New prime K-tuple theorem (6)

$$P, P+4^n (n=1,\cdots,k)$$

Chun-Xuan Jiang (蒋春暄)

Institute for Basic Research, Palm Harbor, FL34682-1577, USA
And: P. O. Box 3924, Beijing 100854, China (蒋春暄，北京 3924 信箱，100854)
jiangchunxuan@sohu.com, cxjiang@mail.bcf.net.cn, jcxuan@sina.com, Jiangchunxuan@vip.sohu.com, jcxxxx@163.com

Abstract: Using Jiang function we prove that for every positive integer k there exist infinitely many primes P such that each of $P+4^n$ is prime.

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Theorem

$$P, P+4^n (n=1,\cdots,k). \tag{1}$$

For every positive integer k there exist infinitely many primes P such that each of $P+4^n$ is prime.

Proof. We have Jiang function [1]

$$J_2(\omega)=\prod_P [P-1-\chi(P)], \tag{2}$$

where $\omega=\prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{n=1}^{k}[q+4^n]\equiv 0 \pmod P, \tag{3}$$

where $q=1,\cdots,P-1.$

From (3) we have

If $P<2k$ then $\chi(P)=\frac{P-1}{2}$, if $2k<P$ then $\chi(P)=k$.

From (3) and (2) we have

$$J_3(\omega)=\prod_{P=3}^{P<2k}\frac{P-1}{2}\prod_{2k<P}(P-1-k)\neq 0. \tag{4}$$

We prove that for every positive integer k there exist infinitely many primes p such that each of $P+4^n$ is prime.

We have the best asymptotic formula [1, 2]

$$\pi_{k+1}(N,2)=\left|\left\{P\le N: P+4^n=prime\right\}\right|\sim\frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)}\frac{N}{\log^{k+1}N}. \tag{5}$$

where $\phi(\omega)=\prod_P(P-1)$.

References

Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (http://www. wbabin. net/math/xuan2. pdf)

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