interfaces 20 (1), 65–82.
3. Deuermeyer, B.L., Schwarz, L.B., 1981. A model for the analysis of system service level in warehouse-retailer distribution systems: The identical retailer case. TIMS Studies in the Management Sciences 16, 163–193.
4. Dıaz, A., Fu, M.C., 1997. Models for Multi-echelon repairable item inventory systems with limited repair capacity. European Journal of Operational Research 97, 480–492.
5. Graves, S.C., 1985. A multi-echelon inventory model for a repairable item with one-for-one replenishment. Management Science 31 (10), 1247–1256.
6. Hadley, G., Whitin, T.M., 1963. Analysis of Inventory Systems. Prentice-Hall, Inc., Englewood Cliffs, NJ. Hopp, W.J.,
7. Spearman, M.L., Zhang, R.Q., 1997. Easily implementable inventory control policies. Operations Re- search 45 (3), 327– 340.
8. Hopp, W.J., Spearman, M.L., 2001. Factory Physics, second ed. McGraw-Hill, New York.
9. Hopp, W.J., Zhang, R.Q., Spearman, M.L., 1999. An easily implementable hierarchical heuristic for a two-echelon spare parts distribution system. IIE Transactions 31, 977–988.
10. Sherbrooke, C.C., 1968. METRIC: A multi-echelon technique for recoverable item control. Operations Research 16, 122–141.
11. Svoronos, A., Zipkin, P,1988. Estimating the performance of multi-level inventory systems. Operations Research 36 (1), 57–72.
12. Torab, P., Kamen, E., 2001. On approximate renewal models for the superposition o f renewal processes. IEEE, 2901–2906.
13. Zipkin, P.H., 2002. Foundations of Inventory Management. McGraw-Hill Companions, Inc., New York.
14. M.H.Al-Rafai,M.D.Rossetti,An efficient heuristic optimization algorithm for a two- echelon (R,Q) inventory system.Int.J.production Economics 109(2007)195-213.

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